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$\gamma\gamma$ and $g\,g$ decays of χ_{c0} and χ_{c2}

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ABSTRACT

We present a calculation of the two-photon and the two-gluon decay widths of P-wave charmonia χ_{c0} and χ_{c2}. Using the phenomenological relativistic harmonic model for quarks we obtained the masses of the states χ_{c0} and χ_{c2}. The full Hamiltonian used in the investigation has Lorentz scalar plus vector confinement potential, along with the confined one gluon exchange potential (COGEP). The COGEP consists of a central part, a spin-orbit part and a tensor part. To obtain the masses the Hamiltonian matrix is constructed in the basis of harmonic oscillator states and diagonalized. The parameters and the wave function used for the prediction of the mass spectrum are used to calculate their partial decay widths.
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Key Words: Relativistic harmonic model, Confined-one-gluon-exchange potential, Two-photon and Two-gluon decay widths.

INTRODUCTION

The study of heavy quarkonium systems has played an important role in the development of quantum chromodynamics (QCD). There are different models in literature which describes the quarkonium spectra [1- 8]. The prediction of mass spectrum in accordance with the experimental results doesn't guarantee the validity of the model for describing hadronic interactions. This is because different potentials have been proposed which reproduce the same spectra. Therefore using the model, one must be able to calculate other observables like the radiative decay widths, the leptonic decay widths, the two-photon decay widths, etc. The leptonic decay widths are a probe of the compactness of the quarkonium system and provide important information complementary to level spacings [9]. The decay of a heavy quark-antiquark pair into final states involving leptons, photons and light quarks can provide useful information on the strong coupling constant (α_s) [10, 11]. Heavy quarkonium decays provide a deeper insight on the exact nature of the inter-quark forces and decay mechanisms. In the present work we have calculated the two-photon and the two-gluon decay widths of ground state P wave charmonia χ_{c0} and χ_{c2}. In Sec. 2, we give a brief description of RHM and COGEP. The parameters used in our model are dicussed. A brief description of the two-photon and two-gluon decay widths are also given. The results of the calculations are presented in Sec. 3. Conclusions are given in Sec. 4.

THE MODEL

In our present work, we have used the relativistic harmonic model (RHM) [12] along with the confined one gluon exchange potential (COGEP) [13] to calculate the masses of the states χ_{c0} and χ_{c2}. RHM was highly successful in explaining various properties of hadrons: light meson spectrum, baryon magnetic moments, N-N scattering phase shifts, etc. [14-16]. In RHM quarks are Dirac particles subjected to Lorentz scalar plus vector potentials. A pure vector potential would produce only $Q\overline{Q}$ bound states, whereas a scalar potential provides an attractive force for both $Q\overline{Q}$ and QQ states. Thus, for the confinement of quarks, a scalar plus vector potential is a more appropriate choice. In RHM [12], quarks in a hadron are confined through the action of Lorentz scalar plus a vector harmonic oscillator potential,

$$V_c = \frac{1}{2}(1+\gamma_0)A^2r^2 + M \qquad (1)$$

where γ_0 is the Dirac matrix, M is the quark mass and A^2 the confinement strength. The energy eigenvalue of the single particle Dirac equation with the interaction potential given by Eq. (1) is given by [12, 16]:

$$E_n^2 = M^2 + (2n+1)\Omega_n^2,$$

where Ω_n is an energy dependent parameter,

$$\Omega_n^2 = A(E_n + M)^{1/2}$$

The total energy of the hadron is obtained by adding the individual contributions of the quarks. The quark-antiquark interaction potential is given by the confined one gluon exchange potential (COGEP) [13]. The essential new ingredient in our investigation of the mesonic states is to take into account the confinement of gluons. For the confinement of gluons we have made use of the current confinement model (CCM) which was developed in the spirit of the RHM [17, 18]. The CCM has been quite successful in describing the glue-ball spectra. The confined gluon propagators (CGP) are derived in CCM. Using CGP the confined one gluon exchange potential (COGEP) was obtained [13].

