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On transitive and primitive dihedral groups of degree 2^r ($r \geq 2$)

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ABSTRACT

In this paper, we discussed transitivity and primitivity of all dihedral groups of degree 2^r ($r \geq 2$). The method adopted uses the concepts of group theory and groups algorithms and programming (GAP) to obtain our results.

INTRODUCTION

In mathematics, a dihedral group is the group of symmetries of a regular polygon, including both rotations and reflections. Dihedral groups are good examples of finite groups and have series of applications especially in sciences and engineering.

There are two competing notations for dihedral group associated to a polygon with n sides or degree. In geometry the group is denoted by D_{2n} to indicate the number of elements. In this article, D_n refers to the symmetry of a regular polygon with n sides or degree.

Conventionally, we write

$$D_{n:=} \langle x,y | x^n=y^2=1, yx=x^{n-1}y=x^{-1}y \rangle$$

And we say that D_n is the group generated by the elements x,y subject to the conditions $x^n=y^2=1; yx=x^{n-1}y=x^{-1}y$, and the 2n distinct elements of D_n are $1,x,x^2,\ldots,x^{n-1},y,xy,x^2y,\ldots,x^{n-1}y$. Here x is a rotation about the centre of the polygon through angle $2\pi^c/n$ and y is a reflection about an axis of symmetry of the polygon.

In Section 2 we give some basic concepts and results which are required here. In Section 3 we applied groups, algorithms and programming (**GAP**) to discuss transitivity and primitivity on some dihedral groups of degree 2^r ($r \geq 2$) . The main result of this paper covering all the dihedral groups of degree 2^r ($r \geq 2$) are stated in Section 4.

Preliminaries

Throughout this paper, Ω is a finite set with n elements and G is a permutation group on Ω. Then $G \leq Sym(\Omega)$ – the symmetric group on Ω.

For a point α of Ω, we define αG (where G consists of mappings of Ω) by $\alpha G := \{\alpha g \mid g \in G\}$and this is called the G-orbit that contains α.

Definition 2.1

The set of elements in G which fix a specified point α is called the stabilizer of α in G and is denoted by G_α. Thus

$$G_\alpha := \{g \in G \mid \alpha^g = \alpha\}$$

A group G acting on a set Ω is said to be transitive on Ω if it has only one orbit, and so $\alpha^G = \Omega$ for all $\alpha \in \Omega$. Equivalently, G is transitive if for every pair of points $\alpha, \beta \in \Omega$ there exists $g \in G$ such that $\alpha^g = \beta$. A group which is not transitive is called intransitive.

If $G_\alpha = 1$ for all $\alpha \in \Omega$, we say that G is semi-regular on Ω.

A group G acting transitively on a set Ω is said to act regularly if $G_\alpha = 1$ for each $\alpha \in \Omega$, that is only the identity fixes any point.

Clearly subgroups of semi-regular groups are semi-regular; 1 is semi-regular. As

$$|G_\alpha|\,|\alpha^G| = |G| \text{ for any } \alpha \in \Omega,$$

we get that in a semi-regular group G all orbits have the same size, namely, $|G|$, and hence, the order of G divides the degree of G. Furthermore, in a regular group G we have that,

$$|G| = |\alpha^G| = |\Omega|, \alpha \in \Omega$$

and so the order and the degree of G coincide.

Definition 2.2

Let G be a transitive Group. A subset X of Ω is said to be a set of *imprimitivity* for the action of G on Ω, if for each $g \in G$ either $Xg = X$ or Xg and X are disjoint. In particular, Ω itself, 1-element subsets of Ω and the empty set are obviously sets of imprimitivity of every group G on Ω; these are called trivial sets of imprimitivity. We say that G is *primitive* on Ω if the only sets of imprimitivity are the trivial ones; otherwise G is *imprimitive* on Ω.

We note that the size of the block of a transitive group G divides the degree of G. Hence all transitive groups of degree p, p a prime, are primitive groups since in this case each block must be trivial.

The notion of primitivity has important consequences on the subgroup structure of a transitive group G. In terms of the stabilizers, it can be shown that:

The transitive permutation group G on Ω is primitive if and only if the stabilizer G_α (for$\alpha \in \Omega$, $|\Omega| > 1$) is a maximal subgroup of G (Audu,M.S. 2003).

