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Numerical modelling for the modified Power-law fluid in stenotic capillary-tissue diffusion phenomena

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Abstract

The present study focuses on the behavior of diffusion phenomenon in stenosed capillary-tissue exchange system where the rheology of flowing blood in the capillary is characterized by the generalized Power-law fluid model. Assessment of the severity of the disease could be made possible through the variation of a parameter named as retention parameter. The concentration profile and associated physiological diffusion variable involved in the study for diseased state have been analyzed. The model is also employed to study the effect of shape of stenosis on flow characteristics. An extensive quantitative analysis is performed through numerical computations of the desired quantities having physiological relevance through their graphical representations so as to validate the applicability of the present model.
Keywords: Powe-law fluid, Capillary-tissue exchange, Diffusion, Resistance to flow, Stenosis.

Introduction

Diseases of the heart and circulatory system are still a major cause of death in the world. Blood flow characteristics in arteries can be altered significantly by arterial disease, such as stenosis and aneurysm [1, 2]. The actual reason for formation of stenosis is not known, but its effect over the flow characteristics has been studied by many research workers, [3, 4, 5]. Several attempts have been made in the literature to study the effect of stenosis on the blood flow characteristics, including the important contribution of [6, 7, 8]. In an arterial constriction blood viscosity increases due to the conservation of mass, producing increased wall shear stress in the region of the blood acceleration. Flow and diffusion through capillary-tissue exchange system has been identified as one of the thrust areas of research. In narrow capillaries, at times, the arterial transport become much larger as compared to axial transport and it contributes to the development of atherosclerotic plaques, greatly reducing the capillary diameter. The problem of flow and diffusion become much more difficult through a capillary with stenosis at some region. The response of blood flow through an artery under stenotic conditions has been attempted by [7,

8]. Accordingly, considerable effort has been expended studying the fluid mechanics of flow through a stenosis [9, 10, 11]. Several workers [12, 13, 14] proposed various representative models for blood in narrow capillaries. Viscosity depending on the local variation of the concentration of the suspended cells has been introduced by [15, 16]. Perkkio and Keskinnen [17] studied the effect of concentration on viscosity and the effect of the concentration on blood flow through a vessel with stenosis and found it an important aspect from physiological point of view. Kang and Erigen [18] have also discussed the effect of the variation of concentration of the suspended cells of blood.

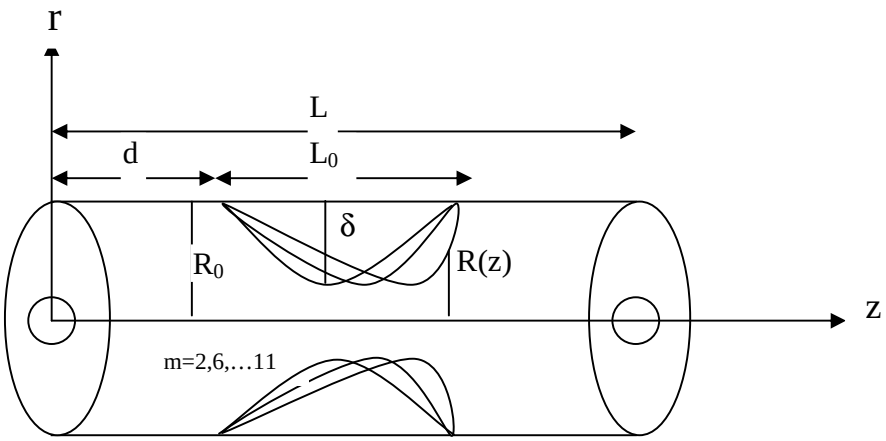

Fig (1) Geometry of Stenosis

Power-law fluid

Non-Newtonian fluid is that of power-law fluid which have constitutive equation,

$$\left.\begin{aligned} \left(-\frac{du}{dr}\right) &= \left(\frac{\tau}{\mu}\right)^{1/n} = f(\tau), \\ \text{where } \tau &= \left(-\frac{dp}{dz}\right)\frac{R_c}{2} \end{aligned}\right\} \qquad (1)$$

μ is the viscosity of fluid, (-dp/dz) is the pressure gradient and n is the flow behaviour index of the fluid, τ is shear stress, R_c is the radius of the plug-flow region, u is the axial velocity along the z direction. The relations correspond to the vanishing of the velocity gradients in regions.