$$V_{COGEP} = V_{COGEP}^{cent} + V_{COGEP}^{L.S} + V_{COGEP}^{Ten}$$

The central part of COGEP is:

$$V_{COGEP}^{cent} = \frac{\alpha_s}{4}N^4\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j[D_0(r) + \frac{1}{(E+M)^2}[4\pi\delta^3(r) - c^4r^2D_1(r)][1-\frac{2}{3}\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j]]$$

Here the first term is the residual Coulomb energy and the second and the third are the chromomagnetic interaction leading to the hyperfine splittings. $D_0(r)$ and $D_1(r)$ are propagators of the CCM. The spin-orbit part of COGEP is given by:

$$V_{COGEP}^{L.S} = \frac{\alpha_s}{4}\frac{N^4}{(E+M)^2}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\frac{1}{2r}[\{(r\times(p_i-p_j)\cdot(\sigma_i+\sigma_j))(D_0^{'}(r)+2D_1^{'}(r))\} + \{(r\times(p_i+p_j)\cdot(\sigma_i-\sigma_j))(D_0^{'}(r)-D_1^{'}(r))\}]$$

The tensor part of COGEP is given by:

$$V_{COGEP}^{Ten} = -\frac{\alpha_s}{4}\frac{N^4}{(E+M)^2}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \left[\frac{D_1^{''}}{3} - \frac{D_1^{'}}{3r}\right]\hat{S}_{ij},$$

where $\hat{S}_{ij} = [3(\sigma_i \cdot \hat{r})(\sigma_j \cdot \hat{r}) - (\sigma_i \cdot \sigma_j)]$. Both spin-orbit and tensor forces affect states with $L > 0$. The spin-orbit and the tensor terms describe the fine structure of the states while the spin-spin term gives the spin singlet-triplet splittings. The $Q\overline{Q}$ wave function for each meson is expressed in terms of oscillator wave functions corresponding to the CM and relative coordinates. The oscillator quantum number for the CM wave function is restricted to $N_{CM} = 0$. The Hilbert space of relative wave functions is truncated at radial quantum number $n = 5$. The Hamiltonian matrix is constructed in the basis $|N_{CM} = 0, L_{CM} = 0; n\ {}^{2S+1}L_J>$ and diagonalized. The diagonal values give the masses of the ground and the radially excited states.

The ${}^1S_0$, ${}^3P_0$ and ${}^3P_2$ levels of charmonium and the upsilon system can decay into two photons. These same states can also decay into two gluons, which accounts for a substantial portion of the hadronic decays for states below the c$\bar{c}$ or b$\bar{b}$ threshold [19].

The decays of the positive C-parity states $({}^3P_J)$ into two photons are of significance because at the lowest order this decay is a pure QED process similar to the two-photon decay of positronium. Their study can shed light on higher order relativistic and QCD radiative corrections [20]. The decay width for the scalar state χ_{c0} and the tensor state χ_{c2} to decay into photons is given by [10]:

$$\Gamma_{\chi_{c0}\to\gamma\gamma} = \frac{27e_Q^4\alpha^2}{m_Q^4}|R_P^{'}(0)|^2\,[1+0.2\alpha_s/\pi] \quad (2)$$

$$\Gamma_{\chi_{c2}\to\gamma\gamma} = \frac{36e_Q^4\alpha^2}{5m_Q^4}|R_P^{'}(0)|^2\,[1-16\alpha_s/3\pi] \quad (3)$$

where $e_q = 2/3$ is the quark charge, α is the QED coupling constant, m_Q is the quark mass, α_s is the QCD coupling constant and $|\psi(0)|$ is the meson wave function calculated at the origin. The term in the parenthesis is the first order QCD correction factor. The decay width for two-gluon decay is given by [10]:

$$\Gamma_{\chi_{c0}\to gg} = \frac{6\alpha_s}{m_Q^4}|R_P^{'}(0)|^2\,[1+9.5\alpha_s/\pi] \quad (4)$$

$$\Gamma_{\chi_{c2}\to gg} = \frac{8\alpha_s^2}{5m_Q^4}|R_P^{'}(0)|^2\,[1-2.2\alpha_s/\pi] \quad (5)$$