We state without proof here that:

If the transitive group G contains an intransitive normal subgroup N different from 1, then G is imprimitive.The orbits of N form a complete block system of G. This is in fact a sufficient condition for the imprimitivity of a transitive group (Audu,M.S. 2003).

Theorem 2.1 (Passman, 1968)
Let G be a non-trivial transitive permutation group on Ω. Then G is primitive if and only if G_α, $\alpha \in \Omega$ is a maximal subgroup of G or equivalently G is imprimitive if and only if there is a subgroup H of G properly lying between $G_\alpha(\alpha \in \Omega)$ and G.

Lemma 2.1
Let G be a dihedral group of any order, then G is transitive.

Proof
For given α_i, α_j as any two vertices of the regular polygon with i < j, we readily see that $(\alpha_1\alpha_2 \ldots \alpha_i \ldots \alpha_j \ldots \alpha_n)^{j-i}$ is the rotation about the centre of the polygon through angle $2\pi^c/n$, (where n is the number of edges of the polygon) which takes α_i to α_j. As such G is transitive.

3. DIHEDRAL GROUPS OF DEGREE 2^r ($r \geq 2$).
We shall now determine using Groups, Algorithms and Programming (**GAP**) some Dihedral groups of Degree 2^r ($r \geq 2$) (4,8,16,32,64,128,256,512,1024,2048,4096) and discuss their transitivity and primitivity which will guide us to obtain our result.

```
gap> # Groups, Algorithms and Programming- GAP 4.4.10
gap> GROUPS ALGORITHMS AND PROGRAMMING (GAP)
 gap> D4:=DihedralGroup(IsGroup,8);
Group([ (1,2,3,4), (2,4) ])
gap> for i in D4 do
> Print(i,"\n");
> od;
(1)
(1,3)(2,4)
(1,4,3,2)
(1,2,3,4)
(2,4)
(1,3)
(1,4)(2,3)
(1,2)(3,4)
gap> IsTransitive(D4);
true
gap> IsPrimitive(D4);
false
gap> D8:=DihedralGroup(IsGroup,16);
Group([ (1,2,3,4,5,6,7,8), (2,8)(3,7)(4,6) ])
gap> for i in D8 do
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```
> Print(i,"\n");
> IsTransitive(D8);
> od;
(1)
(1,5)(2,6)(3,7)(4,8)
(1,7,5,3)(2,8,6,4)
(1,3,5,7)(2,4,6,8)
(1,8,7,6,5,4,3,2)
(1,4,7,2,5,8,3,6)
(1,6,3,8,5,2,7,4)
(1,2,3,4,5,6,7,8)
(2,8)(3,7)(4,6)
(1,5)(2,4)(6,8)
(1,7)(2,6)(3,5)
(1,3)(4,8)(5,7)
(1,8)(2,7)(3,6)(4,5)
(1,4)(2,3)(5,8)(6,7)
(1,6)(2,5)(3,4)(7,8)
(1,2)(3,8)(4,7)(5,6)
gap> IsTransitive(D8);
true
gap> IsPrimitive(D8);
false
gap> D16:=DihedralGroup(IsGroup,32);
Group([ (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16),
(2,16)(3,15)(4,14)(5,13)(6,12)(7,11)(8,10) ])
gap> for i in D16 do
> Print(i,"\n");
> od;
(1)
( 1, 9)( 2,10)( 3,11)( 4,12)( 5,13)( 6,14)( 7,15)( 8,16)
( 1,13, 9, 5)( 2,14,10, 6)( 3,15,11, 7)( 4,16,12, 8)
( 1, 5, 9,13)( 2, 6,10,14)( 3, 7,11,15)( 4, 8,12,16)
( 1,15,13,11, 9, 7, 5, 3)( 2,16,14,12,10, 8, 6, 4)
( 1, 7,13, 3, 9,15, 5,11)( 2, 8,14, 4,10,16, 6,12)
( 1,11, 5,15, 9, 3,13, 7)( 2,12, 6,16,10, 4,14, 8)
( 1, 3, 5, 7, 9,11,13,15)( 2, 4, 6, 8,10,12,14,16)
( 1,16,15,14,13,12,11,10, 9, 8, 7, 6, 5, 4, 3, 2)