In the present analysis, it is assumed that the stenosis develops in the arterial wall in an axially non-symmetric but radially symmetric manner and depends upon the axial distance z and the height of its growth. The geometry of the stenosis, which is assumed to be manifested in the capillary segment, is described as [Fig (1)];

$$\left.\begin{aligned} \frac{R(z)}{R_0} &= 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], && d \le z \le d + L_0 \\ &= 1, && \text{otherwise,} \end{aligned}\right\} \qquad (2)$$

Where the parameter $A = \frac{\delta}{R_0 L_0^{\,m}} \frac{m^{m/(m-1)}}{(m-1)}$

where δ denotes the maximum height of stenosis at $z = (d + L_0 / m^{m/(m-1)})$. R(z) and R_0 are the radius of the capillary with and without stenosis respectively. L_0 is the stenosis length, d represents the location of stenosis and m is stenosis shape parameter.

Formulation of the problem

Considering the axisymmetric laminar steady flow of blood, the general constitutive equation in case of stenosis subject to the additional conditions:

$$\begin{aligned} 0 &= -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial (rz)}{\partial r} \\ 0 &= -\frac{\partial P}{\partial r} \end{aligned} \qquad (3)$$

where p is the fluid pressure, $(-\partial p / \partial z)$ is pressure gradient in artery, where (z, r) are (axial, radial) co-ordinates with z measured along the axis and r measured normal to the axis.

The concentration equation for the solute is expressed by

$$u\frac{\partial C}{\partial z} = D\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) \qquad (4)$$

Where C represents the concentration of the solute, u is the axial velocity and D the diffusion coefficient for the solute under consideration in the blood.

Boundary conditions

To solve the above system of equations, the following boundary conditions are introduced:

$$\begin{aligned} &\frac{\partial u}{\partial r} = 0 \quad \text{at } r = 0, \quad u = 0 \quad \text{at } r = R(z) \\ &P = P_0 \quad \text{at } z = 0, \quad P = P_L \quad \text{at } z = L \\ &\frac{\partial C}{\partial r} = 0 \quad \text{at } r = 0, \quad D\frac{\partial C}{\partial r} = VNC \quad \text{at } r = R. \end{aligned} \qquad (5)$$

Where N is retention parameter, C is concentration; u is the axial velocity and D the diffusion coefficient.

Solution of the problem

Solving for u from equation (1), (3) and using the boundary conditions (5), we have,

$$\frac{du}{dr} = \left(\frac{P}{2\mu}\right)^{1/n} [(r - R_C)^{1/n}], \tag{6}$$

The volumetric flow rate Q can be defined as,

$$Q = \int_0^R 2\pi u r\, dr = \pi \int_0^R r\left(-\frac{du}{dr}\right) dr, \tag{7}$$

By the help of equations (6) and (7) obtains,

$$Q = \left(\frac{P}{2\mu}\right)^{1/n} \left(\frac{n\pi}{(3n+1)}\right) (R)^{[(1/n)+1]} \tag{8}$$

From equation (8) pressure gradient is written as follows,

$$\frac{dp}{dz} = -2\mu \left(\frac{(3n+1)}{n\pi} Q\right)^n \frac{1}{(R)^{3n+1}} \tag{9}$$

Integrating equation (9) using the condition $P = P_0$ at $z = 0$ and $P = P_L$ at $z = L$. We have,

$$P_L - P_0 = \left(\frac{(3n+1)}{n\pi} Q\right)^n \frac{2\mu}{(R_0)^{3n+1}} \int_0^L \frac{dz}{\left(R/R_0\right)^{1+3n}} \tag{10}$$

