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Development of Methods for Analysis and Design of Prestressed Concrete Slabs

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Abstract

A new approach for analysis and design of pre-stressed concrete slabs is presented in this paper. Cable profile is modelled by B-spline to represent the cable shape. For finite element analysis of pre-stressed concrete slabs, tendon and concrete are modelled by 3 noded curved bar and 20 noded brick elements respectively. Using vector calculus formulae, the cable concrete interactions are precisely accounted. Further an efficient algorithm is proposed for cable layout design. Using the proposed techniques several prestressed concrete slabs are successfully analysed and designed.

Key-words: Pre-stressed concrete; B-spline; Cable layout design; Finite element analysis.

Introduction

Prestressed concrete is one of the most widely used construction material. In prestressed concrete, external pre-stressing force is applied on the concrete to reduce or eliminate the tensile stresses. Hence, cracking is prevented or reduced and pre-stressed concrete section becomes stiffer than reinforced concrete section. The downward deflection is reduced or eliminated due to upward force which is imposed on the concrete in curved tendons. Prestressed concrete structures has become extremely useful in the construction in liquid retaining structures and nuclear containment structures where absolutely no leakage is acceptable.

Analysis of three dimensional prestressed concrete structures is a complicated task due to complex nature of cable concrete interaction. The layout of the prestressing cable plays very important role in the stress distribution in prestressed concrete structures. The cable layout designs of prestressed concrete structures, as reported in text books, have been worked out on the basis of limiting eccentricities. In these texts, cables are modelled as parabola and their eccentricities are varied to reduce tensile stresses of the concrete. For realistic analysis of prestressed concrete structures, an advanced computing technique such as finite element

method is employed. Linear finite element analysis (FEA) of pre-stressed concrete structures has been reported by Pandey et al. [1], Buragohian et al [2,3], Pathak et al. [4]. Non linear analysis of the same are reported by Povoas et al. [5], Kang et al. [6], Roca et al. [7], Greunen et al. [8], Figueiras et al. [9], Vanzyl et al. [10] and Elwi et al. [11]. Jirousek et al. [12] and Buragohain et al. [2,3] have considered cable as parabolic and cubic curve in shell and semiloof shell elements, whereas Pandey et al. [1] considered the cable as parabola in 20 node brick element. Pathak et al. [4] considered the cable as cubic spline curve in nine node Lagrangean element. Saleem Akhtar et al. [13] modelled cable as B-spline for two dimensional finite element analysis.

In advanced attempts of cable layout design, Brandt et al. [14] and Kirsch et al. [15] [16] carried out cable layout optimization using mathematical programming methods. Utrilla and Smartin et al. [17], Quiroga et al. [18] obtained optimum cable layout in bridge decks using linear and non-linear programming respectively. Lounis et al. [19] carried out multi objective optimization of prestressed concrete beam and bridge girder using Lagrangean algorithm. Kuyucular et al. [20] obtained optimum cable profile of prestressed concrete slabs using elastic theory and finite element method. The cable profile was assumed to be the combination of parabola, third degree curve, fourth degree curve and combination of parabola and third degree curve having common tangent at the junctions. These studies suffer from large degree of complexity in data preparation as well as in computations. To overcome these limitations, in this study, a holistic approach has been proposed for analysis and design of prestressed concrete structures. It has following features-

- (a). Cable profile is modelled by B-spline which is very suitable for these applications.
- (b). Concrete and cable are modelled by twenty noded brick elements and three noded curved bar elements respectively.

In this paper, theoretical aspects of cable modelling, finite element formulation etc., used in the software development, are described. The proposed software is validated with analytical results of a simply supported beam and found to be in close match.

Cable Modelling

The cable profile plays very important role during analysis of prestressed concrete structure as pressure on concrete depends on the cable profile. To analyse prestressed concrete structures using analytical approach given in the text books (Raju, [21], Lin and Burns, [22], cable profile is modeled by parabola. This brings about constant curvature and simplifies the solution. But profile, thus modeled, becomes discontinuous at supports (Figure 1).

