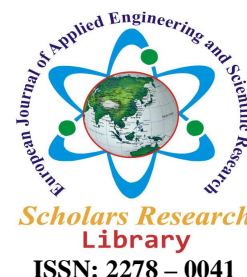




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## The Mechanism and Kinematics of a Pantograph Milling Machine

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### ABSTRACT

*The paper is paying attention on a 2-Revolute & 1-Prismatic (RRP) kind of manipulator kinematically. The manipulator is based on a parallelogram linkage mechanism and translates along horizontal directions and z-axis motion i.e. vertical movement is provided by effective stylus length. At the end-effector a palm router installed with milling cutter is mounted. Compared to conventional milling machine it can traverse the de-scaled profile traversed by stylus. The forward kinematic equations have been formulated. The simulation results by solid works software approximately matches the computation formulation derived in this paper. A prototype is made-up to perform milling operation on any contour.*

**Key words:** Pantograph milling machine, mechanism, computational analysis, Theoretical formulation, kinematic analysis

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### INTRODUCTION

Traditional milling machine were able to mill only on a plain surface or we can say only along the straight paths and could not generate the replica of already existing object. This kind of manipulator has large workspace, high sleight and good maneuverability; it can be widely used in field of painting, welding, assembly and wood/metal engraving<sup>[1]</sup>. However due to its cantilever type structure, the manipulator is inherently not very rigid and thus the link connecting the assembly to the bed is the most vulnerable to failure due to bending load. It is poor in dynamic performance in high speed and heavy duty operations. Hence suitable for light duty and slow speed operations [1, 10].

Since the parallel mechanism was first invented in 1960's and nowadays it is being used in many industrial applications. Light Rail vehicles (LRVs) are equipped with one pantograph that collects current from the contact wire. When the LRVs operate, the pantograph pan-head is in contact with the wire, and electricity is transferred to the vehicle [2, 7, 8, 9].

The mechanical efficiency of pantograph mechanisms and conventional open-chain and closed-chain type manipulators are studied and evaluated using the concept of modified geometric work. The kinematics of 6-DOF, pantograph type manipulators are studied and special: mechanisms which simplify the kinematics are introduced. The computational complexity of both Cartesian and cylindrical type pantograph manipulators are evaluated and compared with a PUMA type manipulator [3, 5]

Operation analysis of a Chebyshev-Pantograph leg mechanism is presented for a single degree of freedom (DOF) biped robot. The proposed leg mechanism is composed of a Chebyshev four-bar linkage and a pantograph mechanism. In contrast to general fully actuated anthropomorphic leg mechanisms, the proposed leg mechanism has peculiar features like compactness, low-cost, and easy-operation [4, 6].

## 2. Objective

The pantograph mechanism is used to design and fabricate a milling machine which could traverse on any contour provided that stylus is moved along the same on any already existing object. Using such kind of manipulator we can generate the de-scaled replica of the object or we can say it to be a copying machine which can be employed in mass production with economical production/machining cost. The 3-DOF in this manipulator adds a feature to increase or decrease the depth of engrave and thus can be used in metal engraving industries or wood carving industries to copy the engraved wooden design. At the end-effector we can replace the cutter by a welding torch or a paint brush to perform the desired typical operation with very ease and accuracy. The only constraints are its poor dynamic performance but our kinematic analysis shows it to be perfect for light and slow speed operation.

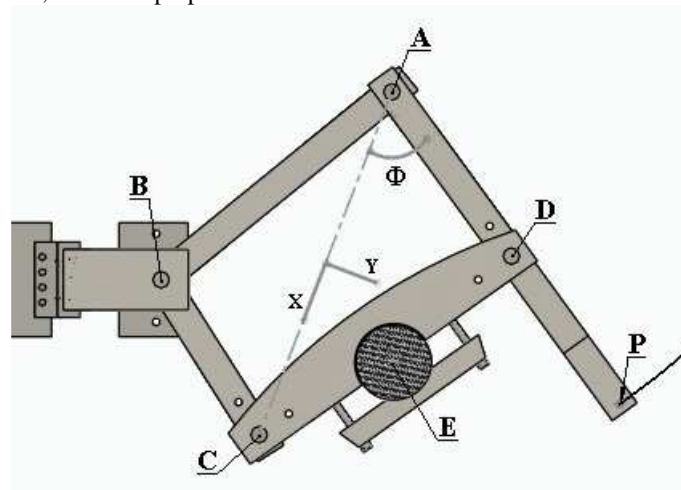
## 3. Mechanism Description

By adjusting the length of two linear links AD and BE using magnification holes on the corresponding links, we can control the movement of point E in the horizontal plain. When the point D is moved to trace a given contour, the point E(end-effector) will trace the similar profile, which is de-scale a factor  $k$  ( $k=AD/AE$ ). This is analogous to inverse of RPP (Revolute-2prismatic robotic arm) serial manipulator kinematically [1].