The resistance to flow is denoted by λ and defined as follows:

$$\lambda = \frac{P_L - P_0}{Q} \tag{11}$$

The resistance to flow from equation (10) using equations (11) can write as:

$$\lambda_0 = \left(\frac{(3n+1)Q}{n\pi}\right)^n \frac{2\mu}{Q R_0^{3n+1}} \left(\int_0^d dz + \int_0^{d+L_0} \frac{dz}{\left(R/R_0\right)^{3n+1}} + \int_{d+L_0}^L dz \right) \tag{12}$$

When there is no stenosis in artery then $R = R_0$, the resistance to flow,

$$\lambda_N = \left(\frac{(3n+1)}{n\pi} Q\right)^n \frac{2\mu}{Q R_0^{3n+1}} L \tag{13}$$

From equation (12) and (13) the ratio of (λ_0 / λ_N) is given as;

$$\lambda = \frac{\lambda_0}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{1}{L} \int_d^{d+L_0} \frac{dz}{(R/R_0)^{3n+1}} \tag{14}$$

Now the ratio of shearing stress at the wall can be written as;

$$\frac{\tau_R}{\tau_N} = \left(\frac{R_0}{R}\right)^{-3n}$$

$$\tau = \frac{\tau_R}{\tau_N} = \frac{1}{\left(1-\frac{\delta}{R_0}\right)^{3n}} \tag{15}$$

To solve the Eq. (4) takes the form:

$$\frac{\nu R_0^2}{D_1 L}\frac{\partial C_1}{\partial x} = \frac{\partial^2 C_1}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial C_1}{\partial \eta} \tag{16}$$

The boundary conditions are:

$$\frac{\partial C_1}{\partial \eta} = 0 \qquad \text{at} \quad \eta = 0,$$

$$D_1 \frac{\partial C_1}{\partial \eta} = VNC_1 \qquad \text{at} \quad \eta = \frac{R}{R_0} \tag{17}$$

$$\bar{u} = \frac{R_0^2}{4\mu L}\left(\frac{dp}{dx}\right)\left[2R'^n\eta - \left(\frac{n+1}{n}\right)\eta_c^n R_c^{1/n} + \frac{2n+3}{n}\eta_c^2\right] \tag{18}$$

On using Eq. (16) the solution for concentration subject to the boundary conditions (17) is given as:

$$C_1 = \frac{R_0^{\,n}}{2\mu L^2 D_1}\left(-\frac{dp}{dx}\right)\left(\frac{\partial C_1}{\partial x}\right)\left[-\left(\frac{\eta^4(2n+1)}{(1+1/n)}\left(\frac{R^{5n/3+1}\eta^2}{n} - \frac{54\eta^{n+5/2}}{(2n+1)}\right)\right)\right] - \left(\frac{\partial C_1}{\partial z}\right) + M \tag{19}$$

Where

$$M = \left[\frac{R_0^4}{4\mu L^2}\left(-\frac{dp}{dz}\right)\left\{\frac{R'^{3n+1}}{4}\left(1-\frac{VNR_0R'}{D_1(2n+1)}\right)^n - \left(\frac{(5n+8)}{\eta_c} - \frac{VNR_0R'}{D_1}\right)\right\}\right.$$
$$\left. + \frac{\eta_c}{R'^{3n+1}}\left(1+\frac{V^{(2n+1)}R_0}{D_1}\right) - \frac{R'R_0^2}{L}\bar{u}\left(1-\frac{V^nNR_0R'}{D_1}\right)\right\}\Bigg]$$

Results and Discussion

In order to have estimate of the quantitative effects of various parameters involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, concentration profile and associated physiological diffusion variables for normal and diseased system associated with stenosis due to the local deposition of lipids have been determine. The results are shown in Fig 2-7 by using the values of parameter based on experimental data in capillary.

