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Role of spatial string tension under the QCD phenomenology

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ABSTRACT

The spatial string tension has a widespread influence upon various distinctive features of Quantum-Chromodynamics (QCD). These include the behavioural implications of spatial string tension correponding to varying temperatures under different QCD phases. An exclusive study has been performed for visualising this variation both above and below the critical deconfinement temperatures T_c. Furthermore, we have pinpointed the observational difference between the QCD approach corresponding to N_c=1 and N_c=3 limits. Also, we have tried to study the effective spatial string tension in quenched SU(N_c) QCD under the gluon chain model when temperatures are considered below T_c. The spatial string tension is also visualized within a five dimensional AdS/QCD framework. We observed that the temperature dependence of string tension is very soft below T_c and sharp above T_c.

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INTRODUCTION

A broad spectrum of research areas such as Cosmology, Astrophysics and Heavy-Ion-Phenomenology are implicitly dependent upon the calculation of QCD thermodynamics from the first principle. Recently, lattice QCD [1] has proven to be the richest source to perform such calculations and the interest in QCD at temperatures larger than a few hundred MeV witnesses effective triggering for both experimental as well as pure theoretical reasons. We have studied the thermodynamics of QCD, both below and above the deconfinement temperature.

The heavy ion collisions aim at creating quark-gluon plasma in the laboratory and recent RHIC experiments [2,3] suggest that the quark-gluon plasma may be more than a perfect liquid and spatial string tensions prevail in this high temperature regime. Basically, the spatial string tension verifies the theoretical concept of dimensional reduction at high temperatures [4] and serves as a classic non-perturbative probe for the convergence of the weak coupling expansion in this high temperature domain. Thus, we can say that the study of QCD at temperatures larger than (a few) hundred MeV helps in probing the property of asymptotic freedom. The spatial string tension thereby projects out as a classic non-perturbative probe for the convergence of weak coupling expansion at high temperatures. However, we are well aware of the fact that at the confinement temperature T_c, the physical string tension becomes zero. Using a simple argument on the behaviour of horizontal Wilson loops at high temperature, a general relation between deconfinement point T_c and string tension can be obtained. The Wilson loop serves as an important tool for studying confinement in gauge theories. The gauge string/duality is itself useful to calculate the Wilson loop from string configurations[5].

The spatial string tension can be easily extracted as the coefficient in the area law of a large rectangular Wilson loops [5]. In the non-Abelian gauge theories, the Wilson loop for large space like contours obeys the area law at arbitrary temperature [6]. This phenomenon is known as magnetic confinement and yields non-zero string tension. This can be dynamically understood by the fact that when the temperature of the system is increased, there is reduction in the phase space of the colour flux tube until it fills the whole space. Now, the flux tube begins to be squeezed between the two opposite sides of the temporal box. At temperatures far above the deconfinement temperature, the distribution of the colour flux tube along the temporal axis becomes uniform. Thus, the translational invariance in the time direction is restored and the Goldstone field describing the field fluctuations disappear. Even when the deconfinement phase transition was investigated by numerical simulations on lattice for pure SU(3) gauge theory[7], the data demonstrated strong suppression of the electric component of the correlator above T_c and subsequent persistence of the magnetic component. The contribution of the magnetic correlator remains visible even across the phase transition temperature. On the contrary, the electric part suddenly vanishes above T_c making the electric condensate drop to zero at the deconfining phase transition point [8].

Mathematically, the spatial string tension can be expressed as the coefficient in the area law of a large rectangular Wilson loop $W_s(R_1,R_2)$ in the (x_1,x_2) plane and can be expressed as [9]:-

$$\sigma_s = -\lim_{R\to\infty} \lim_{R\to\infty} \frac{1}{R_1R_2} lnW_s(R_1,R_2) \tag{1.1}$$

Lattice simulations [10] indicated that at $T \geq 2T_c$, the magnetic fields as determined by spatial string tension starts growing quadratically as $\sigma_s(T)\sim T^2$ which projects forth the advent of a new visualization, called the dimensional reduction. Under this particular framework, the temporal direction is squeezed and the higher Matsubara frequencies are suppressed. This leads to the effective reduction of dynamics to three dimensional gluodynamics [11]. Thus, three dimensional lattice calculations help in the determination of physical quantities such as $\sigma_s(T)$.

We have studied the spatial string tension under two different models, which portray its behaviour both above and below the deconfinement temperature. One is the gluon chain model [12], where quenched $SU(N_c)$ QCD approximations are utilized. Here, the spatial string tension behaviour is exclusively studied for temperatures less than the critical temperature and visible behavioural differences between the N_c=1 and N_c=3 limits are projected out. For temperatures greater than the critical temperature, we have studied the spatial string tension within a five dimensional framework, known as AdS/QCD [13].