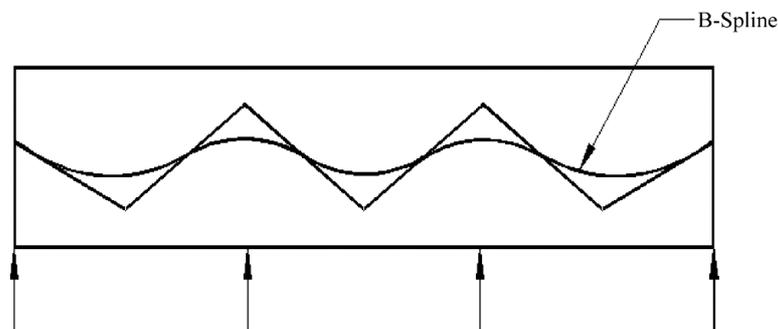


Figure 1 : Actual cable and parabolic profile

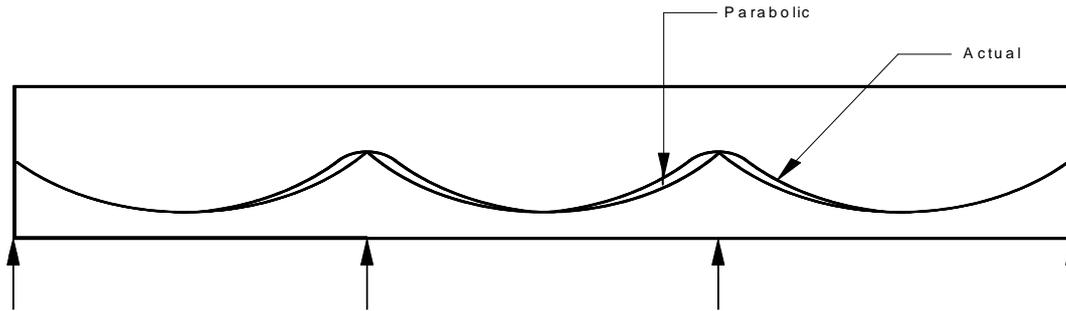


Figure 2: B-spline profile

If cable profile, to maintain continuity, is modeled by higher order polynomial, it results in a very zigzag shape. In order to overcome these difficulties, in this study, cable profile is modeled by B-spline. A B-spline is a typical curve of the CAD philosophy [Qing and Liu [23]; Rogers and Adams [24]]. It models a smooth curve between the given ordinates (Figure 2). The theory of the B-spline was first suggested by Schoenberg [25]. A recursive definition useful for numerical computation was independently discovered by Cox and by de Boor [Rogers and Adams [24]]. Gordon and Riesenfeld [26] applied the B-spline basis to curve definition. The brief definition of B-spline curve is given below and detailed account of this can be found in Rogers and Adams [24].

$$P(t) = \sum P_i \cdot N_{i,k}(t) \quad 0 < t < n - k + 2, 2 \leq k < n \quad \dots\dots\dots(1)$$

where,

$$N_{i,1} = \begin{cases} 1 & \text{if } x_i \leq t \leq x_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

and

$$N_{i,k}(t) = \frac{(t - x_i) N_{i,k-1}(t)}{(x_{i+k-1} - x_i)} + \frac{(x_{i+k} - t) N_{i+1,k-1}(t)}{(x_{i+k} - x_{i+1})} \quad \dots\dots\dots(2)$$

In above equations, P_i 's are the $n+1$ defining polygon vertices, k is the order of the B spline and $N_{i,k}(t)$ is called the weighing function. x is the additional knot vector which is used for B-spline curve to account for the inherent added flexibility. A knot vector is simply a series of real integers x_i such that $x_i \leq x_{i+1}$ for all x_i . They are used to indicate the parameter t used to generate a B-spline. The curve generally follows the shape of the defining polygon and the curve is transformed by transforming the defining polygonal vertices. The order of the resulting curve can be changed without changing the number of defining polygon vertices. When a B-spline curve is used, the geometrical regularity is automatically taken into account.

The friction between the cable and duct gradually reduces tension along the length of the cable. The bar element is assumed to have two Gauss points GP1 and GP2. In calculation of friction loss the radius of curvature R is used. Cable tension at different locations along cable profile can be obtained.

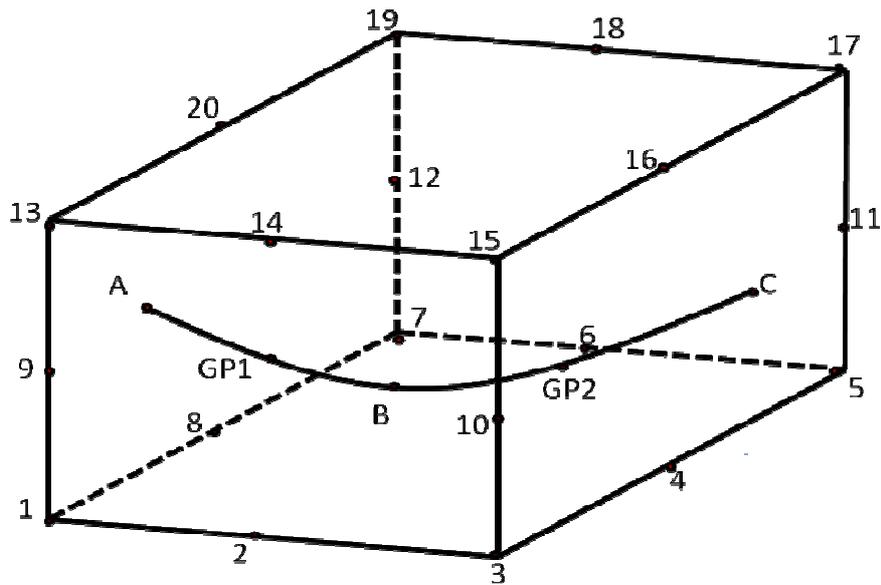


Figure 3: Brick and curved bar element

If radius of curvatures at GP1 and GP2 be R_1 and R_2 (Figure 3), cable tensions at B and C after friction loss will be –

$$T_B = T_A e^{(-\mu\alpha_1 - KL_{A-B})} \dots\dots\dots(3)$$

$$T_C = T_B e^{(-\mu\alpha_2 - KL_{B-C})}$$

where,

T_A = Tension at jacking end for first element

For subsequent elements T_A will become T_C of their previous elements.

$$\alpha_1 = L_{A-B}/R_1 \dots\dots\dots(4)$$

$$\alpha_2 = L_{B-C}/R_2$$

μ = coefficient of friction

K = wobble coefficient

The length of the cable between A-B and B-C are calculated as

$$\begin{aligned} L_{A-B} &= \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2} \\ L_{B-C} &= \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2 + (z_B - z_C)^2} \end{aligned} \dots\dots\dots(5)$$

Analysis and design methods

(a) Analysis of prestressed structure

Because of the convex hull properties of the B-spline, it is very suitable to represent cable profile in continuous span structures. Braibant and Fleury [27], Pourazady et al. [28] and Ghoddosian [29] have used this curve in shape optimization problems and cable geometry is being modeled as B-spline in this study. For finite element analysis, cable is modeled by three noded bar element and concrete by 20 noded brick element (Zienkiewicz and Taylor) [30]. In Figure 3, a 3 node curved bar element is shown which is embedded in

three dimensional 20 node concrete element. The cable exerts tangential and normal forces on concrete due to interactions between contacting surfaces and curvature of the cable as shown in Figure 4.

