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A comparative study of parameters of semiclassical theory of laser

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ABSTRACT

Semiclassical treatment of lasers gives the details of theoretical methodology followed along with the semiclassical theory of laser action as preceded by Lamb and co-workers in this work. This theory has explained successfully about Lamb dip which led a variety of stabilization schemes. A medium with population inversion is capable of amplification. A feedback of energy into the system is brought about by placing the active medium between a pair of mirrors which are facing each other. The semiclassical theory has been discussed with the complex conjugate terms only and also with both complex conjugate terms and real terms. Gain and dispersion relations are worked out. It has been shown that the depth of the spatial hole increases as the value of the dimensionless intensity at a particular value of z versus the normalized population difference. This observation was not indicated in the original work of Lamb. Considering the complex conjugate terms in calculation of the basic equations some interesting result was found, which leads to a meaningful physical interpretation of the calculation. A direct relationship between polarization and refractive index of the lasing medium is established. It may be inferred from the calculations that the complex conjugate terms do have some effect which leads to a different value. As the gain and dispersion relations are changed, it can be utilized to calculate other parameters of different quantum electronic devices like Ring, Zeeman lasers etc. related to semi classical theory of laser.

Key words: Semiclassical theory of laser, gain, dispersion, quantum electronic device.

INTRODUCTION

Scope

Today in different fields (research, medicine, detection etc.) it is realized that implementation of laser in devices may further improve their reliability, performance and usefulness. Semiclassical theory by Lamb and co-workers [1- 5] can be used to derive gain and dispersion relations of different quantum electronic devices. Lamb dip which led a variety of stabilization schemes [6, 7] can be explained by it. The quantum electronics is a field concerns with the interaction of radiation and matter, particularly those interactions involving quantum energy levels and resonance phenomena [8], and especially those involving lasers and masers. Quantum electronics is usually understood to refer to only those devices in which stimulated transitions between discrete quantum energy levels are important, together with related devices and physical phenomena which are excited or explored using lasers. Zeeman effects [9] are important in quantum electronics because they make possible a method by which the value of the energy level can be slightly changed, causing fine tuning of emission or absorption frequencies in optical and infrared regions. They also provide the distinct energy levels whose separations are in the microwave or radio frequency range ideal for lasers. Magnetic field strength of a surface can be calculated with Zeeman splitting technique. Ring cavities [10-12] can sustain the oscillation of traveling waves. The traveling waves sheltered by ring resonators can oscillate unidirectional, used in intra-cavity acousto-optic modulator. Here using the same procedure as followed by Lamb and co-workers we have derived gain and dispersion relations for complex conjugate terms and also for considering both simultaneously which are different as by Lamb and co-workers for real terms only. This theoretical approach of derivation of gain and dispersion relations with real (only), complex conjugate (only) and both simultaneously related can be utilized to derive and to compare different parameters of quantum electronic devices like Zeeman and Ring lasers.

Objectives:

- In earlier calculation of gain and dispersion relations the complex conjugate terms are ignored. So to derive equations with these terms
- To verify whether complex conjugate terms have a significant effect as the real term.
- To calculate overall effect that may change the parameters related to gain and dispersion.

MATERIALS AND METHODS

Consider a homogeneously broadened medium with two level atoms and a Fabry-Perot cavity with Brewster Windows which ensures linearly polarized field and a scalar field. Electromagnetic field of the cavity mode produces a macroscopic polarization of the medium. The polarization calculated by quantum treatment acts a source for the field in the cavity. Here in cavity electromagnetic field equations are describe by Maxwell's equations. The real part of susceptibility calculated is responsible for additional phase shifts due to medium leads mode puling. The imaginary part is responsible for gain or loss due to the medium. Here the effects due to spontaneous emission are not considered as it is a semiclassical approach. The wave equation obtained from the Maxwell's equations for the scalar Electric field $E(z,t)$ is

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}+\mu_0\,\sigma\frac{\partial\vec{E}}{\partial t}+\mu_0\,\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=-\mu_0\frac{\partial^2\vec{P}}{\partial t^2} \qquad \text{--- (1)}$$

Where σ is the conductivity introduced in the lossy medium and this conductivity can be adjusted to give damping due to diffraction and the reflection transmission.

