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Vacuum fluctuation in quantum theory and analogy with classical and non-physics cases

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ABSTRACT

Several problems in quantum optics in which spontaneous emission or vacuum fluctuations play can be found and discussed. However there are some classical examples and also some non physical cases can be found which are quite analogues to zero point fluctuation or vacuum fluctuation. In the present work we investigate such cases for the sake of analogy only. Examples are triode oscillator, tetrode oscillators, heartbeat; build up of longitudinal growth of biological samples. Another example is the growth of dimensionless intensity in a Laser cavity. We have discussed about fifty such cases where fluctuations are involved. It may however be noted that the vacuum fluctuation is the outcome of a fully quantized theory of radiation. And there is no actual analogy that exists in classical or semi-classical field.

Keyword: Quantum optics, vacuum fluctuation, spontaneous emission, Laser cavity

INTRODUCTION

An interesting consequence of the quantization of radiation in quantum theory is the fluctuations associated with zero-point energy or the so called vacuum fluctuations. These fluctuations have no classical analogy and are responsible for many interesting phenomena in quantum optics. It is worthwhile to note that a semi classical theory of atom field interaction in which only the atom is quantized and the field is treated classically can explain many of the phenomena which we observe in modern optics. As for example the semi classical theory of laser as formulated by Lamb[1,2] can explain a large number of laser behaviours particularly in the gaseous phase. Spatial hole burning and Lamb Dip are few important examples. But quantization of radiation field is needed to explain effects such as spontaneous emission, the Lamb shift, the laser line width, the Casimir effect and full photon statistics of laser. In fact each of these effects can be understood from the point of view of vacuum fluctuations perturbing the atom. The existence of irreducible field fluctuations in vacuum is an important prediction of quantum theory. In the present work we consider analogous situations in classical physics and to some extent in some non physics contents where fluctuations at the initial level are responsible for building up oscillations until they reach the stage of saturation. Nature provides us with numerous situations where these fluctuations may be found. At present one really does not know how these fluctuations are responsible for building up oscillations in a particular system. We consider few such cases for our discussion.

2. Van der Pole’s triode oscillation:

Van der Pole obtained an equation of motion in his treatment of the triode oscillator as

$$\ddot{v} - d/dt (a V - \beta v^3) + w^2 v = 0 \tag{1}$$

Here a is the Linear net gain (i.e. gain in excess of losses), β' is the saturation co-efficient and w is the resonance frequency in the absence of dissipation or gain. Equation (1) may be solved approximately by an important technique called the method of slowly varying amplitudes. Writing

$$v(t) = 1/2 V(t) \exp(-i\omega t) + c.c \tag{2}$$

Where $V(t)$ is the amplitude. The slowly varying equation of motion is obtained as

$$V(t) = 1/2 V(t) (a - \beta V^2) \tag{3}$$

For sufficiently large $V(t)$ saturation sets in and the steady state condition is

$$V(t) = a/\beta$$

It may be noted that equation (3) does not build up from Zero amplitude .In practice some fluctuations gets things going.

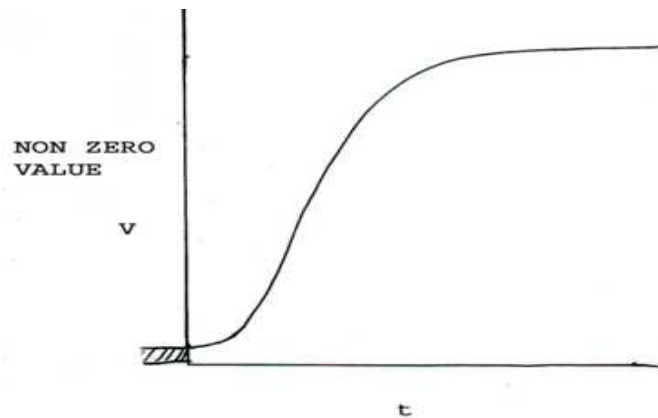


Fig.1 Nature of buildup of voltage in Vander-Pol Oscillation

It may be noted that the type of fluctuations indicated in this case is not clearly defined.

3 .Semi classical theory of laser and build up of dimensionless intensity

Semi classical theory of laser as formulated by Lamb (1, 2) has explained a large number of laser behaviours. Those who have studied this theory must have come across some important parameters associated with the theory. As for example the parameter known as “dimensionless intensity” as an important parameter which appears in the semi classical theory.

The dimensionless intensity equation is given by

$$\dot{I}_n = 2 I_n (a_n - \beta_n I_n) \tag{5}$$

Where the term

- Linear gain $a_n = \int (\omega - \nu_n) - 1/2 \nu/Q_n$
- Saturation $\beta_n = \int^2 (\omega - \nu_n) F_3$
- Third order factor $F_3 = 2/3 \gamma_{ab} / \gamma F_1$
- First order factor $F_1 = 1/2 \nu \sum^2 (\epsilon_0 \hbar \gamma)^{-1} \bar{N}$

$\bar{N} = \sum^* V_{ab}$ is the dipole moment matrix element $\gamma_{ab} = 1/2 (\gamma_a + \gamma_b)$, Where γ_a is the decay rate from the level **a** and γ_b is the decay rate from the level **b**

Equation (5) can be integrated with result

$$I_n(t) = \frac{a_n \left[\frac{I_0}{a_n - \beta_n I_0} \right] \exp(2a_n t)}{1 + \beta_n \left[\frac{I_0}{a_n - \beta_n I_0} \right] \exp(2a_n t)} \quad (6)$$

Where $I_0 = I_n(0)$. The buildup of dimensionless intensity shown by this equation indicates that this yields exponential gain