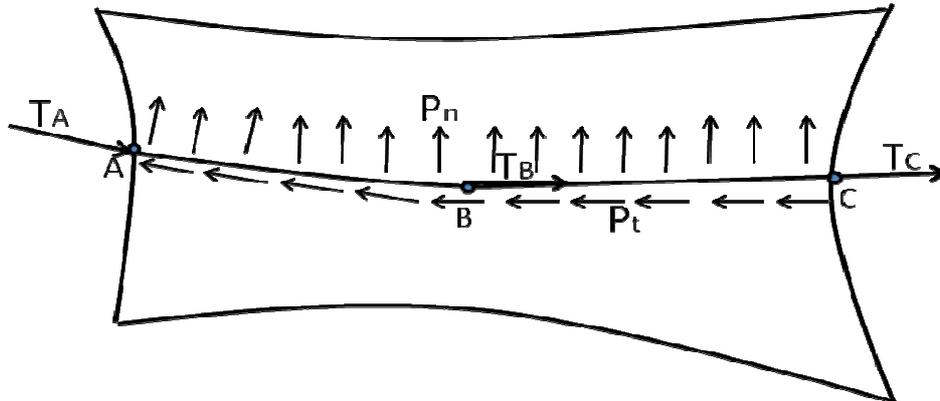


Figure 4: Forces on concrete due to cable tension

Tangential and normal forces can be given by -

$$P_t = \frac{dT_n}{dX} = \frac{1}{|T|} \frac{dT_n}{d\rho} \dots\dots\dots(6)$$

$$P_n = \frac{T_n}{R} \dots\dots\dots(7)$$

Now the resultant force can be computed by -

$$\bar{P} = P_t \bar{t} + P_n \bar{n} \dots\dots\dots(8)$$

Where \$T_n\$ is the tension in the cable and \$T\$ is the tangent vector. \$\bar{t}\$ and \$\bar{n}\$ are unit tangent and normal vector.

Using the principle of virtual work, these loads can be transferred to the nodes of brick elements. The equivalent nodal force vector for brick element is given by-

$$\{P_L\} = \int_{-1}^1 [N]^T \{\bar{P}\} |T| d\rho \dots\dots\dots(9)$$

Cable reaction acts as concentrated loads on the concrete at the ends, where cable is anchored. The anchorage end point forces can be calculated by-

$$\{P_A\} = [N]^T \{T_{end}\} \dots\dots\dots(10)$$

where, \$T_{end}\$ is the cable tension at the end points and \$[N]\$ is the shape functions of 20 node brick elements.

Now total load vector due to interaction of concrete and cable is obtained by-

$$\{P_T\} = \{P_L\} + \{P_A\} \dots\dots\dots(11)$$

This nodal load vector is applied on the three dimensional finite element model along with live and dead load vectors to include pre-stressing effects.

Local coordinates are required for known global coordinates for calculation of the anchorage end point forces as described below.

Let (x,y,z) is the global coordinate and (ξ,η,ζ) be the corresponding local co-ordinate. Numerical computations of local co-ordinates can be obtained using following iterative relationship-

$$\begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}_{i+1} = \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}_i + \begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \zeta} \end{pmatrix}^{-1} \begin{Bmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \\ z_{i+1} - z_i \end{Bmatrix} \dots\dots\dots(12)$$

In above equation inverse matrix is the jacobian matrix. $(x_{i+1}, y_{i+1}, z_{i+1})$ and (x_i, y_i, z_i) are the known and computed values of the global co-ordinates. Starting values of ξ, η, ζ are considered as 0, 0, 0. The prescribed tolerance is 0.01.

Incorporating above mentioned formulations, a FORTRAN codes named as PRECON3D have been developed. An automatic mesh generator has been developed to create finite element mesh. For this, coordinates of 20 seed points of the main block are required. For efficient debugging, flags have been used between major computational steps. The output file stores results like pre-stress cable profile, pre-stress loss along the cable, displacements, strain and stresses.

(b) Design of prestressed structure

An algorithm has been developed to find the cable layout so that stresses in the structural element be below the limiting tensile stress. Based on stress results obtained from the finite element analysis, cable profile is changed in iterative manner.

Criterion for cable layout design

The convex hull property of the B-spline ensures the curve to be convex or concave depending upon the ordinates of the polygon. The objective of the prestressed concrete design is to get rid of the tensile stresses from the concrete produced due to different loading conditions. In Figure 5, a prestressed concrete beam is shown.

