*Available online a*t www.scholarsresearchlibrary.com

Scholars Research Library

Archives of Physics Research, 2013, 4 (2):87-96
(http://scholarsresearchlibrary.com/archive.html)

Scholars Research Library

ISSN : 0976-0970
CODEN (USA): APRRC7

Propagation of electromagnetic waves in curved triangular waveguide

Pravin R. Chaudhari

Microwave Research Laboratory, Z. B. Patil College, Dhule, India

ABSTRACT

The curved waveguides of various cross sections are very useful in building a waveguide system and are required in many communication applications, especially in satellite communications. While designing curved waveguides it is necessary to take care about exact position of axis and angle of bending for proper distribution of electric and magnetic fields. So that electromagnetic waves can propagate through waveguide in the desired manner. The problem of propagation of dominant TE mode in curved waveguide of right isosceles triangular cross-section is analyzed up to third order perturbation terms. The cylindrical coordinate system is used for mathematical formulation of the problem and perturbation technique is used in obtaining the solution. The information about the percentage contribution up to third order terms is worked out. It is shown that the propagation of electromagnetic waves through triangular waveguide and its phase velocity depends on the sense of bending of waveguide and frequency.

Keywords: Triangular waveguide, curved waveguide, perturbation technique.

INTRODUCTION

Waveguide is normally rigid and therefore it is often necessary to direct the waveguide in a particular direction. Using waveguide bends and twists it is possible to arrange the waveguide into the positions required. Regular straight hollow waveguides have phase velocities greater than the free-space speed of light for propagating electromagnetic waves. Conventional slow wave structures used for accelerating charged particles and other applications employ reactive loadings in hollow straight waveguides to reduce the phase velocity of electromagnetic fields in the specific mode to be used.

The problem of propagation of electromagnetic waves in curved rectangular waveguide has been solved L. Lewin [1,2] He used perturbation method and obtained formula for guide wavelength and showed that how radius of curvature of the waveguide affects the propagation is independent on the sense of bending of the guide.

The propagation of electromagnetic waves in curved structures has also been studied by Lewin, Chang and Kuester [3]. They solved this problem by starting with two coupled modes and showed that the combination of the two modes at any cross-section involves a phase change depending on the radius of curvature.

Various methods for the analysis of curved waveguides have been studied in the literature. The propagation of general-order modes in curved rectangular waveguide examined by using asymptotic expansion method [4]. Several methods of investigation of propagation were developed for study of empty curved waveguide and bends [5-8]. Rectangular waveguides with curved corners are analyzed using super quadric functions [9]. A novel high order finite difference method is introduced for optical waveguides with smoothly curved perfectly electric conducting boundaries [10]. The propagation of dominant mode in twisted waveguide of right isosceles triangular cross-section is analyzed by using helical co ordinate system and perturbation technique [11].

A good design of a curved waveguide is intended to limit the pure radiation losses and the transition losses between the straight and the bent waveguides. Not properly designed bends may lead to the excitation of higher-order leaky modes in the plane of the bend. Accordingly waveguide bend and waveguide twist sections are manufactured specifically to allow the waveguide direction to be altered without unduly destroying the field patterns and introducing loss.

Here we analyzed the propagation of dominant mode in curved waveguide of right isosceles triangular cross-section. The cylindrical co-ordinate system is used for the mathematical formulation of the problem and perturbation technique is used in obtaining the solution. The problem is worked out up to third order perturbation terms and obtained solutions. The effect of radius of curvature, on the propagation properties of a wave, is then studied with the use of these solutions. This study will help in designing the lossless curved waveguides of triangular section.

MATHEMATICAL FORMULATION

1. The wave Equation:

Figure 1 show the isosceles right triangular waveguide of dimension ‘a’ curved about the guide axis. The cylindrical co-ordinates ρ, ∅, z, are replaced by a new co-ordinate system (x’, y’, z’). The relation between these two co-ordinate systems is given by

$$\left.\begin{aligned} x' &= z \\ y' &= \rho - R \\ \text{and } z' &= R\emptyset \end{aligned}\right\} \qquad \text{.........1.1}$$

Where R is radius of curvature of guide axis.