Considering, $\vec{\nabla}.\vec{D}=0$ and $\vec{\nabla}.\vec{P}=0$ for laser medium. On the basis of the fact that only certain discrete modes have achieved appreciable magnitude the variation in the electromagnetic field transverse to the laser axis is ignored, thus

$$\vec{\nabla}\times\vec{\nabla}\times\vec{\mathrm{E}} = -\frac{\partial^2\vec{\mathrm{E}}}{\partial z^2} \qquad \text{--- (2)}$$

Where, z is the direction along the optical path of the laser. The transverse variations of the field are neglected. Inside the cavity only certain discrete modes achieve appreciable magnitude whose circular frequency is

$$\Omega_n = \frac{n\pi c}{L} = K_n c \qquad \text{--- (3)}$$

Where L is the length of the cavity, c is the velocity of light, n is a large integer in our discussion we take normal modes to have sinusoidal z dependence.

The electric field can be expressed as a sum of modes, ie.

$$U_n(z) = \exp(\mathrm{i}\,K_n z) \quad , K_n = n\frac{\pi}{L}$$

The single polarization component of the electric field is written as

$$E(z,t) = \frac{1}{2}\sum_n \mathbf{E}_n(t)\exp\{-\mathrm{i}(\upsilon_n t + \phi_n)\}U_n(z) + c.c \qquad \text{--- (4)}$$

$$P(z,t) = \frac{1}{2}\sum_n \mathbf{P}_n(t)\exp\{-\mathrm{i}(\upsilon_n t + \phi_n)\}U_n(z) + c.c \qquad \text{--- (5)}$$

Here, the amplitude coefficient E_n and complex polarization component P_n very little in an optical frequency period. The real part of polarization is in phase with the electric field leads to dispersion due to medium. The imaginary part is in quadrature with the electric field and results in gain or loss. Now using values in equation (1), (after doing calculation by algebraic method)

$$\Rightarrow \Omega_n^2\mathbf{E}_n - \mathrm{i}\frac{\sigma}{\varepsilon_0}\upsilon_n\mathbf{E}_n - 2\,\mathrm{i}\,\upsilon_n\dot{\mathbf{E}}_n - (\upsilon_n + \dot{\phi}_n)^2\mathbf{E}_n = \upsilon_n^2\,\varepsilon_0^{-1}\mathbf{P}_n \qquad \text{--- (6)}$$

Adjusting the fictional conductivity σ to create the desired value of Q of the mode

$$\sigma = \varepsilon_0\frac{\upsilon_n}{Q_n}$$

From equation (6) equating real and imaginary part we finally get

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = -\frac{1}{2}\upsilon_n\,\varepsilon_0^{-1}\,\mathrm{Im\ part\ of}\ \mathbf{P}_n \qquad \text{--- (7)}$$

$$\upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\upsilon_n \varepsilon_0^{-1}\mathbf{E}_n^{-1} \text{ Re part of } \mathbf{P}_n \quad \text{--- (8)}$$

These are basic equations of the semiclassical theory of laser. In absence of active medium P_n = 0

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = 0$$

and $$\upsilon_n + \dot{\phi}_n = \Omega_n$$

Using the same procedure for the complex conjugate terms, we get the gain and dispersion theorem as

$$\dot{\mathbf{E}}_n + \frac{\upsilon}{2Q_n}\mathbf{E}_n = \frac{1}{2}\upsilon_n \varepsilon_0^{-1} \text{Im part of } \mathbf{P}_n \quad \text{--- (9)}$$

$$\{2\upsilon_n - \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\} = \Omega_n + \frac{1}{2}\upsilon_n \varepsilon_0^{-1}\mathbf{E}_n^{-1} \text{Re part of } \mathbf{P}_n \quad \text{--- (10)}$$