( 1, 8,15, 6,13, 4,11, 2, 9,16, 7,14, 5,12, 3,10)
( 1,12, 7, 2,13, 8, 3,14, 9, 4,15,10, 5,16,11, 6)
( 1, 4, 7,10,13,16, 3, 6, 9,12,15, 2, 5, 8,11,14)
( 1,14,11, 8, 5, 2,15,12, 9, 6, 3,16,13,10, 7, 4)
( 1, 6,11,16, 5,10,15, 4, 9,14, 3, 8,13, 2, 7,12)
( 1,10, 3,12, 5,14, 7,16, 9, 2,11, 4,13, 6,15, 8)
( 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15,16)
( 2,16)( 3,15)( 4,14)( 5,13)( 6,12)( 7,11)( 8,10)
( 1, 9)( 2, 8)( 3, 7)( 4, 6)(10,16)(11,15)(12,14)
( 1,13)( 2,12)( 3,11)( 4,10)( 5, 9)( 6, 8)(14,16)
( 1, 5)( 2, 4)( 6,16)( 7,15)( 8,14)( 9,13)(10,12)
( 1,15)( 2,14)( 3,13)( 4,12)( 5,11)( 6,10)( 7, 9)
( 1, 7)( 2, 6)( 3, 5)( 8,16)( 9,15)(10,14)(11,13)
( 1,11)( 2,10)( 3, 9)( 4, 8)( 5, 7)(12,16)(13,15)
( 1, 3)( 4,16)( 5,15)( 6,14)( 7,13)( 8,12)( 9,11)
( 1,16)( 2,15)( 3,14)( 4,13)( 5,12)( 6,11)( 7,10)( 8, 9)
( 1, 8)( 2, 7)( 3, 6)( 4, 5)( 9,16)(10,15)(11,14)(12,13)
( 1,12)( 2,11)( 3,10)( 4, 9)( 5, 8)( 6, 7)(13,16)(14,15)
( 1, 4)( 2, 3)( 5,16)( 6,15)( 7,14)( 8,13)( 9,12)(10,11)
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( 1,14)( 2,13)( 3,12)( 4,11)( 5,10)( 6, 9)( 7, 8)(15,16)
( 1, 6)( 2, 5)( 3, 4)( 7,16)( 8,15)( 9,14)(10,13)(11,12)
( 1,10)( 2, 9)( 3, 8)( 4, 7)( 5, 6)(11,16)(12,15)(13,14)
( 1, 2)( 3,16)( 4,15)( 5,14)( 6,13)( 7,12)( 8,11)( 9,10)
gap> IsTransitive(D16);
true

gap> IsPrimitive(D16);
false
gap> D32:=DihedralGroup(IsGroup,64);
Group([
(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,2
9,30,31,32), (2,32)(3,31)(4,30)(5,29)(6,28)(7,27)(8,26)(9,25)
(10,24)(11,23)(12,22)(13,21)(14,20)(15,19)(16,18) ])
gap> for i in D32 do
> Print(i,"\n");
> od;
(1)
( 1,17)( 2,18)( 3,19)( 4,20)( 5,21)( 6,22)( 7,23)( 8,24)
( 9,25)(10,26)(11,27)(12,28)(13,29)(14,30)(15,31)(16,32)
( 1,25,17, 9)( 2,26,18,10)( 3,27,19,11)( 4,28,20,12)( 5,29,21,13)      (
6,30,22,14)( 7,31,23,15)( 8,32,24,16)( 1, 9,17,25)( 2,10,18,26)      (
3,11,19,27)( 4,12,20,28)   ( 5,13,21,29)( 6,14,22,30)( 7,15,23,31)    (
8,16,24,32)( 1,29,25,21,17,13, 9, 5)( 2,30,26,22,18,14,10, 6)        (
3,31,27,23,19,15,11, 7)( 4,32,28,24,20,16,12, 8)( 1,13,25, 5,17,29, 9,21)(
2,14,26, 6,18,30,10,22)( 3,15,27, 7,19,31,11,23)               ( 4,16,28,
8,20,32,12,24)( 1,21, 9,29,17, 5,25,13)                       (
2,22,10,30,18, 6,26,14)( 3,23,11,31,19, 7,27,15)                     (
4,24,12,32,20, 8,28,16)( 1, 5, 9,13,17,21,25,29)
( 2, 6,10,14,18,22,26,30)( 3, 7,11,15,19,23,27,31)
( 4, 8,12,16,20,24,28,32)
( 1,31,29,27,25,23,21,19,17,15,13,11, 9, 7, 5, 3)
( 2,32,30,28,26,24,22,20,18,16,14,12,10, 8, 6, 4)
( 1,15,29,11,25, 7,21, 3,17,31,13,27, 9,23, 5,19)                      (
2,16,30,12,26, 8,22, 4,18,32,14,28,10,24, 6,20)
( 1,23,13, 3,25,15, 5,27,17, 7,29,19, 9,31,21,11)
( 2,24,14, 4,26,16, 6,28,18, 8,30,20,10,32,22,12)
( 1, 7,13,19,25,31, 5,11,17,23,29, 3, 9,15,21,27)
( 2, 8,14,20,26,32, 6,12,18,24,30, 4,10,16,22,28)