The wave equation in cylindrical co-ordinate system is

$$\rho^2 \frac{\partial^2 \Psi}{\partial \rho^2} + \rho \frac{\partial \Psi}{\partial \rho} + \frac{\partial^2 \Psi}{\partial \phi^2} + \rho^2 \frac{\partial^2 \Psi}{\partial z^2} + \rho^2 K^2 \Psi = 0 \qquad \text{.........1.2}$$

Using transformation equations in equation 1, the wave equation 2 can be written in terms of x’, y’ and z’ as

$$\left(1+\frac{y\prime}{R}\right)^2 \frac{\partial^2 \Psi}{\partial x\prime^2} + \left(1+\frac{y\prime}{R}\right)^2 \frac{\partial^2 \Psi}{\partial y\prime^2} + \left(1+\frac{y\prime}{R}\right)^2 . \frac{1}{R} . \frac{\partial \Psi}{\partial y\prime} + \frac{\partial^2 \Psi}{\partial z\prime^2} + \left(1+\frac{y\prime}{R}\right)^2 K^2 \Psi = 0 \qquad \text{.........1.3}$$

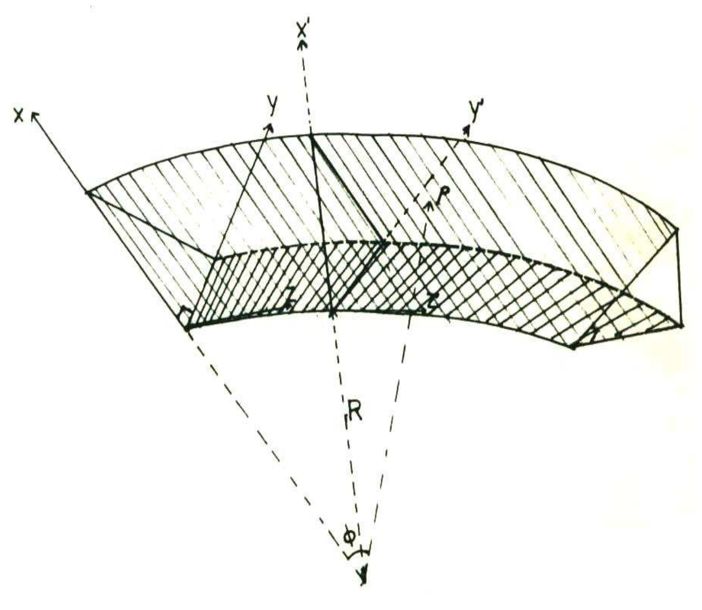


Figure 1: Isosceles Right Triangular Waveguide

The boundary conditions for triangular waveguide are

$$\left.\begin{array}{ll} \frac{\partial\Psi}{\partial x'} = & at\ x' = \ 0 \\ \frac{\partial\Psi}{\partial y'} = & at\ y' = \ 0 \\ \frac{\partial\Psi}{\partial x'} + \frac{\partial\Psi}{\partial y'} & at\ x' + y' = a \end{array}\right\} \quad \ldots\ldots\ldots 1.4$$

2. Zeroth Order Solutions:
We assume a solution Ψ of equation (1.3), of the form

$$\Psi = \left(\Psi_0 + \frac{\Psi_1}{R} + \frac{\Psi_2}{R^2} + \ \ldots\ldots\right) Ae^{-j\beta z\prime} \quad \ldots\ldots\ldots 2.1$$

Where

$$\beta^2 = \mathrm{K}'^2 \left(1 + \frac{B_1}{R} + \frac{B_2}{R^2} + \frac{B_3}{R^3 +} \ldots\ldots\right) \quad \ldots\ldots\ldots 2.2$$

$$\mathrm{K}'^2 \ = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2} \quad \ldots\ldots\ldots 2.3$$

B_1, B_2, B_3 ……… are the unknowns to be determined.

The zeroth order equation is obtained by substituting equations 2.1 and 2.2 into wave equation 1.3 and retaining the terms of order $1/R^0$.

$$\frac{\partial^2\Psi_0}{\alpha x'^2} + \frac{\partial^2\Psi_0}{\alpha y'^2} + \left(\frac{\pi}{a}\right)^2 \Psi_0 = 0 \quad \ldots\ldots\ldots 2.4$$

Solving equation for Ψ_0 and applying boundary conditions in equation 1.4, the solution obtained is of the form

$$\Psi_0 \ = cos\left(\frac{\pi x'}{a}\right) - \ cos\left(\frac{\pi y'}{a}\right) \quad \ldots\ldots\ldots\ldots 2.5$$

To study the effect of bending, it is necessary to obtain the solutions of higher order terms.