Since it is very important to keep the orientation of end-effector steady in practical tasks, a parallelogram mechanism is used. In 3-DOF, parallel manipulator, the transmission of motion and power is in the parallel form. Hence, the joint errors are not cumulative in series and the position accuracy is improved.

## 4. Theoretical Formulation

The kinematics of the pantograph can be defined as: Given the length, velocity and acceleration of the links of pantograph mechanism, compute the position, velocity and acceleration of the milling cutter mounted on one of the link. The coordinate frame OXY is defined in the diagram. O lies at midpoint of line joining A & C, and X axis is along the line joining A & C, Y-axis is perpendicular to AC.



Based on the geometrical analysis of the mechanism, position equations are developed.

Let  $L_a$ ,  $L_b$ ,  $L_c$  are respectively the length of the link AD, DC and distance joining A & C. and  $\beta$  = angle between  $L_c$  and  $L_a$ .  $S_{Ex}$ ,  $S_{Ey}$  are the coordinate position of cutter(point E) along X and Y axis respectively. Amplification factor is defined by  $k = AD/AE$ .

$$\Phi = \cos^{-1} [(L_a^2 + L_c^2 - L_b^2) / (2 \cdot L_b \cdot L_c)]$$

$$\begin{cases} S_{Ex} = k \cdot (L_a \cdot \cos\Phi + L_c/2) - (k-1) \cdot A_x \\ S_{Ey} = k \cdot (L_a \cdot \sin\Phi) \end{cases}$$

$$S_E = \sqrt{(S_{Ex})^2 + (S_{Ey})^2}$$

Thus, we get

$$S_E = k \cdot [ (L_a)^2 + (L_c/2)^2 - 2 \cdot L_a \cdot L_c \cdot \cos\Phi + (1-1/k)^2 \cdot (A_x)^2 - 2 \cdot (1-1/k) \cdot A_x \cdot (L_a \cdot \cos\Phi + L_c/2) ]^{1/2}$$

Where  $A_x$  be the X-coordinate position of point A.

Let  $V_a$  and  $V_b$  is the velocity of link  $L_a$  and  $L_b$  respectively. Thus by differentiating the position equations, we can obtain the velocity equations.

$$\begin{cases} V_{Ex} = k \cdot [ V_a \cdot \cos\Phi - L_a \cdot \sin\Phi \cdot \omega ] \\ V_{Ey} = k \cdot [ V_a \cdot \sin\Phi + L_a \cdot \cos\Phi \cdot \omega ] \end{cases}$$

$$V_E = \sqrt{(V_{Ex})^2 + (V_{Ey})^2}$$

$$V_E = k \cdot [ (V_a)^2 + (L_a)^2 \cdot (\omega^2 \cdot \sin^2\Phi + \cos^2\Phi) + (1-\omega) \cdot V_a \cdot L_a \cdot \sin 2\Phi ]^{1/2}$$

Where  $V_E$  ( $V_{Ex}$ ,  $V_{Ey}$ ) are velocity of point E and  $\omega$ , angular velocity of link AD is constant ( $\omega=1.12$  rad/s)

Let  $a_a$ ,  $a_b$  denotes acceleration of AD & DC respectively. So, by differentiating velocity equations we get:

$$\begin{cases} a_{Ex} = k \cdot ( a_a \cdot \cos\Phi - 2 \cdot \omega \cdot V_a \cdot \sin\Phi - L_a \cdot \omega^2 \cdot \cos\Phi - \alpha \cdot L_a \cdot \sin\Phi ) \\ a_{Ey} = k \cdot ( a_a \cdot \sin\Phi + 2 \cdot \omega \cdot V_a \cdot \cos\Phi - L_a \cdot \omega^2 \cdot \sin\Phi + \alpha \cdot L_a \cdot \cos\Phi ) \end{cases}$$