Gluon chain model and the effective spatial string tension

String dynamics itself help in determining the deconfinement critical temperature T_c. When the string connecting heavy quark-antiquark pair passes through heavy valence gluons (forming a gluon chain), very high entropy is generated in this system. It provides a viable mechanism for predicting the value of T_c and also helps in studying the critical behaviour of string tension below T_c [12].

Soft stochastic background gluonic fields lead to the production of quark-antiquark strings, which sweeps out the flat surface of the corresponding Wilson loop. Moreover, string vibrations are produced by the fluctuations of the gauge field. These fluctuations could be related to the valence gluons through which the quark-antiquark string passes. The string may pass through many valence gluons leading to the production of the gluon chain. The energy of a single string bit between two nearest gluons in a chain is constant. It is worth noting that as long as thermal mass of a valence gluon is smaller (at low temperatures) than this energy, the general global dynamics of the string is unaffected and the gluons move together with the string. Further, when the system is heated, and at a certain temperature T_0, the gluons thermal mass (α T) becomes larger than the free energy of the string bit. Now, there is a drastic change in the configuration of the system and the gluons become nearly static from the strings standpoint. Thereby, at $T_0<T<T_c$, the gluons chain behaves as a sequence of static nodes with adjoint charges linked by independently fluctuating string bits. It is here that the entropy of the system becomes large. This occurs due to the fact that the gluon chain originating from a quark randomly walks over the lattice of static nodes towards an antiquark. The entropy of the system increases due to the fact that colour may change from one node to another during this random walk. This implies that every string bit may transport each of the N_c colour. This increase in the entropy of the system leads to the deconfinement phase transition.

The total free energy of the system is the sum of the usual linear potential and the free energy of the random walk. The entire procedural approach starts with the calculation of partition function for the gluon chain and the effective string tension, which is dependent upon the partition function is given as

$$\sigma(T) = \sigma - T\ln\frac{Z(R,T)}{Z(R,T_0)}\bigg|\, R\to\infty \tag{2.1}$$

where Z(R,T) is the partition function of the random walk and is given by

$$Z(R,T) = \sum_{n=-\infty}^{\infty}\int_0^{\infty}\frac{ds}{(4\pi s)^2}\exp\left[-\frac{R^2+(\beta n)^2}{4s}-\frac{s}{a}\left(\beta\sigma-\frac{\ln Nc}{a}\right)\right] \tag{2.2}$$

Here s=aL is the Schwinger proper time (a is the length of one bit of string and L is the length of the gluon chain), β =1/T , σ is the zero temperature string tension and n is the number of a Matsubara mode. The n=0 term is significant at asymptotically large R's, which is basically the region of interest and then

$$\sigma(T) = \sigma + T\left[\sqrt{\frac{\sigma\beta}{a}\left(1-T\frac{\ln Nc}{\sigma a}\right)}-\sqrt{\frac{\sigma\beta_0}{a}\left(1-T_0\frac{\ln Nc}{\sigma a}\right)}\right] \tag{2.3}$$

The value of T_c is estimated from the condition that the argument of the first square root vanishes and its value comes out to be T_c= 270 MeV [14] for N_c=3 and the effective length of one string bit is a ~ 0.31fm. T_0 can be evaluated from the formula:

$$T_0 = \frac{T_c}{\ln N_c + 1} \sim 130 MeV \qquad (2.4)$$

Now, considering the limiting case, when the string bits cannot change colour, then N_c=1 and equation 1 yields

$$\sigma(T) = \sigma + \sqrt{\frac{\sigma T}{a}}\left(1 - \frac{T}{T_0}\right) \qquad (2.5)$$

In this particular case, T_0 can be determined directly from the equality of the gluons thermal mass in QCD to the free energy of one string bit [12],

$$T_0 = \frac{\sigma a}{g} \qquad (2.6)$$

For a=0.22fm and g=2.5, giving T_0= 85MeV and the critical temperature [12] T_c=290 MeV.