3. First Order Solution:
Substitute equations (3.1) and (3.2) into wave equation (1.3), and examine the terms of order 1/R, we get first order equation.

$$\frac{\alpha^2\Psi_1}{\alpha x'^2} + \frac{\alpha^2\Psi_1}{\alpha x'^2} + \left(\frac{\pi}{\alpha}\right)^2 \Psi_1 = F\,(x', y') \quad \ldots\ldots\ldots\ldots 3.1$$

Where

$$F(x', y') = \ 2y'\left(\frac{\partial^2\Psi_0}{\partial x'^2} + \frac{\partial^2\Psi_0}{\partial x'^2}\right) - \ \frac{\partial\Psi_0}{\partial y'} - 2y'K^2\Psi_0 + K'^2 B_1\Psi_0 \quad \ldots\ldots\ldots\ldots 3.2$$

With the substitution of Ψ_0 from equation (2.5), equation (3.2) becomes

$$\begin{aligned} &F\,(x', y') \\ &= \ -2K'^2 y'\left(cos\left(\frac{\pi x'}{a}\right) - \cos\left(\frac{\pi y'}{a}\right) + K'^2 B_1\, cos\left(\frac{\pi x'}{a}\right) - \cos\left(\frac{\pi y'}{a}\right)\right) \\ &- \left(\frac{\pi}{a}\right)\cos\left(\frac{\pi y'}{a}\right) \end{aligned} \quad \ldots\ldots\ldots\ldots 3.3$$

The most general solution of (3.1) satisfying the boundary conditions (1.4), is

$$\begin{aligned} \Psi_1 = \ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \mathrm{A_{mn}}.\Bigg\{&\cos\left[(\mathrm{m}+\mathrm{n})\frac{\pi \mathrm{x}'}{\mathrm{a}}\right].\cos\left(\frac{\mathrm{n}\pi \mathrm{y}'}{\mathrm{a}}\right) \\ &+ (-1)^{\mathrm{m}}\cos\left[(\mathrm{m}+\mathrm{n})\frac{\pi \mathrm{y}'}{\mathrm{a}}\right].\cos\left(\frac{\mathrm{n}\pi \mathrm{x}'}{\mathrm{a}}\right)\Bigg\} \end{aligned} \quad \ldots\ldots\ldots\ldots 3.4$$

Substituting equation (3.4) into the left side of equation (3.1), we get

$$-\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}.\left[(m+n)^2+n^2-1\right]\left(\frac{\pi}{a}\right)^2\left\{\cos\left[(m+n)\frac{\pi x'}{a}\right].\cos\left(\frac{n\pi y'}{a}\right) + (-1)^m\cos\left[(m+n)\frac{\pi y'}{a}\right].\cos\left(\frac{n\pi x'}{a}\right)\right\} = F(x',y') \quad \ldots\ldots\ldots\ldots 3.5$$

The LHS of above equation vanishes for m = 1 and n = 0, we can find value of B_1, with the usual process of determining coefficients by Fourier analysis [4]. For this, multiply to the right side of equation 5 by $\left[\cos\left(\frac{\pi x'}{a}\right) - cos\left(\frac{\pi y'}{a}\right)\right]$ and equate with zero after integration.

$$\therefore \int_0^a\int_0^{a-x'} F(x',y').\left[cos\left(\frac{\pi x'}{a}\right) - cos\left(\frac{\pi y'}{a}\right)\right]dx'dy' = 0 \quad \ldots\ldots\ldots 3.6$$

Substitute F (x', y') from equation (3.3) and write (3.6), as

$$2K'^2 I_1 - K'^2 B_1 I_2 + \left(\frac{\pi}{a}\right) I_3 = 0 \quad \ldots\ldots\ldots\ldots 3.7$$

Solve integrals I_1, I_2 *and* I_3 one by one by parts and substitute into (3.7), we get value of B_1.

$$B_1 = \left(\frac{2}{3} + \frac{3}{2\pi^2}\right)a + \frac{1}{2aK'^2} \quad \ldots\ldots\ldots\ldots 3.8$$

4. Evaluation of Coefficients A_{mn}:
In order to obtain the coefficients A_{mn}, multiply on both sides of equation (3.5) by

$$\left\{\cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right) + (-1)^m cos\left[(m+n)\frac{\pi y'}{a}\right].cos\left(\frac{n\pi s'}{a}\right)\right\}$$ and integrate.

$$\therefore A_{mn}\left[(m+n)^2+n^2-1\right]\left(\frac{\pi}{a}\right)^2.\int_0^a\int_0^{a-x'}\left\{\cos\left[(m+n)\frac{\pi x'}{a}\right].\cos\left(\frac{n\pi y'}{a}\right) + (-1)^m\cos\left[(m+n)\frac{\pi y'}{a}\right].\cos\left(\frac{n\pi x'}{a}\right)\right\}^2 dx'dy' = -\int_0^a\int_0^{a-x'} F(x',y').$$
$$.\left\{\cos\left[(m+n)\frac{\pi x'}{a}\right].\cos\left(\frac{n\pi y'}{a}\right) + (-1)^m\cos\left[(m+n)\frac{\pi y'}{a}\right].\cos\left(\frac{n\pi x'}{a}\right)\right\}dx'dy' \quad \ldots\ldots\ldots 4.1$$

For simplicity we write this equation as

$$I_L = I_R \quad \ldots\ldots\ldots\ldots 4.2$$

After solving equation for I_L it becomes

$$I_L = A_{mn}\left[(m+n)^2+n^2-1\right].\frac{\pi^2}{4} \quad \ldots\ldots\ldots\ldots 4.3$$

Let us now consider I_R of equation (4.2)

$$I_R = 2K'^2 I_1 - K'^2 B_1 I_2 + \left(\frac{\pi x'}{a}\right) I_3 \qquad \text{… … … … 4.4}$$

Now solve these integrals one by one, by parts and substitute values of I_L and I_R into equation (4.1), we can obtain value of coefficients A_{mn}.