Fig.2 and Fig.3 shows the results for resistance to flow for different values of stenosis size, stenosis shape parameter and flow index behaviour. Resistance to flow decreases as stenosis shape parameter m increases and increases as stenosis size increases. Resistance to flow increase as stenosis grows or radius of artery decreases. This referred to as Fahraeus-Lindquist effect in very thin tubes. The present results are therefore consistent with the observation of Haldar [5, 4, 15].

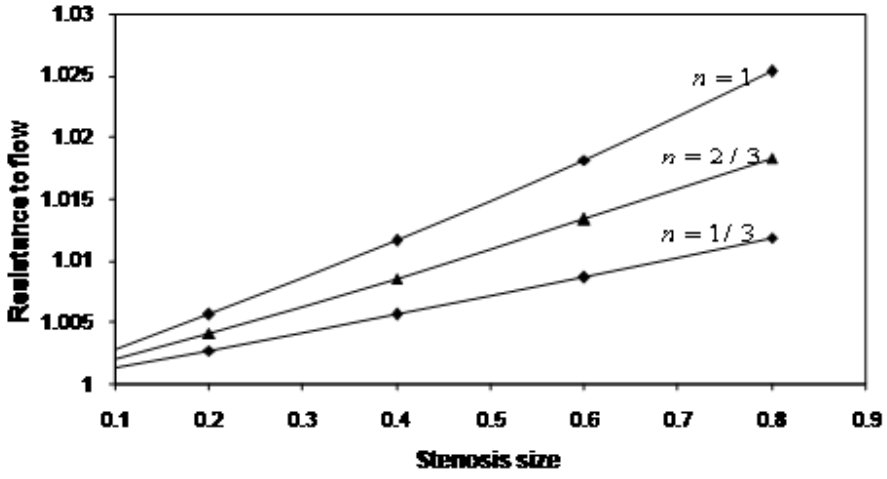

Fig.2 Variation of resistance to flow with stenosis size for different values of n

Fig.4 and Fig.5 shows the results for wall shear stress for different values of stenosis shape parameter, stenosis length, flow index behaviour and stenosis size. Wall shear stress increases as stenosis size, stenosis length increases and decreases as stenosis shape parameter increases and results are compared with [15, 17]. It is clear that the resistance to flow and wall shear stress increases as stenosis grows. But the same is not true in the absence of stenosis. In capillary flow, the viscosity of blood flow found to vary with the radius of the capillary. The development of stenosis accelerates the velocity of plasma between the cells. This in turn increases the concentration of red cell and viscosity of blood in stenotic region, therefore increases.

Fig.6 shows the diffution of large and small molecular weight nutrients within the capillary region for different values of stenosis size. Large molecular weight nutrients within the capillary region face more resistance to diffuse into the tissue and therefore the cells of the the deeper region are deprived of getting sufficient nutrition. This result is consistent with result of Tandon et al. [15]. Fig.7 represents the effects of retention parameter (N) on concentration in blood flow capillary region. Increasing values of retention parameter described the increase in retention of solute within the blood flow in the capillary region. The value of retention parameter (N=1) implies the complete retention. No solute or fluid diffuses and as retention parameter decreases from 1 to 0.4 more solute diffuses, which in turns, decreases the solute concentration in the capillary region. The variation of the values of retention parameter in the stenotic region may also be associated with the type of plaques deposited on the walls: calcified, fibrous or fatty plaque.