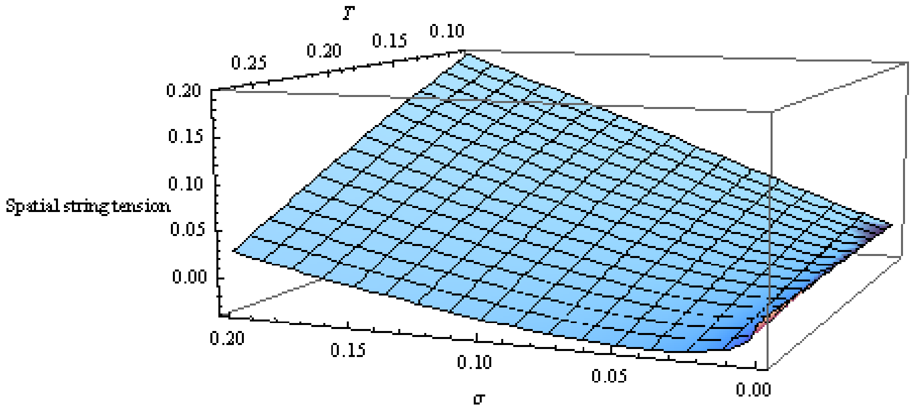

Figure 1: Spreadsheet displaying the variation of spatial string tension(in GeV^2) with temperature(in MeV) and physical string tension(inGeV^2) for N_c=1.

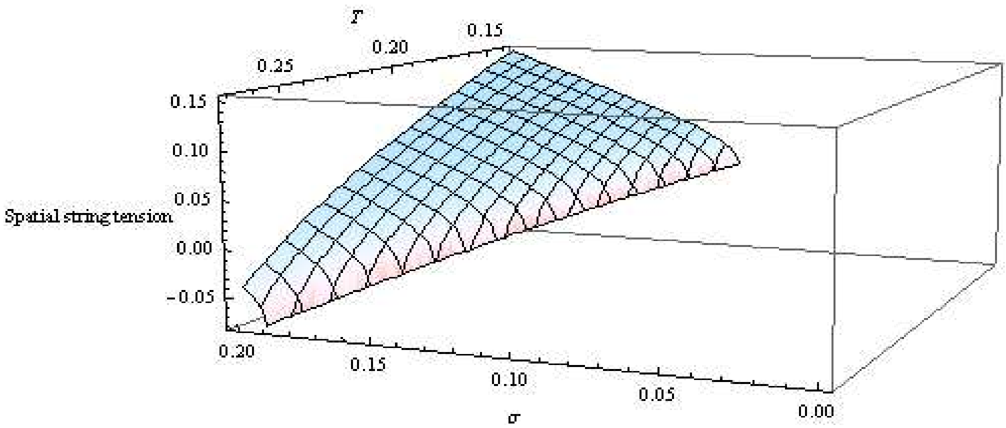

Figure 2: Spreadsheet displaying the variation of spatial string tension(in GeV^2) with temperature(in MeV) and physical string tension(inGeV^2) for N_c=3.

Thus, for temperatures T_0<T<T_c, the spatial string tension behaviourism is implicitly dependent upon two prominent features that is the physical string tension and temperature parameter itself. This particular analysis exhibits the fact that spatial string tension variation pattern can be effectively determined by varying these two parameters within their specified range. The physical string tension is varied within a range 0 to 0.2GeV^2. We have plotted a three dimensional spreadsheet which clearly helps in visualising the spatial variation. Figures 1 and 2 display the variation pattern for N_c=1 and N_c=3 respectively. Both the plots display observable differences and provide a platform for demonstrating evident difference between the N_c=1 and N_c=3 limit. It may however be pointed out that for temperatures far above the critical temperature (more than a few hundred MeV), the spatial string tension displays a linear variation with the temperature. The spreadsheet for N_c=1 limit covers the entire available spatial dimensions, whereas the N_c=3 three dimensional plot is restricted within a smaller domain and the spreadsheet possesses an effective curvature.

3. Modelling of spatial string tension within the five dimensional AdS/QCD framework

We have scrutinized the modelling of temperature dependence of spatial string tension within a five dimensional framework, known as AdS/QCD[13]. The SU(N) gauge theories undergo a phase transition to a deconfined phase at high temperature. The pseudo-potential extracted from spatial Wilson loops does not exhibit quantative drastic change at deconfinement temperature T_c. This owes to the fact that certain confining properties survive in the high temperature phase. High temperature perturbation theory is helpful in determining the behaviourism of pseudo potential for temperatures well above T_c. However, near the phase transition point, the non-perturbative effects pose difficulties in the computation of the pseudo potentials. At this point the AdS/QCD approach came to the rescue which deals with a string description of strong interactions.

The five dimensional AdS/QCD approach helps in exploring the temperature dependence of the spatial string tension. Spatial Wilson loops are studied which obey an area law and provide string tension. The whole framework of this approach relies upon c, which is the Regge parameter at zero temperature and its value (c~0.9GeV^2) is fixed from the ρ meson Regge trajectory[15], with the co-efficient of proportionality fixed from the linear term of the Cornell potential.