$$a_E = \sqrt{(a_{Ex})^2 + (a_{Ey})^2}$$

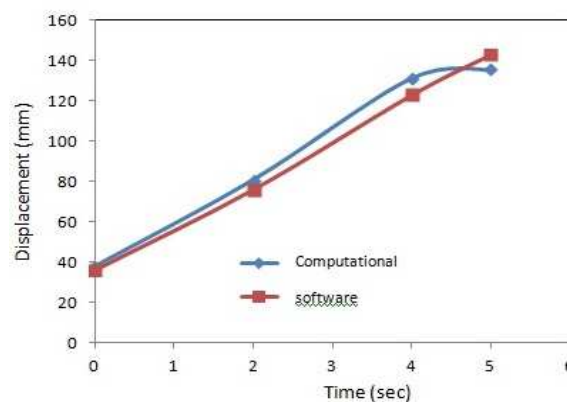
and

$$a_E = k \cdot [ (a_a)^2 + (2 \cdot \omega \cdot V_a)^2 + (\omega^2 \cdot L_a)^2 + (\alpha \cdot L_a)^2 + 2 \times (2 \cdot \omega \cdot \alpha \cdot L_a \cdot V_a - \omega^2 \cdot L_a \cdot a_a) ]^{1/2}$$

where  $a_E$  is the linear acceleration of point E with angular acceleration,  $\alpha=1.07$  rad/s<sup>2</sup>.

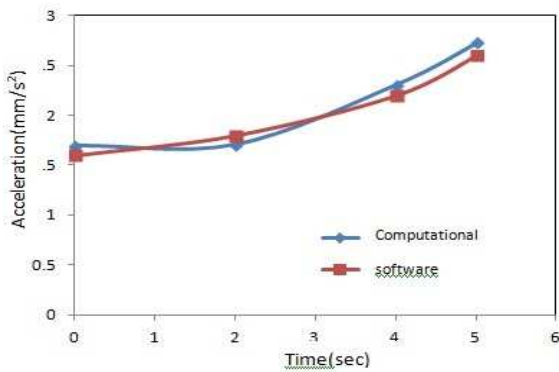
For DOF(degree of freedom) parallel mechanism system, in this work with the help of Wang-Hongguang et al(2003) equations, the equations for pantograph are developed for position, velocity and acceleration of links.

### RESULTS AND DISCUSSION

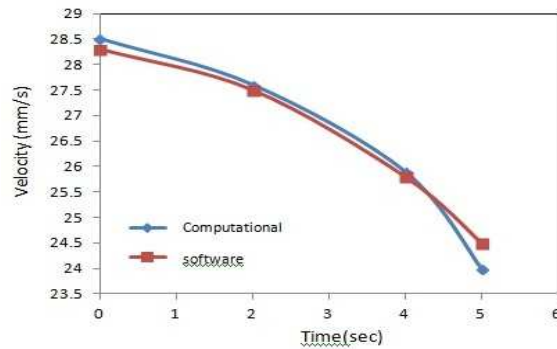


For this pantograph, given parameters are  $AD = L_a = 26.90$  cm,  $DC = L_b = 40.90$  cm,  $AB = 39.40$  cm,  $BC = 24.30$  cm and thus de-scaling factor  $k=AD/AE=0.8$ .

The kinematic formulation results are compared with the results obtained by using solid works software and the graphs are plotted as shown in figure (a), (b) and figure (c).