5. Second Order Solution:

With the substitution of equations (2.1) and (2.2) into wave equation (1.3) and examining the terms of order $1/R^2$, we get second order perturbation equation, which is

$$\frac{\partial^2 \Psi_2}{\partial x'^2} + \frac{\partial^2 \Psi_2}{\partial y'^2} + \left(\frac{\pi}{a}\right)^2 \Psi_2 = F\,(x', y') \qquad \text{… … … … 5.1}$$

Where

$$F(x', y') = -\,y'^2\left(\frac{\partial^2 \Psi_0}{\partial x'^2} + \frac{\partial^2 \Psi_0}{\partial y'^2}\right) - y'\,\frac{\partial \Psi_0}{\partial y'} - 2y'\left(\frac{\partial^2 \Psi_1}{\partial x'^2} + \frac{\partial^2 \Psi_1}{\partial x'^2}\right) - \frac{\partial \Psi_1}{\partial y'}\, 2y'\Psi_1 K^2 - y'^2 \Psi_0 K^2 + K'^2 B_1 \Psi_1 + K'^2 B_2 \Psi_0 \qquad \text{… … … … 5.2}$$

The most general solution of equation (5.1), satisfying boundary conditions (1.4), is

$$\Psi_2 = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn.} \left\{ cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right) + (-1)^m\, cos\left[(m+n)\frac{\pi y'}{a}\right].cos\left(\frac{n\pi x'}{a}\right)\right\} \qquad \text{… … … … 5.3}$$

Substitute Ψ_2 from above equation to the left of equation (5.1),

$$-\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} B_{mn.}\,[(m+n)^2 + n^2 - 1]\left(\frac{\pi}{a}\right)^2 . \left\{ cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right) + (-1)^m\, cos\left[(m+n)\frac{\pi y'}{a}\right].cos\left(\frac{n\pi x'}{a}\right)\right\} = F\,(x', y') \qquad \text{… 5.4}$$

On the left, the coefficient of $\left[cos\left(\frac{\pi x'}{a}\right) - cos\left(\frac{\pi y'}{a}\right)\right]$ vanishes when m = 1 and n = 0. Hence by the usual process of determining coefficients by Fourier analysis, we can obtain constant B_2. Thus according to Fourier analysis multiply to the right side of equation (5.4) by $\left[cos\left(\frac{\pi x'}{a}\right) - cos\left(\frac{\pi y'}{a}\right)\right]$ and equate with zero after integration.

$$\therefore \int_0^a\int_0^{a-x} F(x', y')\left[cos\left(\frac{\pi x'}{a}\right) - \cos\left(\frac{\pi y'}{a}\right) dx'dy'\right] = 0 \qquad \text{… … … … 5.5}$$

Substitute F (x’, y’) from equation (5.2) and write equation (5.5), in the simple form as

$$(I_1 + I_2) + I_3 + 2\,(I_4 + I_5) + I_6 + 2K^2 I_7 + K^2 I_8 - K'^2 B_1 I_9 - K'^2 B_2 I_{10} = 0 \qquad \text{… … … 5.6}$$

$$\therefore I_{10} = \frac{a^2}{2} \qquad \text{… … . … 5.7}$$

Substitute value of integral I_{10} form equation (5.7) and solve for B_2, we get

$$B_2 = \frac{2}{a^2K^2}\{I_1 + I_2 + I_3 + 2(I_4 I_5) + I_6 + 2K^2 + I_7 + K^2 I_8 K'^2 B_1 I_9\} \qquad \text{… … … … 5.8}$$

After substituting the value of integrals I_1 to I_9, from respective equations, we get value of B_2.