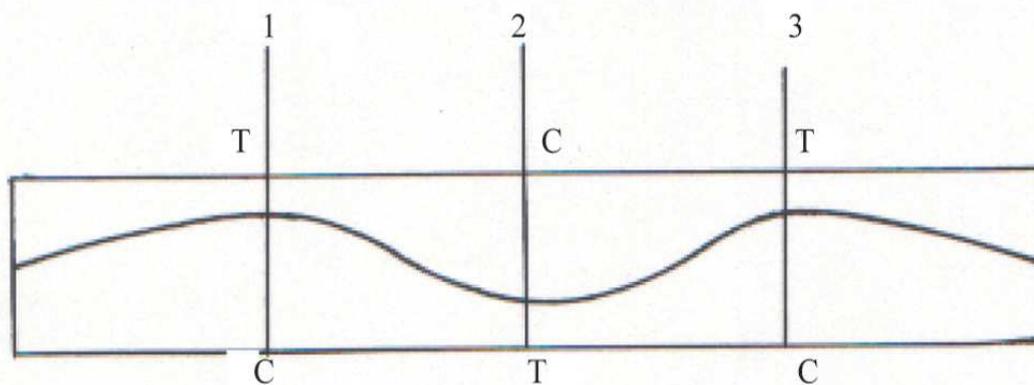


Figure 5: Required cable profile

It is assumed that top fiber at sections 1 and 3 are in tension and bottom fiber at section 2 is in tension. The shape of the cable to eliminate the tension should be as shown in the same figure. The shape of the cable can be accurately represented by a B-spline (Figure 2). By varying the ordinates of the B-spline, cable shape is changed to get the desired profile. In this way any 3D prestressed concrete structures can be handled to obtain the optimum cable profile.

Algorithm for layout design

In Figure 6 a concrete structure with typical loading is selected for cable layout design. Assume the cable to be straight between anchorage ends and initial prestressing force be P . Now finite element analysis of the concrete structure is carried out for external loads and initial prestressing force P acting at the ends.

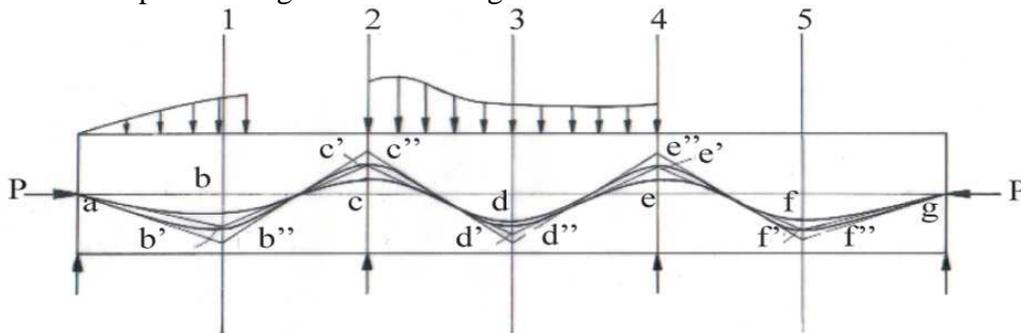


Figure 6: Variation of cable profile

Let sections 1,2,3,4 and 5 be such that bottom fibre of 1, 3 and 5 are in tension and top fibre of 2 and 4 are in tension. Assume top and bottom stresses at these sections be σ_{it} and σ_{ib} , where i varies from 1 to 5.

Since stresses at bottom fiber at 1,3 and 5 are in tension, the cable should be concave there, whereas at 2 and 4 it should be convex. The ordinates of B-spline will move downward at 1,3 and 5 and will move upward at 2 and 4. Let the initial y-coordinates of these points be y_1, y_2, y_3, y_4 and y_5 . Depending upon top and bottom fibre stresses at a point, following four cases may be detected-

- (a). Top fibre is in tension and bottom fibre is in compression.
- (b). Bottom fibre is in tension and top fibre is in compression.
- (c). Both fibres are in tension.
- (d). Both fibres are in compression.

(a) Top fibre is in tension and bottom fibre is in compression

Let σ_{it} and σ_{ib} be top and bottom stresses, then a ratio R is defined as

$$R = \frac{\sigma_{it}}{\sigma_{it} + abs(\sigma_{ib})} \dots\dots\dots(13)$$

where abs is the absolute value. New y-coordinates of the B-spline ordinates are calculated as-

$$y_{i+1} = (1 + R) y_i \dots\dots\dots(14)$$

here y_i represents the y-coordinate of the previous iteration.

