



A genetic Algorithm method for the inventory routing and optimal pricing in a two-echelon supply chain with demand function

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ABSTRACT

This paper deals with the operational issues of a two-echelon supply chain under linear demand function for each buyer. The operational parameters to the model are sales price and transportation quantity that determine the channel cost of the supply chain transportation. To find out the optimal price and the optimal transportation quantity, for each buyer from several suppliers, a mathematical model is formulated. For using the genetic algorithm, the formulated model is modified and then the best value of the parameters is derived. Finally, a numerical example is given to illustrate the model.

Key words: Genetic algorithms, inventory routing, supply chain, demand function.

INTRODUCTION

A two echelon supply chain model, involved some suppliers and buyers that have relationship with each other. Each buyer has demand that can be explained with demand function or constant value that satisfied by suppliers. The inventory routing problem (IRP) in a supply chain (SC) is to determine delivery routes from suppliers to some geographically dispersed retailers and inventory policy for retailers. It is consisted of two sub-problems: inventory problem for retailers and vehicle routing problem (VRP) for suppliers. The IRP considering inventory and routing simultaneously has gained attentions since the coordination of the inventory and routing decisions between the supplier and retailers leads to a better overall performance [1] According to the literature [2], the pricing and demand decisions seem ignored and assumed known in most IRP researches. Since the pricing decision affects the demand decision and then both inventory and routing decisions, it should be made in the IRP simultaneously to achieve the objective of maximal profit in the supply chain. For example, higher pricing causes lower demand then lower order quantity and lower inventory. In contrast, lower pricing causes higher demand and then higher order quantity and higher inventory. Since the pricing decision is interrelated to inventory routing decisions, the profit may decrease when they are made separately. Hence, how to determine inventory, routing and price simultaneously becomes an important issue in supply chain management. Because the inventory routing and pricing problem (IRPP) is a NP-hard problem (Since inventory routing decisions is a NP-hard problem [3], the IRPP is more complex than the IRP.), a heuristic method is adopted to resolve this problem. Until now, there are few researches about IRPP. Hence, this paper presented a survey for two related areas: inventory routing problem and pricing problem, in the following. Bell, Dalberto, and Fisher[4] adopted an optimization method to resolve the IRP. After that, some other optimization methods were developed to resolve the IRP [5,6,7,8,9,10]. Since the IRP is an NP-hard problem, heuristic methods are needed. Federgruen and Zipkin[11] developed a nonlinear integer programming model and adopted an exchange method to resolve the IRP. Golden, Assad and Dahl (1984) [12] adopted an insertion method to resolve the IRP. Viswanathan and Mathur [13] adopted a stationary nested joint replenishment policy heuristic (SNJRP) to resolve the IRP. The results show the method simultaneously making inventory and routing decisions is better than that making inventory and routing decisions separately. Campbell and Savelsbergh[14] adopted a two-

phase method to resolve the IRP. The first phase adopted an integer programming method to obtain the initial solution. The second phase adopted an insertion method to improve the initial solution. Gaur and Fisher [15] adopted a randomized sequential matching algorithm (RSMA) to resolve the IRP. An insertion method was adopted to obtain the initial solution. Then a cross-over method was adopted to improve the initial solution. Sindhuchoo, Romeijn, Akcali, and Boondiskulchok [16] adopted a two-phase method for the IRP. The first phase adopted a column generation method to obtain the initial solution. The second phase adopted very large-scale neighborhood search (VLSN) to improve the initial solution. Lee, Jung, and Lee [17] adopted a tabu search method to resolve the IRP. Raa and Aghezzaf [18] adopted a heuristic method to resolve the IRP. A column generation method was adopted to find the initial solution. Then a saving heuristic method was adopted to improve the initial solution. Zhao et al. [19] adopted a heuristic method to resolve the IRP. The initial solution was generated randomly. Then a tabu search method adopting the GENI neighborhood search was used to improve the initial solution. Zhao, Chen, and Zang [20] adopted a variable large neighborhood search (VLNS) method to resolve the three-echelon (suppliers, distributors, retailers) IRP. The results show the proposed method is better than the tabu search method. In summary, tabu search (TS) adopting the GENI neighborhood search approach and VLNS have been adopted to find the optimal solution for the inventory routing problem effectively and efficiently [15, 19, 20].