6. Evaluation of Coefficients B_{mn}:

To determine coefficients B_{mn}, multiply on both sides of equation (5.4) by

$$\left\{ cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right) + (-1)^m \cos\left[(m+n)\frac{\pi y'}{a}\right].cos\left(\frac{n\pi x'}{a}\right)\right\}\ and$$

Integrate.

$$B_{mn}[(m+n)^2+n^2-1]\left(\frac{\pi}{a}\right)^2.\int_0^a\int_0^{a-x'}\left\{cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right)+(-1)^m cos\left[(m+n)\frac{\pi y'}{a}\right].cos\left(\frac{n\pi x'}{a}\right)\right\}^2 dx'dy' = -\int_0^a\int_0^{a-x'}F(x',y').$$
$$\left\{cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right)+(-1)^m cos\left[(m+n)\frac{\pi y'}{a}\right].cos\left(\frac{n\pi x'}{a}\right)\right\}^2 dx'dy' \quad \text{… … … … 6.1}$$

Wire above equation as

$$I_L = I_R \quad \text{… … … … 6.2}$$

Above equation is solved for I_L and is equal to

$$\therefore\ I_L = B_{mn}[(m+n)^2+n^2-1].\ \frac{\pi^2}{4} \quad \text{… … … … 6.3}$$

Let us consider I_R of equation (6.2).

After substituting F (x', y') from equation (5.2) we may write equation for I_R as

$$I_R = (I_1+I_2)+I_3+2\,(I_4+I_5)+I_6+2K^2I_7-K^2I_8-K'^2-B_1I_9-K'^2B_2I_{10} \quad \text{… … … … 6.4}$$

Solve the integrals I_1 to I_{10} and substitute their values, we get value of I_R. We can obtain value of coefficients B_{mn} using equation (6.1).

7. Third Order Solution:

Substitute (2.1) and (2.2) into wave equation (1.3) and examining the terms of order $1/R^3$, we get third order perturbation equation.

$$\frac{\partial^2\Psi_3}{\partial x^2}+\frac{\partial^2\Psi_3}{\partial y^2}+\left(\frac{\pi}{a}\right)^2\Psi_3 = F\,(x',y') \quad \text{… … … … 7.1}$$

Where

$$F(x',y') = -y'^2\left(\frac{\partial^2\Psi_1}{\partial x'^2}+\frac{\partial^2\Psi_1}{\partial y'^2}\right)-y'\frac{a\Psi_1}{\partial y'}-2y'\left(\frac{\partial^2\Psi_2}{\partial x'^2}+\frac{\partial^2\Psi_2}{\partial y'^2}\right)-\frac{\partial\Psi_2}{\partial y'}-2y'\Psi_2K^2-y'^2K^2\Psi_1+K'^2B_1\Psi_2+K'^2B_2\Psi_2+K'^2B_3\Psi_0 \quad \text{… … … … 7.2}$$

The most general solution of equation (7.1), satisfying boundary conditions (1.4) is

$$\Psi_3 = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{mn}.\left\{cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right)+\ (-1)^m cos\left[(m+n)\frac{\pi y'}{a}\right].\cos\left(\frac{n\pi x'}{a}\right)\right\} \quad \text{… … … … 7.3}$$

Substitute Ψ_3 into the left side of equation (4.6.1), we get

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}C_{mn}.[(m+n)^2+n^2-1]\left(\frac{\pi}{a}\right)^2.\left\{cos\left[(m+n)\frac{\pi x'}{a}\right].cos\left(\frac{n\pi y'}{a}\right)+\ (-1)^m cos\left[(m+n)\frac{\pi y'}{a}\right].\cos\left(\frac{n\pi x'}{a}\right)\right\} = -F\,(x',y') \quad \text{… … … … 7.4}$$

On LHS the coefficient of $\left[\cos\left(\frac{\pi x'}{a}\right) - \cos\left(\frac{\pi y'}{a}\right)\right]$ vanishes when m = 1 and n = 0. Hence with the process of determining coefficients by Fourier analysis we can obtain constant B_3. In order to get B_3, multiply to RHS of equation (7.4) by $\left[\cos\left(\frac{\pi x'}{a}\right) - \cos\left(\frac{\pi y'}{a}\right)\right]$ and equate with zero after integration.

$$\int_0^a \int_0^{a-x'} F(x',y').\left[\cos\left(\frac{\pi x'}{a}\right) - \cos\left(\frac{\pi y'}{a}\right)\right], dx'dy' = 0 \qquad \ldots\ldots\ldots\ldots 7.5$$

After substituting F (x', y') from equation (7.2), we may write equation (7.5), in the simple form as

$$(I_1 + I_2) + I_3 + 2\,(I_4 + I_5) + I_6 + 2K^2 I_7 - K^2 I_8 - K'^2 - B_1 I_9 - K'^2 B_2 I_{10} - K'^2 B_3\, I_{11} = 0 \qquad \ldots\ldots\ldots\ldots 7.6$$

Now solve these integrals I_1 to I_{11} one by one by parts.

$$\therefore\ I_{11} = \frac{a^2}{2} \qquad \ldots\ldots\ldots\ldots 7.7$$

Substitute value of integral I_{11} from equation (7.7) into equation (7.6) and write equation for B_3 as

$$B_3 = \frac{2}{a^2 K'^2}\,(I_1 + I_2) + I_3 + 2\,(I_4 + I_5) + I_6 + 2K^2 I_7 - K^2 I_8 - K'^2 - B_1 I_9 - K'^2 B_2 I_{10} \qquad \ldots\ldots\ldots\ldots 7.8$$

After solving integrals I_1 to I_{10} and substitute in above equation we get the value of B_3.