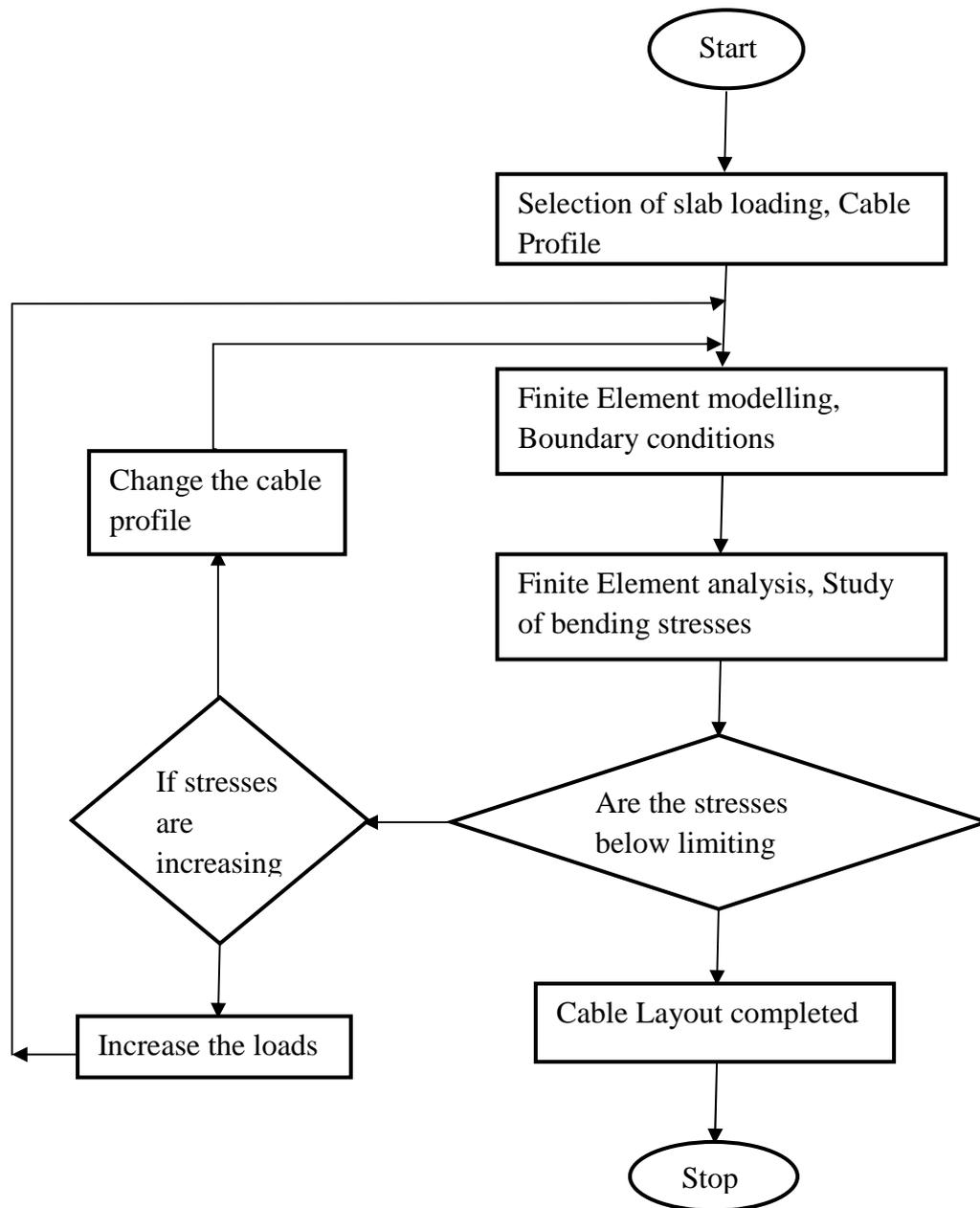


Figure 7: Flowchart for cable layout design

(b) Bottom fibre is in tension and top fibre is in compression

Like previous case the stress ratio is defined in the following way-

$$R = \frac{\sigma_{ib}}{\sigma_{ib} + abs(\sigma_{it})} \dots\dots\dots(15)$$

once again *abs* represents the absolute value. New y-coordinates of the B-spline ordinates are calculated as:

$$y_{i+1} = (1 - R) y_i \dots\dots\dots(16)$$

A check on the compressive stresses at the top fibre will be made for it not to cross the limiting value.

(c) Both fibres are in tension

If top and bottom both fibres are in tension, then either the cable force should be increased or the section should be redesigned.

(d) Both fibres are in compression

Check the compressive stresses at the top and bottom fibres for them not to cross the limiting value. If they are below, cable profile is not altered at that section otherwise redesign the section.

These steps are repeated till the cable profile for limiting tensile stresses is obtained. Based on this procedure, ordinate movements of B-spline are shown in Figure 6. The flowchart of the proposed algorithm of cable layout design is shown in Figure 7. This algorithm is coded in FORTRAN software named as PRECLAD3D which is used for layout design given below.

4 Numerical Examples

(i) Simply Supported Beam

A simply supported prestressed concrete beam of dimension 8000x800x300mm is analysed using PRECON3D software. The Pre-stressing force of 200000 N is applied on the beam and the cable eccentricity at the mid span is 300 mm. The beam is discretised into 10 twenty node brick elements and there are 128 nodes in FE model. The loading conditions and finite element mesh are shown in Figure 8.

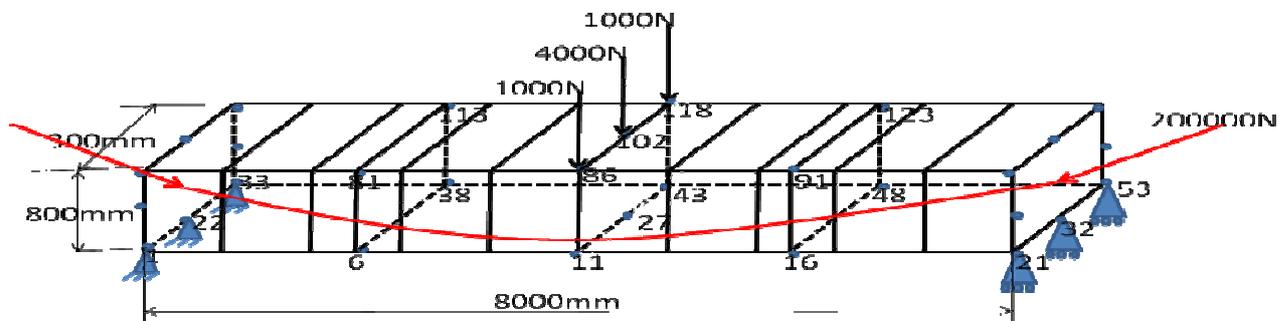


Figure 8: Simply supported beam

Following data is accounted for the analysis purpose:

- (a) Young's modulus of concrete = $2 \times 10^4 \text{ N/mm}^2$
- (b) Coefficient of friction = 0.20
- (c) Poisson's ratio = 0.20
- (d) Density of concrete = 2500 Kg/m^3

Cable profile and eccentricity are given in Table 1. The bending stresses and displacements for simply supported beam obtained from the software are given in Table 2.