A rectangular loop C is considered along two spatial directions (x,y) on the boundary(z=0) of a five dimensional space. One of the direction is taken to be large, say Y→∞ and the quark and antiquark are positioned at x=r/2 and x=-r/2 respectively. The Nambu-Goto action with the world sheet co-ordinates x and y is evaluated and equation of motion for z is determined. The z dependent effective string tension as followed from the AdS metric is viewed simply as

$$\sigma(z) = z^{-2} exp\left\{\frac{1}{2} cz^2\right\} \tag{3.1}$$

The behaviour of potential V=σ(z) shows that it reaches a minimum value at $z=z_c$ ($z_c = \sqrt{\frac{2}{c}}$ and $z_0 = z|x = 0$) , where the repulsive force prevents the string from getting deeper in the z direction. Because the string ends on infinitely heavy quark antiquark pair set at z=0, it faces a minima of potential which can be termed as a wall with condition

$$z_0 < z_c \tag{3.2}$$

Also, in the limit as c goes to 0, z_0 is bounded by a horizon ($z=z_T$) and this gives rise to a wall

$$z_0 < z_T \tag{3.3}$$

Thus, two walls become pertinent in this visualization. This projects out to be the most prominent factor in determining the temperature dependence of the spatial string tension.

Temperature dependence of spatial string tension is determined by evaluating r, which is a continuously growing function of z_0. This means that large distances correspond to a region near the upper endpoint which is the smallest of z_c and z_T and this leads to v~1.

Finally

$$r = -\frac{2z_0}{\sqrt{\beta}} ln\left(1 - \frac{z_0}{z_c}\right)\left(1 - \frac{z_0}{z_T}\right) + O(1) \tag{3.4}$$

Where β is a polynomial in $x = \left(\frac{z_0}{z_T}\right)^4$ and $y = \left(\frac{z_0}{z_c}\right)^2$ and is expressed as

$$\beta = -6 + 22x + 18y - 8y^2 - 34xy + 8xy^2 \tag{3.5}$$

The long distance behaviour (upper endpoint, r→∞) of the energy of the configuration can be expressed as

$$E = -\frac{g e^{\left(\frac{z_0}{z_c}\right)^2}}{\pi z_0 \sqrt{\beta}} \ln\left(1 - \frac{z_0}{z_c}\right)\left(1 - \frac{z_0}{z_T}\right) + O(1) \tag{3.6}$$

From, the long distance pseudo-potential turns out to be linear. The spatial string tension is given by

$$\sigma_s = \begin{cases} \sigma & if\ T \le T_c \\ \sigma\left(\frac{T}{T_c}\right)^2 exp\left\{\left(\frac{T_c}{T}\right)^2 - 1\right\} & if\ T \ge T_c \end{cases} \tag{3.7}$$

where $$T_c = \frac{1}{\pi}\sqrt{\frac{c}{2}} \tag{3.8}$$

This value of critical temperature corresponds to a point when $z_c=z_T$, that is the two walls coincide at the phase transition point and T_c turns out to be ≈ 210 MeV. Also it is found from and that

$$\frac{T_c}{\sqrt{\sigma}} = \sqrt{\frac{2}{e\pi g}} \tag{3.9}$$

Value of g(≈0.94) comes from [15] the linear term of the Cornell potential. The approximation is in agreement with the lattice data for SU(3) gauge theory[7]. We have plotted (Figure3) the spatial string tension as a function of temperature for temperatures above critical temperature T_c. Data points have been plotted for the upper (0.2GeV^2) and lower limits (0.87GeV^2) of the physical string tension[16]. These limits may vary from one system (hadron configurations) to another, but our aim is to study the general behaviour, which should be same for all systems. The spatial string tension increases exponentially with temperature and the observed pattern shows similar behaviourism for the two different values of physical string tension, with the variation being slightly magnified as we proceed from the lower to the upper range.