( 1,27,21,15, 9, 3,29,23,17,11, 5,31,25,19,13, 7)
( 2,28,22,16,10, 4,30,24,18,12, 6,32,26,20,14, 8)
( 1,11,21,31, 9,19,29, 7,17,27, 5,15,25, 3,13,23)
( 2,12,22,32,10,20,30, 8,18,28, 6,16,26, 4,14,24)
( 1,19, 5,23, 9,27,13,31,17, 3,21, 7,25,11,29,15)
( 2,20, 6,24,10,28,14,32,18, 4,22, 8,26,12,30,16)
( 1, 3, 5, 7, 9,11,13,15,17,19,21,23,25,27,29,31)
( 2, 4, 6, 8,10,12,14,16,18,20,22,24,26,28,30,32)
(1,32,31,30,29,28,27,26,25,24,23,22,21,20,19,18,17,16,15,14,13,12,11,10, 9,
8, 7, 6, 5, 4, 3, 2)
( 1,16,31,14,29,12,27,10,25, 8,23, 6,21, 4,19, 2,17,32,15,30,13,28,11,26,
9,24, 7,22, 5,20, 3,18)
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9,32,23,14, 5,28,19,10)
( 1, 8,15,22,29, 4,11,18,25,32, 7,14,21,28, 3,10,17,24,31, 6,13,20,27, 2,
9,16,23,30, 5,12,19,26)
( 1,28,23,18,13, 8, 3,30,25,20,15,10, 5,32,27,22,17,12, 7, 2,29,24,19,14, 9,
4,31,26,21,16,11, 6)
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9,28,15, 2,21, 8,27,14)
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9,12,15,18,21,24,27,30)
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7,32,25,18,11, 4,29,22,15, 8)
( 1,10,19,28, 5,14,23,32, 9,18,27, 4,13,22,31, 8,17,26, 3,12,21,30, 7,16,25,
2,11,20,29, 6,15,24)
( 1,18, 3,20, 5,22, 7,24, 9,26,11,28,13,30,15,32,17, 2,19, 4,21, 6,23,
8,25,10,27,12,29,14,31,16)
( 1, 2, 3, 4, 5, 6, 7, 8,
9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32)
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(19,23)(20,22)
( 1,29)( 2,28)( 3,27)( 4,26)( 5,25)( 6,24)( 7,23)( 8,22)
( 9,21)(10,20)(11,19)(12,18)(13,17)(14,16)(30,32)
( 1,13)( 2,12)( 3,11)( 4,10)( 5, 9)
( 6, 8)(14,32)(15,31)(16,30)(17,29)(18,28)(19,27)(20,26)(21,25)(22,24)
( 1,21)( 2,20)( 3,19)( 4,18)( 5,17)( 6,16)( 7,15)( 8,14)
( 9,13)(10,12)(22,32)(23,31)(24,30)(25,29)(26,28)
( 1, 5)( 2, 4)( 6,32)( 7,31)( 8,30)
( 9,29)(10,28)(11,27)(12,26)(13,25)(14,24)(15,23)(16,22)(17,21)(18,20)
( 1,31)( 2,30)( 3,29)( 4,28)( 5,27)( 6,26)( 7,25)( 8,24)
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( 7, 9)(16,32)(17,31)(18,30)(19,29)(20,28)(21,27)(22,26)(23,25)
( 1,23)( 2,22)( 3,21)( 4,20)( 5,19)( 6,18)( 7,17)( 8,16)
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( 1, 7)( 2, 6)( 3, 5)( 8,32)
(9,31)(10,30)(11,29)(12,28)(13,27)(14,26)(15,25)(16,24)(17,23)(18,22)
(19,21)( 1,27)( 2,26)( 3,25)( 4,24)( 5,23)( 6,22)( 7,21)( 8,20)
( 9,19)(10,18)(11,17)(12,16)(13,15)(28,32)(29,31)
( 1,11)( 2,10)( 3, 9)( 4, 8)
(5,7)(12,32)(13,31)(14,30)(15,29)(16,28)(17,27)(18,26)(19,25)(20,24) (21,23)(
1,19)( 2,18)( 3,17)( 4,16)( 5,15)( 6,14)( 7,13)( 8,12)
( 9,11)(20,32)(21,31)(22,30)(23,29)(24,28)(25,27)
( 1, 3)( 4,32)( 5,31)( 6,30)( 7,29)( 8,28)
( 9,27)(10,26)(11,25)(12,24)(13,23)(14,22)(15,21)(16,20)(17,19)
( 1,32)( 2,31)( 3,30)( 4,29)( 5,28)( 6,27)( 7,26)( 8,25)
( 9,24)(10,23)(11,22)(12,21)(13,20)(14,19)(15,18)(16,17)