8. Evolution of Propagation Constant:
Equation of propagation constant square is given by equation (2.2), which is rewritten here

$$\beta^2 = K'^2\left(1 + \frac{B_1}{R} + \frac{B_2}{R} + \ \ldots\ldots\ldots\ldots\right)$$

From this, the equation for propagation constant of curved triangular waveguide up to third order is written as

$$\beta = K'\left(1 + \frac{B_1}{R} + \frac{B_2}{R^2} + \frac{B_3}{R^3}\right)^{\frac{1}{2}} \qquad \ldots\ldots\ldots\ldots 8.1$$

With the substitution of values of constants B_1, B_2 and B_3 from equations (3.8), (5.8) and (7.8) respectively, in above equation we obtain β.

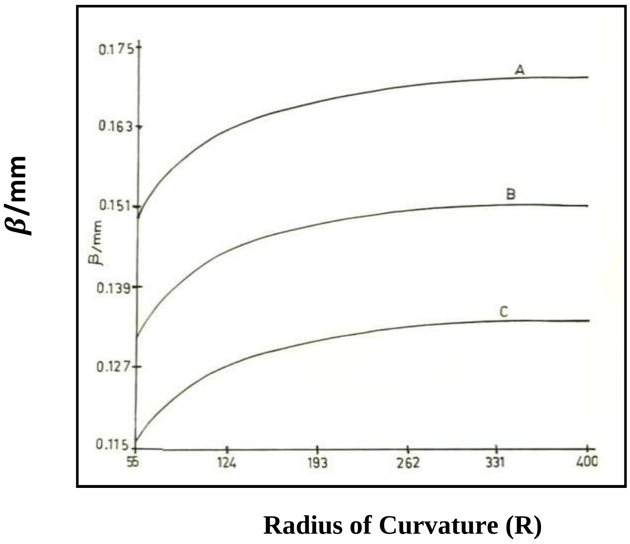
9. Numerical Calculation:
The theoretically analyzed formula of propagation constant in equation (8.1) is used to study the dispersion characteristics of curved isosceles right triangular waveguide. The formula is rewritten below

$$\beta = K'\left(1 + \frac{B_1}{R} + \frac{B_2}{R^2} + \frac{B_3}{R^3}\right)^{\frac{1}{2}}$$

The computer PCAT, 1.2 MB with coprocessor 80287 is used for numerical calculations. The constants B_1, B_2 and B_3 in above formula are evaluated with the help of program 'CWB3' written in Turbo Basic. The constant B_1 is required for calculations of coefficients A_{mn}. The values of B_1 and A_{mn} are used to evaluate constant B_2. The constant B_2 contains summation over m and n. for summation are selected so as to obtain the convergence for calculated values of B_2. The subroutine 'AMNB2' does these calculations of A_{mn} and B_2. These values are then used to obtain coefficients B_{mn}. The constant B_3 is evaluated with the use of A_{mn}, B_{mn}, B_1 and B_2. The constant B_3 also contains summation over m and n, and hence the values of m and n are so chosen that the convergence is obtained for value B_3. This is achieved by subroutine 'BMNB3'. By knowing the values of constants B_1, B_2 and B_3 the value of propagation constant is obtained under different situations (i.e. for different values of radius of curvature and frequency), with the use of above formula.

The propagation constant is evaluated at λ = 28mm and for R varying from 55mm to 400 mm and the graph of propagation constant versus R is plotted as one curve. The process is repeated for λ = 30 mm and λ = 32mm as shown in **Figure 2**.

The **Figure 3** depicts the variation of propagation constant with R, for parameters λ and R same as that of figure 2, but for the waveguide curved in opposite sense. The opposite sense of bending is achieved by replacing R by –R in the formula of propagation constant.