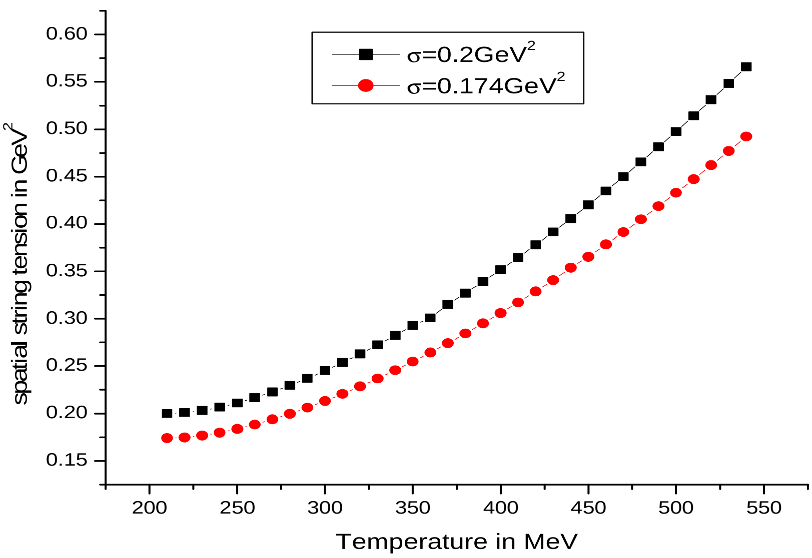

Figure 3: Variation pattern of spatial string tension with temperature for upper and lower values of physical string tension.

When SU(2) gauge theory is considered and temperature dependence of spatial string tension is interrogated and it was found that

$$\frac{\sqrt{\sigma_s}}{T_c} = \frac{\rho_s}{T_c} = 1.44\frac{T}{T_c}exp\left\{\frac{1}{2}\left(\frac{T_c}{T}\right)^2 - \frac{1}{2}\right\} \quad (3.10)$$

(g depends on the number of colours, so its value has to be adjusted to SU(2) by employing fit $\frac{\sqrt{\sigma_s}}{T_c}$ at T=T_c to the data from [17]).

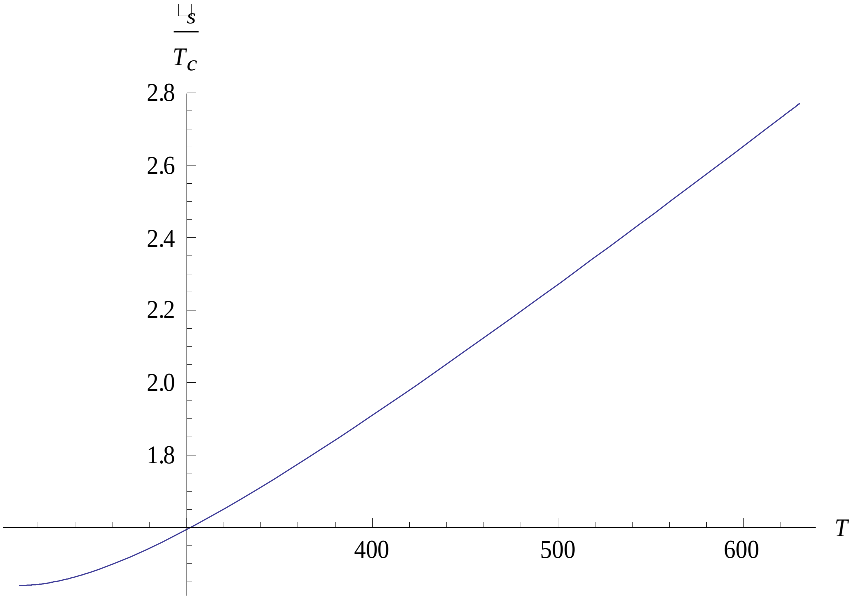

Figure 4: Variation of ρ_s(in units of Tc) with temperature T(in MeV)

The temperature dependence of string tension at high temperature is determined by the β function of gauge theory. Figure 4 demonstrates the variation of ρ_s/T_c with temperature T for temperatures a few MeV above the critical temperature. For high temperatures, that is values greater than around 300 MeV, the plot shows a linear variation. An appreciable curvature is visible at lower values of T, that is near the critical temperature. Thus we can say the temperature dependence of string tension is very soft below T_c and sharp above T_c.

CONCLUSION

Our analysis evidently reveals the intrinsic and intimate connection between the QCD thermodynamics and the spatial string tension parameter. The behavioural implications of the Wilson loops at high temperature helped in probing the deconfinement physics and thereby proved useful in determining the temperature dependence of the spatial string tension under the gluon chain model and within the AdS/QCD picture. Under the gluon chain model, for temperatures $T_0<T<T_c$, the spatial string tension variation attributes its dependency solely to the physical string tension and the temperature parameter and their corresponding variation pattern clearly depicts the fact that N_c=3 limit extensively stands apart from the N_c=1 limit.

Modelling of spatial string tension within the five dimensional AdS/QCD framework reveals that for SU(3) gauge theory, the spatial string tension increases exponentially with temperature for temperature range lying above T_c. However when the same analysis is performed under the SU(2) gauge theory, a linear variation is observed. Thus, the variation pattern of spatial string tension parameter is sensitive to the gauge theory within which we perform our analysis.

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