$I_n(t) = I_0 \exp(2 a_n t)$ for small values of I_0 and as $t \rightarrow \alpha$ it yield steady state result

$$I_n = a_n / \beta_n$$

The buildup of dimensionless intensity I_n for some value may be plotted. I_n for some value may be plotted. In the following Fig 2 we indicate this graph for $a_n / \beta_n = 0.75$ and $I_n(0) = I_0 = 0.01$

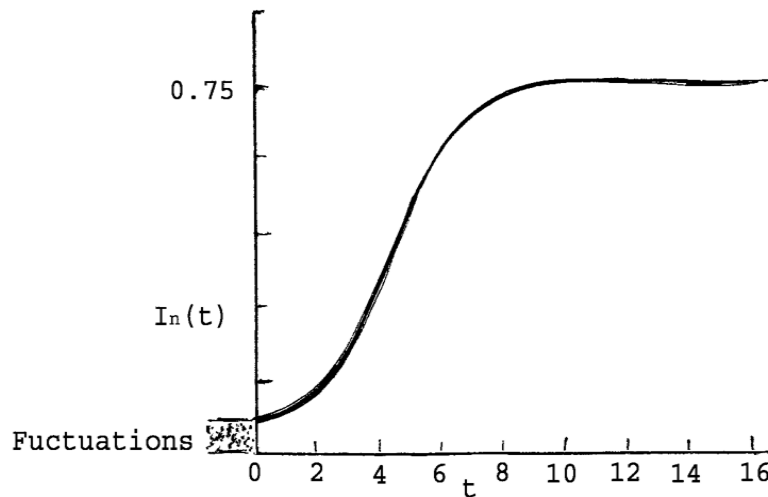


Fig.2 Build up of dimensionless intensity

The peculiarity of equation (6) is that the buildup of intensity never takes place from zero value. According to Willis Lamb Jr.(2) some sort of fluctuations get things going.

4. Non Physics Content

We know turn our attention to widely occurring phenomenon, that is, the growth of biological species. The buildup of dimensionless intensity appearing in equation (6) show up also in non physics contents. There are numerous examples of this type and we have brought under our investigation only about twenty such cases. One of the example is the growth of a well known bacteria known as E-coli. Bacteria can be grown in a liquid medium or on a solid surface. The population growing in a liquid medium is called bacterial culture. A culture is initialed by placing a small amount of bacteria on an inoculums into sterile medium in a tube. If the liquid is a comlex extract of biological material it is called a “froth”. When bacteria are inoculated in a liquid medium they slowly start to grow and divide. After an initial period of very slow growth called the lag phase, they begin a period of rapid growth in which they divide at a fixed time interval called the doubling time. The number of cells per millimeter, the cell density doubles repeatedly giving rise to a logarithmic increase in cell number. This stage of growth of bacterial culture is called the exponential phase. This stage of growth continues until a cell density for E-coli of at most 10^9 cells/ml is reached ,at which point nutrients and O_2 become limiting and the growth rate decreases. Ultimately at a cell density of 2 to 3×10^9 cells/ ml , no further growth is possible and the cell number becomes constant. This stage is called the stationary phase. A typical growth curve is shown in Fig.3. It is to be noted that the exponential growth does not take place from zero value. This is similar to the situation in the growth of the dimensions intensity in the semi classical theory of laser or the growth of voltage in the triode oscillator circuit given by Vunder pol (3). This is the initial fluctuation we are discussing in the present work.

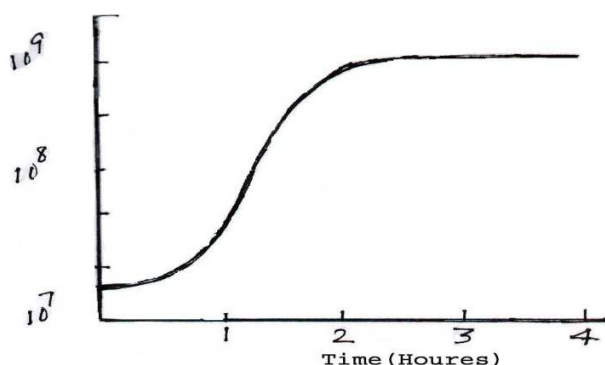


Fig.3 Growth of bacteria (E-Coli) estimated by bacterial culture. (Ref 4)

An extensive investigation of various biological samples has been undertaken and in all the cases of investigations exponential growth in the initial state and saturation in the latter stage have been observed. The characteristic features of these observations are that they all grow from a non zero value. This is termed as fluctuation in the language of Willis Lamb Jr.(2). In the macroscopic world there are thousands of such examples. Indeed it is an universal phenomenon. It is responsible for the visible phenomena exhibiting exponential growth and saturation. The answer is Yes in the sense that this non zero value is a part of the entire curve. The mathematical formulation in the semi classical theory of laser shows that in the buildup of the dimensionless intensity I_n the initial value is $I_n(0) \neq 0$, It may assume any value in the lower scale like .001, .01 or .1,etc. The time scale is also arbitrary. The buildup of dimensionless intensity is concerned with a Fabry Perot Cavity. It is not possible to build up a theory for non physics content. But looking at the similarity of all the biological and other systems at is possible to make some bold assumptions regarding the existence of fluctuations at the initial stages.

CONCLUSION

Vacuum fluctuations arises as a result of quantization of electromagnetic field and responsible for many important phenomena in physics like spontaneous emission, Casimir effect, Lamb shift and even there is a speculation that birth of the universe is the result of large scale vacuum fluctuation. Following similar argument we can speculate that Biological Universe in the macroscopic level is the result of fluctuations. We are macroscopic species living in a quantum universe.

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