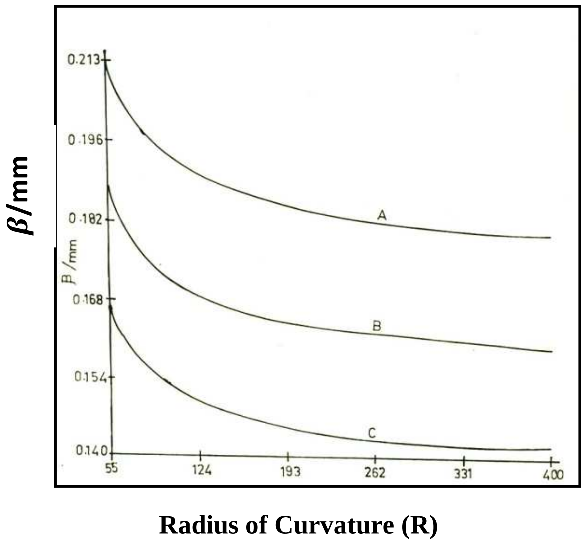


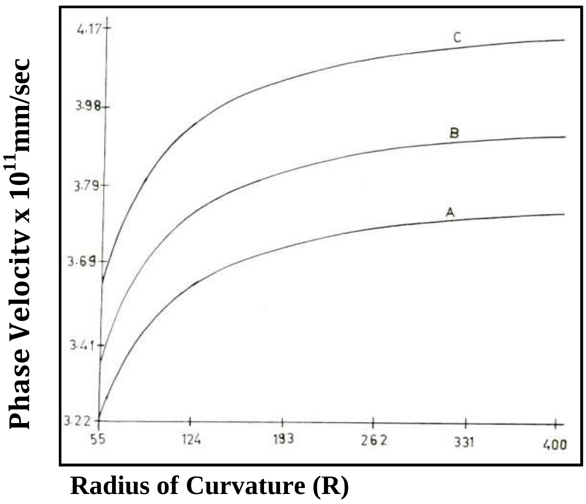
Figure 2: Variation of Propagation Constant (β) with radius of curvature (R) for different values of λ (Curves A, B and C are for λ = 28, 30 and 32 mm respectively)

Figure 3: Variation of Propagation Constant (β) with radius of curvature (R) for different values of λ (Curves A, B and C are for λ = 28,30 and 32 mm respectively)

***Waveguide is curved in opposite sense**

The phase velocity is calculated for R ranging from 55mm to 400mm, at λ = 28mm. The calculations are repeated for λ = 30mm and λ =32mm. A graph of phase velocity as a function R is plotted for each value of λ. The corresponding curves are drawn in **Figure 4**.

The above calculations of phase velocity are also performed for waveguide curved in opposite sense and the respective curves are shown in **Figure 5**. These calculations of propagation constant and phase velocity are carried out with the use of program 'PRCW'.



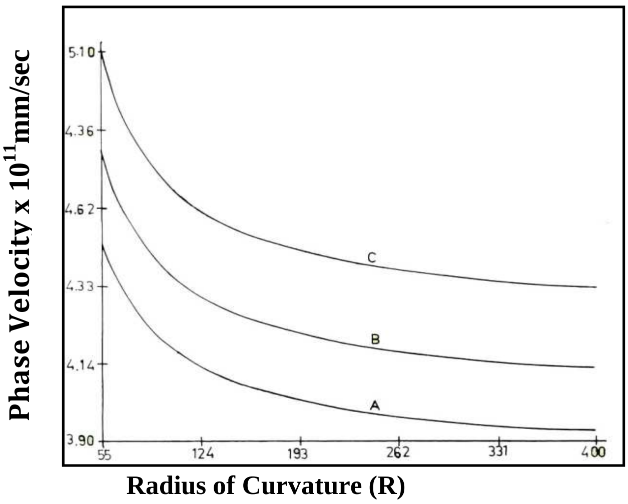


Figure 4: Phase Velocity against radius of curvature (R) for different values of λ (Curves A, B and C are for λ = 28, 30 and 32 mm respectively)

Figure 5: Phase Velocity against radius of curvature (R) for different values of λ (Curves A, B and C are for λ = 28,30 and 32 mm respectively)

***Waveguide is curved in opposite sense**

The propagation constant and phase velocity are evaluated for R = 75mm, over the range of frequency from 8GHz to 12GHz. The calculations are repeated for R = 100mm and 150mm. **Figure 6** depicts the behavior of propagation constant with change in frequency for each R, while the behavior of phase velocity is depicted in **Figure 7**. The program 'PRCWF' is developed and used for evaluation of propagation constant and phase velocity.

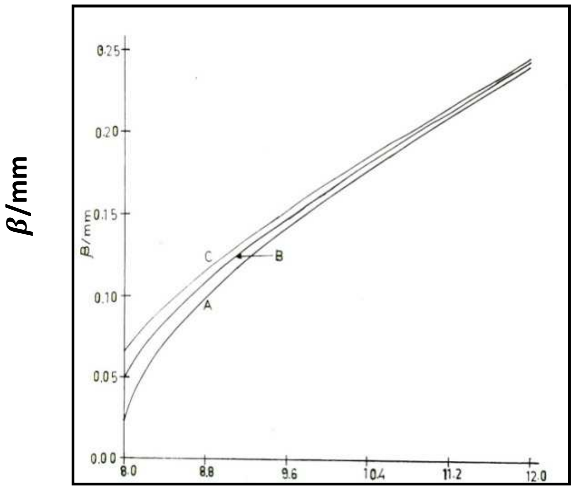


Figure 6: Propagation Constant (β) against Frequency for different values of radius of curvature (R)
(Curves A, B and C are for R=75, 100 and 150 mm respectively)