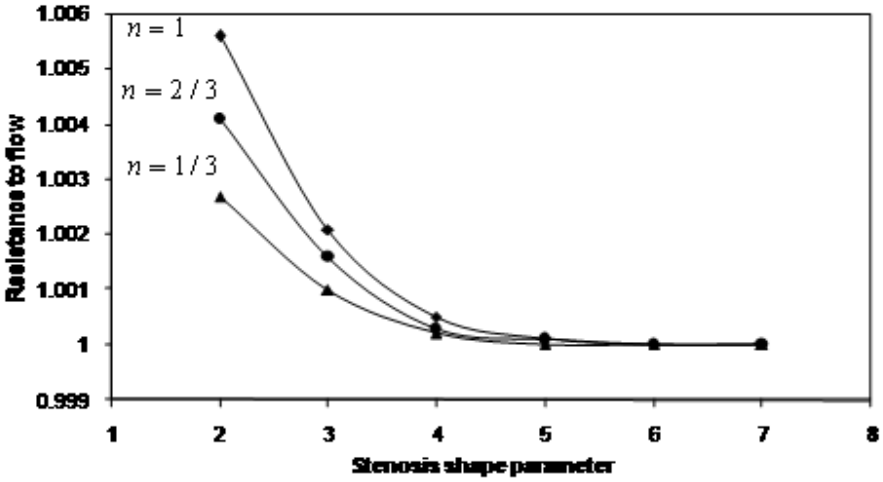

Fig.3 Variation of resistance to flow with stenosis shape parameter for different values of n