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( 1,16)( 2,15)( 3,14)( 4,13)( 5,12)( 6,11)( 7,10)( 8,
9)(17,32)(18,31)(19,30)(20,29)(21,28)(22,27)(23,26)(24,25)
( 1,24)( 2,23)( 3,22)( 4,21)( 5,20)( 6,19)( 7,18)( 8,17)
( 9,16)(10,15)(11,14)(12,13)(25,32)(26,31)(27,30)(28,29)
( 1, 8)( 2, 7)( 3, 6)( 4, 5)
(9,32)(10,31)(11,30)(12,29)(13,28)(14,27)(15,26)(16,25)(17,24)(18,23) (19,22)
(20,21)( 1,28)( 2,27)( 3,26)( 4,25)( 5,24)( 6,23)( 7,22)( 8,21)
( 9,20)(10,19)(11,18)(12,17)(13,16)(14,15)(29,32)(30,31)
( 1,12)( 2,11)( 3,10)( 4, 9)( 5, 8)
(6,7)(13,32)(14,31)(15,30)(16,29)(17,28)(18,27)(19,26)(20,25)(21,24)(22,23)(
1,20)( 2,19)( 3,18)( 4,17)( 5,16)( 6,15)( 7,14)( 8,13)
( 9,12)(10,11)(21,32)(22,31)(23,30)(24,29)(25,28)(26,27)
( 1, 4)( 2, 3)( 5,32)( 6,31)( 7,30)( 8,29)
( 9,28)(10,27)(11,26)(12,25)(13,24)(14,23)(15,22)(16,21)(17,20)(18,19)
( 1,30)( 2,29)( 3,28)( 4,27)( 5,26)( 6,25)( 7,24)( 8,23)
( 9,22)(10,21)(11,20)(12,19)(13,18)(14,17)(15,16)(31,32)
( 1,14)( 2,13)( 3,12)( 4,11)( 5,10)( 6, 9)
( 7, 8)(15,32)(16,31)(17,30)(18,29)(19,28)(20,27)(21,26)(22,25)(23,24)
( 1,22)( 2,21)( 3,20)( 4,19)( 5,18)( 6,17)( 7,16)( 8,15)
( 9,14)(10,13)(11,12)(23,32)(24,31)(25,30)(26,29)(27,28)
( 1, 6)( 2, 5)( 3, 4)( 7,32)( 8,31)
(9,30)(10,29)(11,28)(12,27)(13,26)(14,25)(15,24)(16,23)(17,22)(18,21)
(19,20)( 1,26)( 2,25)( 3,24)( 4,23)( 5,22)( 6,21)( 7,20)( 8,19)
( 9,18)(10,17)(11,16)(12,15)(13,14)(27,32)(28,31)(29,30)
( 1,10)( 2, 9)( 3, 8)( 4, 7)
(5,6)(11,32)(12,31)(13,30)(14,29)(15,28)(16,27)(17,26)(18,25)(19,24) (20,23)
(21,22)( 1,18)( 2,17)( 3,16)( 4,15)( 5,14)( 6,13)( 7,12)       ( 8,11)(
9,10)(19,32)(20,31)(21,30)(22,29)(23,28)(24,27)(25,26)
( 1, 2)( 3,32)( 4,31)( 5,30)( 6,29)( 7,28)( 8,27)
( 9,26)(10,25)(11,24)(12,23)(13,22)(14,21)(15,20)(16,19)(17,18)
gap> IsTransitive(D32);
true
gap> IsPrimitive(D32);
false
gap> D64:=DihedralGroup(IsGroup,128);
<permutation group with 2 generators>
gap> IsTransitive(D64);
true
gap> IsPrimitive(D64);
false
gap> D128:=DihedralGroup(IsGroup,256);
<permutation group with 2 generators>
gap> IsTransitive(D128);
true
gap> IsPrimitive(D128);
false
gap> D256:=DihedralGroup(IsGroup,512);
<permutation group with 2 generators>
gap> IsTransitive(D256);
true
gap> IsPrimitive(D256);
false
gap> D512:=DihedralGroup(IsGroup,1024);
<permutation group with 2 generators>
gap> IsTransitive(D512);
true
gap> IsPrimitive(D512);
```

```
false
gap> D1024:=DihedralGroup(IsGroup,2048);
<permutation group with 2 generators>
gap> IsTransitive(D1024);
true

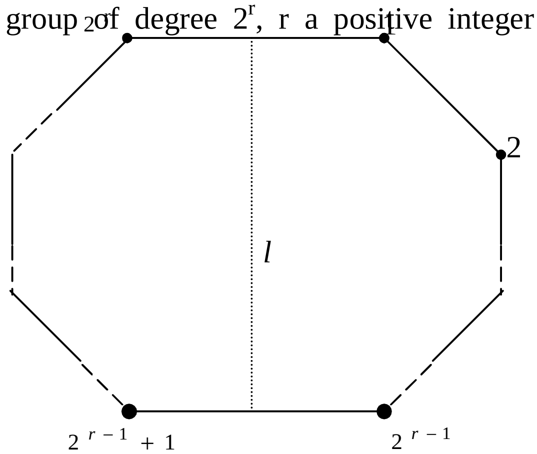
gap> IsPrimitive(D1024);
false
gap> D2048:=DihedralGroup(IsGroup,4096);
<permutation group with 2 generators>
gap> IsTransitive(D2048);
true
gap> IsPrimitive(D2048);
false
gap> D4096:=DihedralGroup(IsGroup,8192);
<permutation group with 2 generators>
gap> IsTransitive(D4096);
true
gap> IsPrimitive(D4096);
false
gap> quit;
```