Table 1: Cable layout and eccentricity (mm) in simply supported beam

Distance from left end (mm)	Poygon coordinates(mm)	B-spline coordinates(mm)	Eccentricity (mm)
0	400	400	0
4000	-200	100	300
8000	400	400	0

Table 2: Bending stresses (MPa) and displacement (mm) for simply supported beam

Distance from leftend (mm)	Coordinates (mm)	Stresses (N/mm ²)	Displacement (mm)
2000	2000,0,0	-2.00	0.94
	2000,300,0	-2.00	0.94
	2000,0,800	+0.39	0.94
	2000,300,800	+0.39	0.94
4000	4000,0,0	-2.31	1.31
	4000,150,0	-2.31	1.31
	4000,300,0	-2.31	1.31
	4000,0,800	+0.66	1.31
	4000,150,800	+0.66	1.31
	4000,300,800	+0.66	1.31
6000	6000,0,0	-2.00	0.94
	6000,300,0	-2.00	0.94
	6000,0,800	+0.39	0.94
	6000,300,800	+0.39	0.94

The stresses are compared with the same obtained using analytical method as using formulae. Top fibre stresses at mid section obtained from software and analytical approaches are 0.66 and 0.67 MPa. The same for bottom fibre stresses are -2.31 and -2.33 MPa. It can be observed that both are in close match.

Stresses as per formulae

Given: Concentrated load $P=6000$ N

Span $L= 8000$ mm

Pre-stress force $P=200$ KN

Cross sectional area $A(b \times h) = 300 \times 800$ mm

and eccentricity at mid section $e = 300$ mm

Top and bottom stresses are-

$$f_{\text{top}} = - \frac{3WL}{2bh^2} - \frac{P}{A} + \frac{6Pe}{bh^2} = 0.67 \text{ N/mm}^2$$

$$f_{\text{bottom}} = + \frac{3WL}{2bh^2} - \frac{P}{A} - \frac{6Pe}{bh^2} = -2.33 \text{ N/mm}^2$$

(ii) Two way prestressed concrete slab

A two way prestressed concrete slab of dimension 4200x4200x300 mm is considered for cable layout design. Fourteen prestressing cable, 7 each in length and width directions are used. The slab is discretised into 49 twenty noded brick elements and 416 nodes. The loading conditions and three dimensional finite element model are shown in Figure 9. Four concentrated loads, PL, of value 270000 N are applied on the slab.

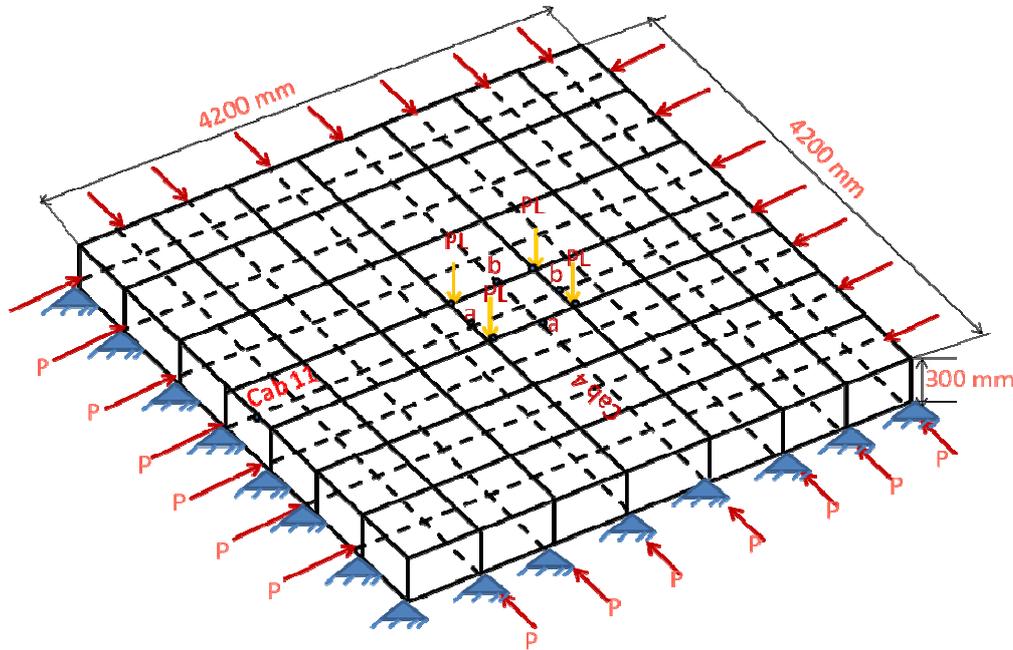


Figure 9: Two way prestressed concrete slab

Following material properties are adopted:

- (a). Young's modulus (concrete) = 2×10^4 N/mm²
- (b). Compressive strength of concrete = 40 N/mm²
- (c). Poisson's ratio = 0.15
- (d). Wobble coefficient = 1×10^{-5} /mm
- (e). Tensile strength of concrete = 0.25 N/mm²
- (f). Density of concrete = 2500 Kg/m³
- (g). Coefficient of friction = 0.20

The cable layout design is taken up for two conditions:

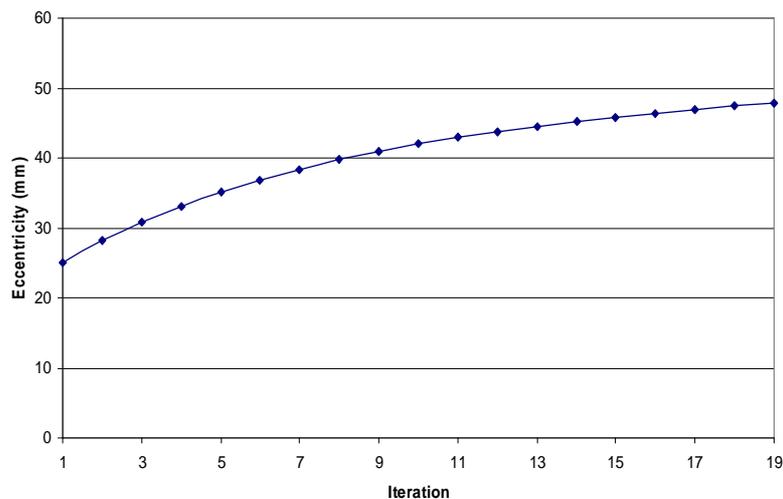
(a). Constant Prestressing Force

A constant prestressing force of 2000000 N is applied on all the fourteen cables. Initially eccentricities of all the cables are kept 25 mm. Maximum compressive and tensile stresses are noted during the design process. It was observed that maximum tensile stresses occur at four locations a, b, c and d which lie on cable 4 and 11. Hence layout designs of these two cables were taken up.

Table 3: Bending stresses (MPa) and eccentricity (mm) for constant prestressing force

Iteration No.	Prestrss Load (KN)	Maximum Bending Stresses (MPa)		Eccentricity (mm)
		Compressive	Tensile	
1.	2000	-17.05	1.15	25.00
2.	2000	-16.91	1.02	28.18
3.	2000	-16.80	0.92	30.86
4.	2000	-16.70	0.82	33.17
5.	2000	-16.62	0.75	35.15
6.	2000	-16.54	0.68	36.88
7.	2000	-16.48	0.62	38.40
8.	2000	-16.42	0.56	39.74
9.	2000	-16.37	0.52	40.91
10.	2000	-16.33	0.48	41.97
11.	2000	--16.29	0.44	42.92
12.	2000	-16.25	0.40	43.77
13.	2000	-16.22	0.37	44.52
14.	2000	--16.19	0.35	45.21
15.	2000	-16.16	0.32	45.84
16.	2000	-16.14	0.30	46.41
17.	2000	-16.12	0.28	46.94
18.	2000	-16.10	0.26	47.43
19.	2000	-16.08	0.247	47.87

It took 19 iterations to converge to maximum tensile stress of 0.247 MPa. Due to friction, cable prestress force got reduced from 2000000 N to 1899567 N. Maximum compressive and tensile stresses along with eccentricities are given in Table 3. Variation of eccentricity with respect to iteration is shown in Figure 10. The variation of maximum tensile stresses with respect to iteration is shown in Figure 11 which shows non-linear trend.

**Figure 10: Eccentricity Vs Iteration**

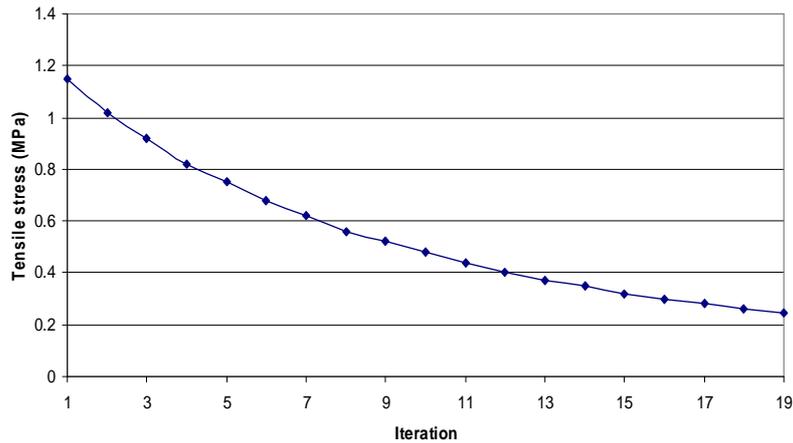


Figure 11: Tensile stresses Vs Iterations

(b). Variable Prestressing Force

In second case of layout design, eccentricities of all the cables are kept constant 25 mm. Cable profile and eccentricity are given in Table 4. Based on the stress analysis results, cable forces are varied. The initial value is kept 2000000 N which is varied during the design process. It is observed that at cable force of 2180000 N maximum tensile stresses are below the limit. The details of the tensile stresses and compressive stresses are given in Table 5.