b) The Velocity v/s time plot for end effector



c) The linear acceleration v/s time plot of end effector

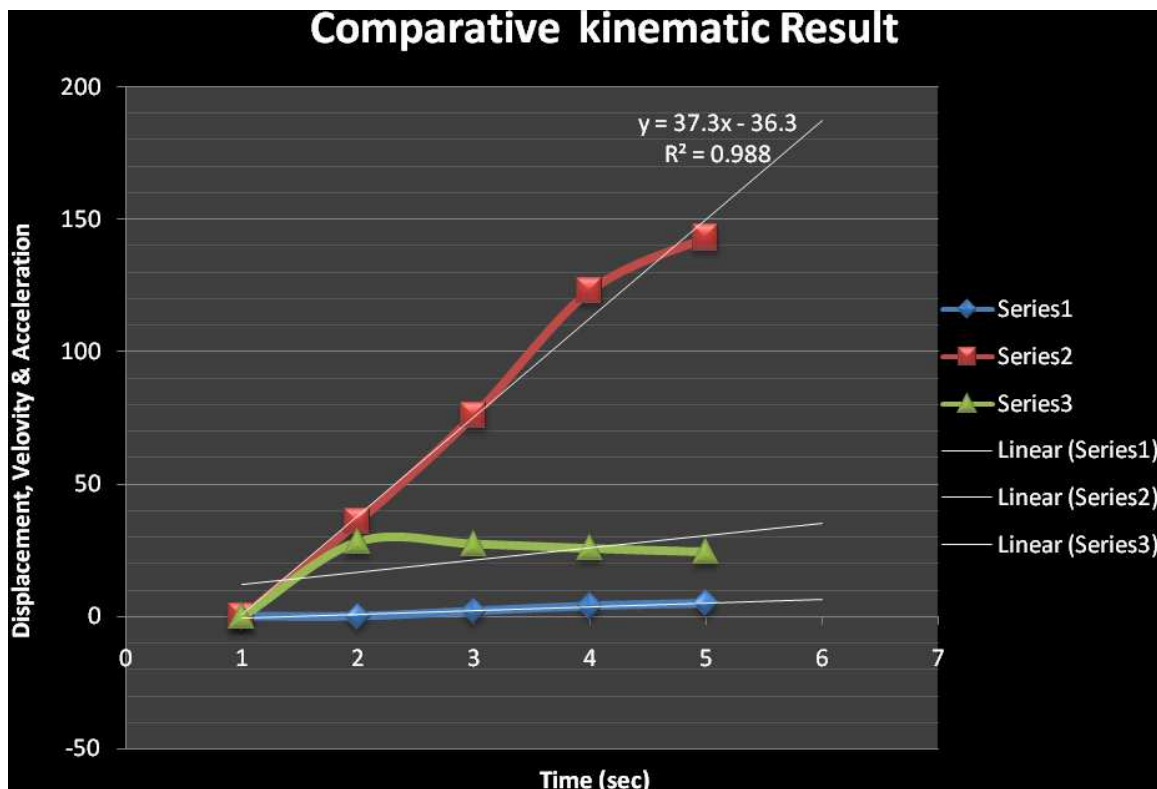


Fig.d (Comparative Kinematic Result)

The results are compared for a time interval of 0-5 seconds. The nature of the displacement v/s time curve for the end effector of pantograph mechanism is increasing approximately linearly as shown in fig. (a).

While, end effector's velocity with respect to time decreases parabolically provided constant angular velocity ( $\omega=1.12$  rad/s) and the result is depicted by fig. (b).

Linear acceleration of end effector with respect to time increases while the angular acceleration is kept nearly constant ( $\alpha=1.07$  rad/s<sup>2</sup>) and the plot is shown in fig. (c).

On the basis of above parameters, in this work with the help of Solid Works software, different parameters such as change in angle( $\Phi$ ), Displacement( $S_E$ ), Velocity( $V_E$ ) and acceleration( $a_E$ ) are nearly same within  $\pm 6\%$  errors.

The value of above parameters such as  $\Phi$ ,  $S_E$ ,  $V_E$ , and  $a_E$  can be find out by mathematical equations which are developed in "The Mechanism and Kinematics of a Pantograph Milling Machine". The graphical comparisons proved the validity of this study.

The nature of Displacement v/s time graph is linearly increasing (approximately) and the generalized equation of this line is of form " $y=m.x+c$ " where time is on x-axis and displacement on y-axis. It is clearly depicted in the cumulative graph.

By differentiating the displacement w.r.t. time, velocity plot is coming out to be nearly constant. But due to some experimental errors involved, there is slight variation in the nature of velocity v/s time plot. Similarly, As the velocity is coming out to be constant, acceleration have to be zero.

Within all the experimental errors which is nearly  $\pm 6\%$  of the software results, the cumulative comparison of these analysis has been plotted in fig.d and are found to be acceptable.

Hence, we can conclude that the computational formulation for this pantograph milling machine are valid and can be further used for any other kind of machines working on the same principle.

## CONCLUSION

The values of these parameters ( $\Phi$ ,  $S_E$ ,  $V_E$ , and  $a_E$ ) calculated by Solid Works software and by mathematical equations computational technique are of same order. Therefore, the theoretical formulation of pantograph is valid.

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