RESULT AND DISCUSSION

From above equations (7) and (9), both are the same except the negative sign. But equations (8) and (10) are different with representing dispersion of the medium. In absence of active medium

$$\dot{\mathbf{E}}_n = \frac{\upsilon}{2Q_n}\mathbf{E}_n$$

and $$\{2\upsilon_n - \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\} = \Omega_n$$

From equation (10) we get two relations, one part is

$$\upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t) \quad \text{--- (11)}$$

This equation is same as the real part of the original basic equation of Laser derived by Lamb. The another part of the equation is,

$$[\upsilon_n - \{\dot{\phi}_n + \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n\}] = \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t) + \frac{1}{2}\frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t)$$

or, $$\upsilon_n - \dot{\phi}_n = \frac{\Omega_n}{\upsilon_n}\dot{\phi}_n + \frac{\upsilon_n}{\varepsilon_0}\mathbf{E}_n^{-1}(t) \text{ Re } \mathbf{P}_n(t) \quad \text{--- (12)}$$

It represents dispersion but with different form.

The above calculations are done either with the complex term or the complex conjugate terms. Proceeding in the same way for the general equations of semiclassical theory of laser, considering both the real and complex conjugate terms simultaneously, we get

$$\Rightarrow K_n^2 \mathbf{E}_n = \mu_0 \upsilon_n^2 \mathbf{P}_n \qquad \text{--- (13)}$$

As $\mathbf{E}_n$, ϕ_n and $\mathbf{P}_n$ are vary little in an optical frequency period and losses are small, the terms involving $\ddot{\mathbf{E}}_n, \ddot{\phi}_n, \ddot{\wp}_n, \dot{E}_n, \dot{\phi}_n, \sigma\dot{E}_n, \sigma\dot{\phi}_n, \phi_n, \wp_n$ and $\dot{\mathbf{P}}_n$ be neglected.

Physical significance:

The complex polarization for complex susceptibility

$$\mathbf{P}_n(t) = \varepsilon_0 \chi_n = \varepsilon_0(\chi_n' + i\chi_n'')\ \mathbf{E}_n(t) \qquad \text{--- (14)}$$

Hence from equation (12) and (14)

$$\dot{\mathbf{E}}_n(t) = -\frac{\upsilon}{2Q_n}\mathbf{E}_n(t) - \frac{1}{2}\upsilon\chi_n''\mathbf{E}_n(t) \qquad \text{--- (15)}$$

and

$$\upsilon_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\upsilon\chi_n' \qquad \text{--- (16)}$$

Equation (16) can be summed

$$\therefore \quad \upsilon_n + \dot{\phi}_n = \frac{\Omega_n}{\eta(\upsilon_n)} \qquad \text{--- (17)}$$

Equation (16) or (17) shows completely difference between gain and classical problem.

CONCLUSION

The equation for mode amplitude $\dot{\mathbf{E}}_n(t) + \frac{\upsilon}{2Q_n}\mathbf{E}_n(t) = -\frac{1}{2}\upsilon\chi_n''\mathbf{E}_n(t)$ concerns about the conservation of energy. The equation (16) determines the associated frequency. For small values of susceptibility, it can rewrite as equation (17), shows dispersive phenomenon of the laser medium. Thus a difference between the gain and the classical absorption value of these equation is noticed as the oscillation frequency $\upsilon_n + \dot{\phi}_n$ is shifted by the medium instead of wavelength. This results from the self-consistency nature of the laser field which requires an integral number of wavelengths in a round trip regardless of the medium characteristics. Again from the frequency determining equation (16) we get a dispersion relation which is different as compared to equation (17).thus mode pulling is affected. From equation (13),

$$\Rightarrow \mathbf{P}_n = \varepsilon_0 \eta^2(\upsilon_n)\mathbf{E}_n$$

i.e. for particular frequency polarization is related with refractive index $\eta(\upsilon_n)$ of the laser medium that further depends on susceptibility. Thus medium of particular material can change value of polarization. So the gain and dispersion relations are affected. The relationship of polarization and refractive index can be utilized to derive different parameters of various laser devices.

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