Table 4: Cable Layout and Cable eccentricity (mm) at different sections

Cable No			Cable 4			Cable 11			Eccentricity (mm)
Distance from end (mm)			0	2100	4200	0	2100	4200	
It No	Load (KN)	Ordinate							
1.	P	Polygon	150	100	150	150	100	150	25
		B-spline	150	125	150	150	125	150	

Table 5: Bending stresses (MPa) for variable prestressing load

Iteration No.	Prestress Load (KN)	Maximum Bending Stresses (MPa)		Final Prestressed Load (N)
		Compressive	Tensile	
1.	2000	-17.05	1.15	1899567
2.	2010	-17.07	1.10	1909065
3.	2020	-17.10	1.05	1918563
4.	2030	-17.12	0.99	1928061
5.	2040	-17.15	0.94	1937559
6.	2050	-17.17	0.89	1947057
7.	2060	-17.20	0.84	1956554

8.	2070	-17.22	0.79	1966052
9.	2080	-17.25	0.74	1975550
10.	2090	-17.27	0.68	1985048
11.	2100	-17.30	0.63	1994546
12.	2110	-17.32	0.58	2004044
13.	2120	-17.35	0.53	2013545
14.	2130	-17.37	0.48	2023039
15.	2140	-17.40	0.43	2032537
16.	2150	-17.42	0.37	2042035
17.	2160	-17.44	0.32	2051533
18.	2170	-17.47	0.27	2061031
19.	2175	-17.48	0.25	2065780
20.	2180	-17.49	0.22	2070529

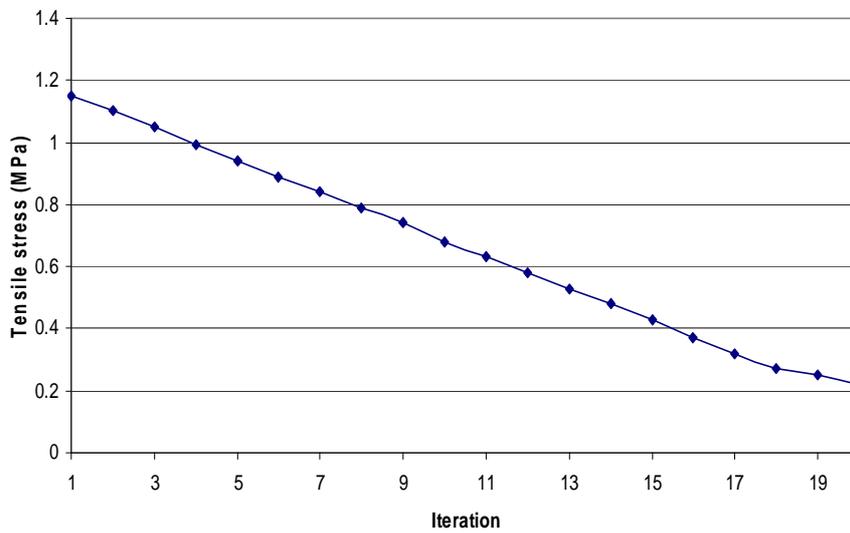


Figure 12: Tensile stresses Vs Iterations

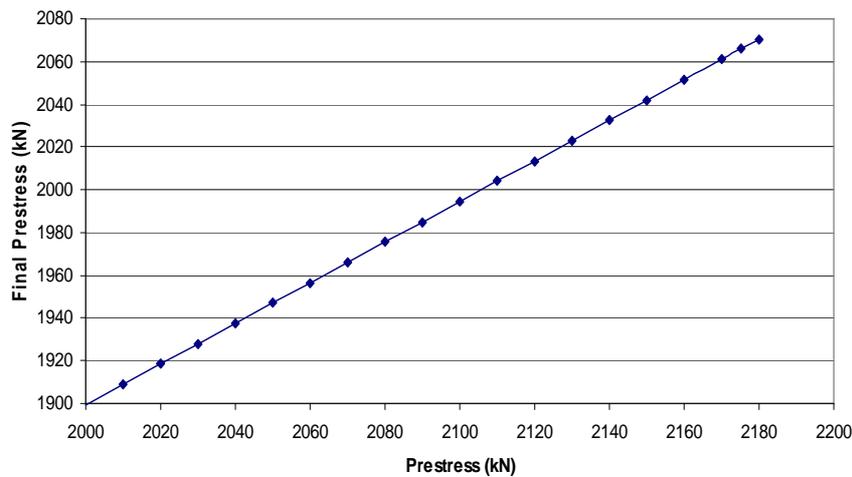


Figure 13: Prestress Load Vs Final Prestress Load

It took 20 iterations to bring the tensile stresses below 0.22 MPa. It is observed that due to friction cable prestress force is reduced at the anchorage end and final prestressing force at the anchorage end obtained from software is given in Table 5. The variation of tensile stresses with respect to iteration is shown in Fig.12 which is almost linear nature. The variation of final prestresses load with respect to prestress load is shown in Fig. 13 which is almost in linear nature and proves the correctness of the software.

Conclusion

In this paper computational methods for analysis and design of prestressed concrete structures are presented. The cable segment in the finite element is modelled by three node bar elements, which is embedded in three dimensional brick elements. The pre-stressing cables are modelled as B-spline curve whose ordinates are taken as design variables. Stress analysis is carried out using 3D finite element analysis. This approach is coded in 3D FE software. Using the proposed methodology, a two way prestressed concrete slab has been successfully designed to satisfy several design criteria. It can be observed that proposed approaches offers a holistic tool for realistic analysis and design of prestressed concrete structures.

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