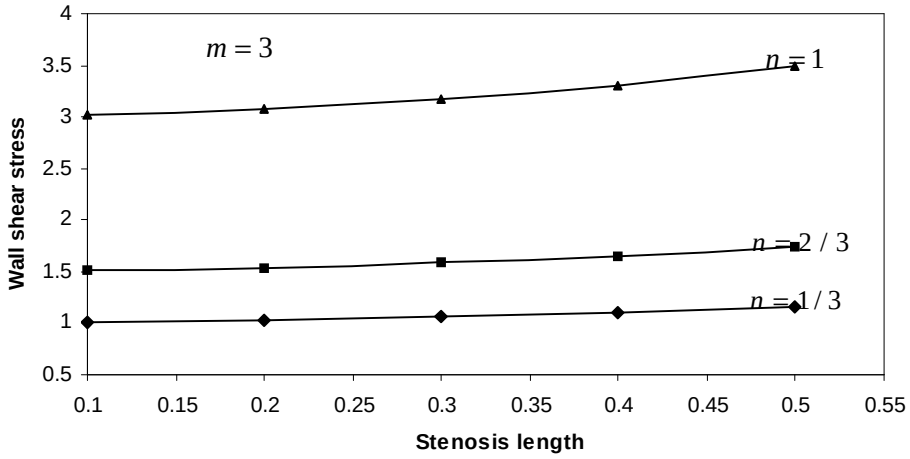

Fig.4 Variation of wall shear stress with stenosis length for different n

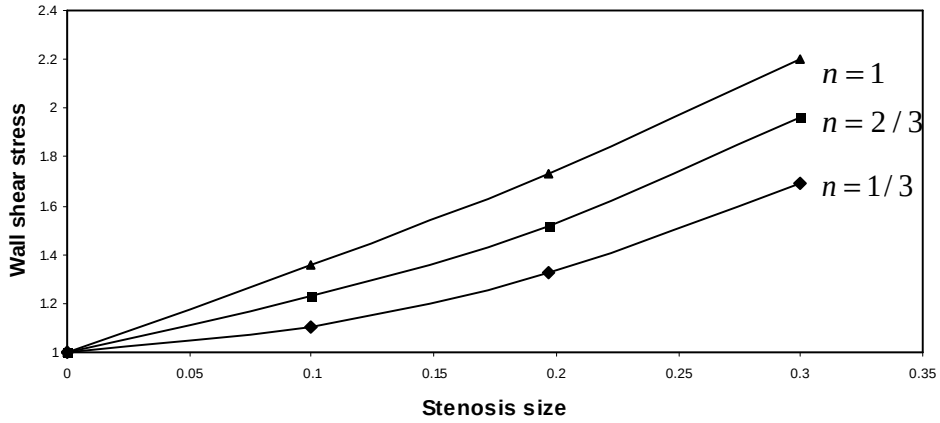

Fig.5 Variation of wall shear stress with stenosis size for different n

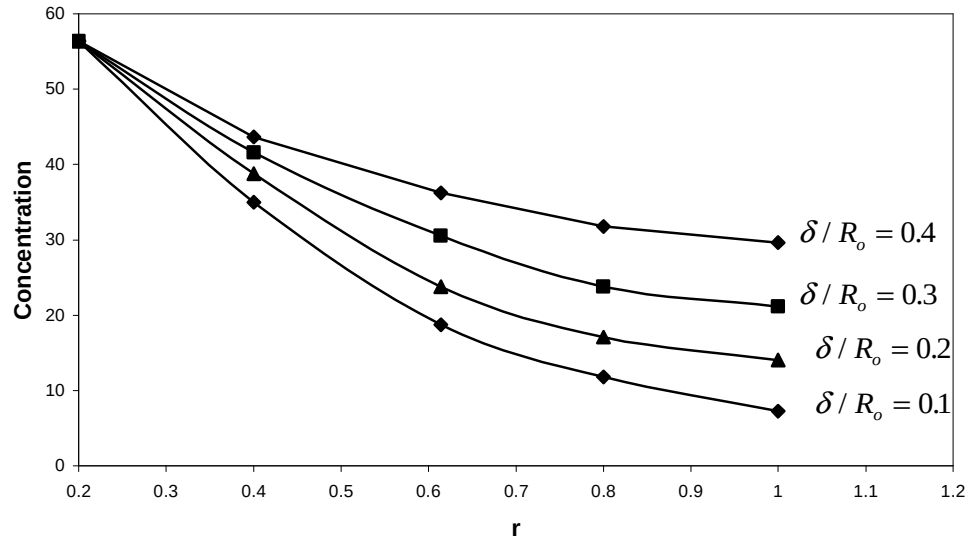

Fig.6 Concentration profile for diffrent values of stenosis size

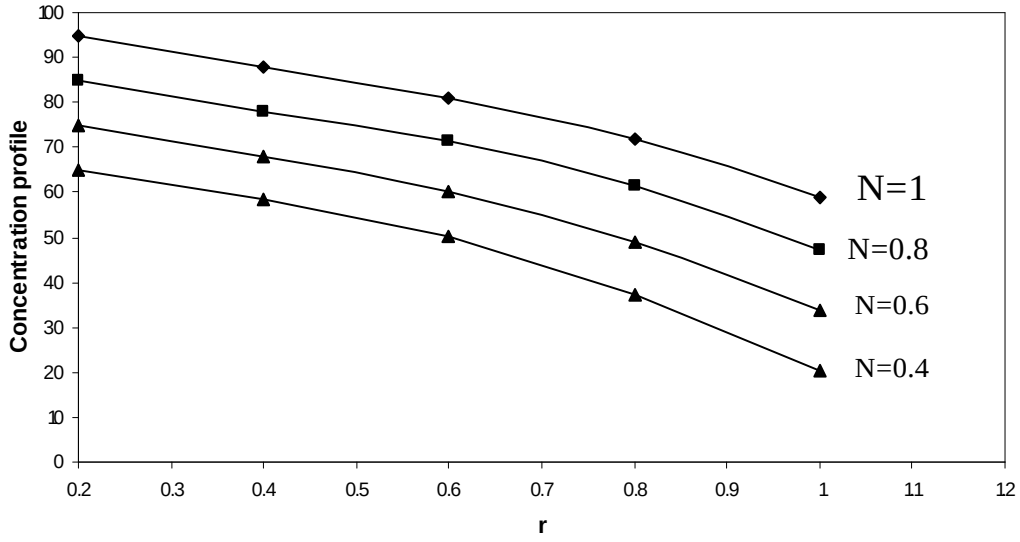

Fig.7. Concentration profile for different values of retention parameter (N)

Concluding remarks

The present study incorporates the more realistic representation for blood in small diameter blood vessels and simultaneous dispersion of solute in capillary in normal and stenotic depending on various parameters including retention parameter. Power-law fluid model appears to be realistic in the sense that the equations are fairly closely to the blood flow and the central core region is easily represented and one more parameter index behavior (non-Newtonian nature of this fluid) is given in the model. The results are more encouraging and correlating well with the experimental observation that deeper region cells are deprived of the nutrients in the stenotic region.

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