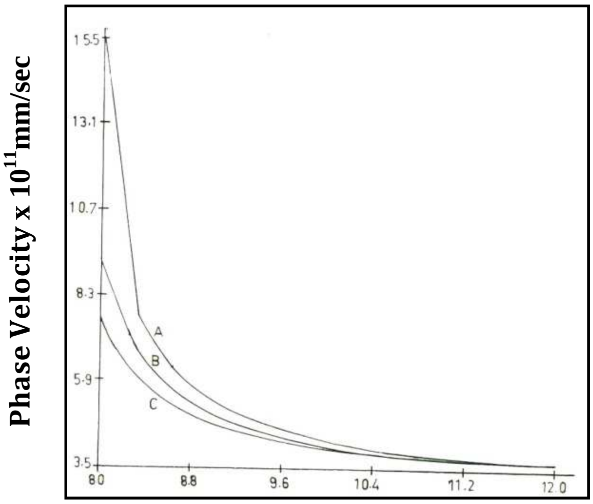


Figure 7: Phase Velocity against Frequency for different values of radius of curvature (R)
(Curves A, B and C are for R=75, 100 and 150 mm respectively)

CONCLUSION

With the help of graphs plotted we arrive to the following conclusions.

1. The increase in radius of curvature of waveguide R, decreases the propagation constant of a wave for λ given and converges to a value equal to the value of propagation constant of straight waveguide at higher values of R, as shown in **Figure 2**.
In case of waveguide curved in opposite sense the propagation constant increases with the increase of R and attains a value equal to the value that for straight waveguide at higher R, which is represented in **Figure 3**.
Thus the propagation in a curved waveguides depends on the sense of bending.
2. **Figure 4** shows that, as we increase the radius of curvature R, the phase velocity of a wave increases for a given λ and for higher R it remains constant at the value same as that for untwisted waveguide.
For a waveguide curved in opposite sense the propagation constant goes on decreasing with the increase of R and converges to a value equal to that for untwisted waveguide at high values of R, as indicated in **Figure 5**.
In other words phase velocity is changed in curved waveguide and its value depends on the sense of bending.
3. From **Figure 6** it seen that, increase in frequency will increase the propagation constant for a given R. It is also observed that as frequency increases the difference between the values of propagation constant for different values of R goes on decreasing and finally at higher frequencies this difference between propagation constant becomes negligibly small.
4. For given R, phase velocity of a wave goes on decreasing with the increase of frequency as shown in **Figure 7**. It can also be seen that the difference between the phase velocities for different values of R decreases as frequency increases and this difference becomes very small at higher value of frequency.

Acknowledgment
I am very thankful to Dr. P. B. Patil (X-Head, Dept. of Physics, Dr. BAM University, Aurangabad) for his valuable guidance in designing the problem of Curved Triangular Waveguide.

REFERENCES

[1] L. Lewin, Theory of Waveguides, London, England, Newnes-Butterworths, **1975,** 91 - 96.
[2] L. Lewin , Proceedings of the IEE - Part B: *Radio and Electronic Engineering*, **1955**, 102(1),75-80.
[3] L. Lewin , D. C.Chang, E. F. Kuester, Electromagnetic waves and curved structures *Stevenage, Herts., England, Peter Peregrinus, Ltd.*, **1977**, *2*, 58-68.
[4] K. Riess, *Quarterly of Applied Mathematics*, **1944**, 1, 328-333.
[5] J. A. Cochran and R. G. Pecina, *Radio Science*, **1966**, 1(6), 679-696.
[6] P. L. Carle, *Electronics Letters*, **1987**, 23(10), 531-532.
[7] A. Weisshaar, S. M. Goodnick, and V. K. Tripathi, *IEEE Transactions* on *Microwave Theory and Techniques*, **1992**, 40(12), 2200-2206.
[8] P. Cornet, R. Duss'eaux, and J. Chandezon, *IEEE Transactions* on *Microwave Theory and Techniques*, **1999**,47, 965-972.
[9] Mahanfar and Alireza , Proceedings, IEEE conference publications, 3rd International Conference on Microwave and Millimeter Wave Technology, **2002,** 837 – 839.
[10] Shan Zhao, *Computer Methods* in *Applied Mechanics and Engineering*, **2010**,199, 2655–2662.
[11] P. R. Chaudhari and P. B. Patil, *Indian journal of Pure and Applied Physics*, **1991**, 29, 566-568.