RESULTS

Based on the trend in Section 3, we proved a theorem which concerns particularly on transitivity and primitivity of all the dihedral groups of degree 2^r ($r \geq 2$). This is the content of the next theorem and therefore it forms an important part of this work.

Theorem

Let G be a dihedral group of degree 2^r, r a positive integer ≥2. Then G is transitive and imprimitive.



Proof

That G is transitive follows easily from Lemma 2.1.

Now, name the vertices of G as 1,2,3, …, 2^r, and let ℓ be the line of symmetry joining the middle of the vertices 1 and 2^r and the middle of the vertices 2^{r-1} and $2^{r-1} + 1$. The reflection in ℓ is given by

$$t = (1\ \ 2^r)\ (2\ \ 2^r - 1)\ (3\ \ 2^r - 2) \ldots (2^{r-1}\ \ 2^{r-1} + 1)$$

Then $G_1 = \{(1), (2\ \ 2^r - 1)\ (3\ \ 2^r - 2) \ldots (2^{r-1}\ \ 2^{r-1} + 1)\}$ is the stabilizer of the point 1.

We readily see that G_1 is a non-identity proper subgroup of G which has

$$B = \{(1), (1\ \ 2^r), (2\ \ 2^r - 1)\ (3\ \ 2^r - 2) \ldots (2^{r-1}\ \ 2^{r-1} + 1),\ t\}$$

as a subgroup properly lying between G_1 and G, i.e., $G_1 < B < G$. The result follows from Theorem *2.1,* showing that G is imprimitive.

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