The mass of the quark m_Q, α_s and the parameters of the quarkonium wave function are fixed from the quarkonium spectrum. In our work $m_c = 1.55$ GeV and $\alpha_s = 0.2$. The parameters of the COGEP are the same as used in [21, 22]. The model parameters and the radial wave functions that have been used to obtain the masses are used to calculate the partial decay widths.

RESULTS AND DISCUSSIONS

The triplet 1P states of charmonium (χ_{cJ}) were first observed in the radiative decay of the $\psi(2S)$ produced through the e^+e^- collisions: $e^+e^- \to \psi(2S) \to \gamma + \chi_{cJ}$. Later the accurate mass measurements of these states were determined in the $p\bar{p}$ collisions [23-25]. The masses of the charmonium states χ_{c0} and χ_{c2} after diagonalization in harmonic oscillator basis are 3415 MeV and 3506 MeV respectively. The PDG [26] values are 3415 MeV and 3556 MeV respectively. The obtained mass agree well with the experimental results. Expressing the meson state by harmonic oscillator wave function, the two-photon and the two-gluon decay widths are calculated using eqns.(2-5). For P states the wave function vanishes at the origin and therefore the partial widths for the two-photon decay and two-gluon decay are proportional to the square of the derivative of the wave function at the origin. Experimental determination of the two photon widths of the χ_{cJ} states of charmonium in their decay into two photons have been reported by the CLEO Collaboratio [27]. From their measurements they obtained the width $\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.36 \pm 0.35 \pm 0.22)$ keV and $\Gamma_{\gamma\gamma}(\chi_{c2}) = (0.66 \pm 0.07 \pm 0.06)$ keV, where the first error is statistical, second is systematic and the third is due to the PDG parameters used. From our calculation we obtain $\Gamma_{\gamma\gamma}(\chi_{c0}) = 5.7$ keV and $\Gamma_{\gamma\gamma}(\chi_{c2}) = 2.8$ keV. The calculated widths are higher than the experimental results. The calculated value of the ratio $R_{\gamma\gamma} = \Gamma_{\gamma\gamma}(\chi_{c2})/\Gamma_{\gamma\gamma}(\chi_{c0})$ is 0.48 while the experimental value is $0.28 \pm 0.05 \pm 0.04$. The calculated widths for the two-gluon decays are $\Gamma_{gg}(\chi_{c0})$ =7.71 MeV and $\Gamma_{gg}(\chi_{c2})$ = 1.10 MeV, and the experimental values are $\Gamma_{gg}(\chi_{c0})$ =10.20 MeV and $\Gamma_{gg}(\chi_{c2})$ = 2.03 MeV. The obtained two-gluon widths agree reasonably well with the experimental values. The results are listed in Table 1 in comparison with experimental and other theoretical values.

Table 1: Decay Widths

Decay	Present	Exp.	[19]
$\chi_{c0} \to \gamma\gamma$	5.7 keV	2.36 keV	5.35 keV
$\chi_{c2} \to \gamma\gamma$	2.8 keV	0.66 keV	1.55 keV
$\chi_{c0} \to gg$	7.71 MeV	10.20 MeV	4.88 MeV
$\chi_{c2} \to gg$	1.10 MeV	2.03 MeV	0.69 MeV

CONCLUSIONS

In the present work we have calculated the two-photon and two-gluon decay widths of the P wave charmonium states χ_{c0} and χ_{c2}. The masses of the charmonium states were obtained in the frame work of RHM along with COGEP. The results are in good agreement with the experiment both for the masses and their partial decay widths. This work could be extended for other heavy meson systems.

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