Hence, they will be adopted to resolve the IRP sub-problem in IRPP in this paper. As for the pricing problem, some researchers [17, 21, 22, 23] determined the prices and demands using calculus according to the known demand function based on the maximal profit criterion. Nachiappan and Jawahar [24] adopted a genetic algorithm (GA) method to find the prices and demands based on the maximal profit criterion in a supply chain. The pricing problem is a nonlinear integer programming (NIP) problem. Searching for the optimal solution is an NP problem. According to the literature [25, 26; 27], genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO) and tabu search (TS) have been adopted to resolve the NIP problem. Since tabu search is adopted to resolve the IRP sub-problem in IRPP mentioned above, if GA, PSO or ACO is adopted to resolve the pricing sub-problem in IRPP, the IRPP would be resolved separately by different methods.

In this paper, the best value of the parameters derived from the GA approach.

Model formulation for the inventory routing and pricing problem

Before the model for the inventory routing and pricing problem is formulated, the relevant information is discussed first.

Revenue

Demand function defines the price and demand quantity relationship. The planning horizon is usually 1 year or half year. The demand function for retailer i : $D_i = a_i(b_i - p_i)$ (The linear demand function is the most popular in the related research [22, 24]). The function becomes as follows:

$$R_i = (1 + \theta)p_i \times D_i \Rightarrow R_i = a_i b_i p_i - p_i^2 + \theta a_i b_i p_i - \theta p_i^2$$

That p_i is the sell price of retailer ' i ', θ is the percentage of the sell profit of retailers, a_i is a positive constant and b_i is the upper bound of price.

Supply chain cost

Transportation and holding cost

The detailed computation is as follows:

$$C = \sum_i \sum_j h_{ij} x_{ij}$$

Where h_{ij} is inventory cost include transportation and holding cost.

2.2.2.

Purchase cost computation is as follows:

$$K = \sum_i p_i \sum_j x_{ij}$$

After the revenue and supply chain cost are discussed, the model for inventory routing and pricing problem is as follows:

$$\text{Max } z = \sum_i (a_i b_i p_i - p_i^2 + \theta a_i b_i p_i - \theta p_i^2) - \sum_i \sum_j h_{ij} x_{ij} - \sum_i p_i \sum_j x_{ij}$$

$$\text{s.t.} \quad \sum_j x_{ij} = a_i (b_i - p_i) \quad (1)$$

$$\sum_i x_{ij} \leq S_i \quad (2)$$

$x_{ij} \geq 0$ (Nonnegative Constraint and Integer)

$0 \leq p_i \leq b_i$ (Nonnegative Constraint and Integer)

The goal of the objective function is to make the supply chain profit maximum. The constraint (1) indicates the purchase for retailer j equal to demand of retailer j . The constraint (2) indicates the sum of i th supplier sell must be less than or equal to supplier capacity S .

The proposed method for the inventory routing and pricing problem

To resolve the IRPP, three major decisions: inventory, routing and pricing, need to be made. Because the IRPP is a NP-hard problem (Since both inventory routing problem and pricing problem are NP problem), this paper proposes a genetic algorithm to improve the solution through inventory routing improvement procedure and pricing improvement procedure

Sodo code for proposed genetic algorithm

Begin:

1. Initialization

1-1. Parameter Setting (Pc, Pm, Stop Criteria, Pop Size, Selection Strategy, Num Gen)

1-2. Initialize Population (Randomly)

2. Fitness Evaluation

Repeat

3. Individual Selection for Mating Pool (Size of Mating Pool =Pop Size)

4. For each consecutive pair apply Crossover (For each consecutive pair apply Crossover with probability pc)

5. Mutate Children (For each new-born apply mutation with probability pm)

6. Replace the Current Population by the resulting Mating Pool

7. Fitness Evaluation

Until Stopping Criteria is met

End.

Stop criteria

The maximum number of generations is achieved.

Crossover

For crossover in this paper we use two kind of approach crossover:

For transportation quantity matrix we use uniform crossover:

$$O_1 = X_1 \psi + X_2 (1 - \psi)$$

$$O_2 = X_1 (1 - \psi) + X_2 \psi$$

Where ψ is the crossover mask; O is offspring and X is parents.

For price vector we use liner compound crossover.

$$O_1 = P_1 \lambda + P_2 (1 - \lambda)$$

$$O_2 = P_1 (1 - \lambda) + P_2 \lambda$$

Where λ is the crossover mask; O is offspring and P is parents.

Mutation

For mutation we use one point mutate for both transportation quantity matrix and price vector.

Elitist strategy

Using an elitist strategy to produce a faster convergence of the algorithm to the optimal solution of the problem. The elitist individual represents the more fit point of the population. The use of elitist individual guarantees that the best fitness individual never increase (Minimization Problem) from one generation to the next generation (Towards the end of the process). Although the GA represents a possible way of solving the models; some problems remain in its implementation.

Feasible solution

Genetic algorithms are derived from an analogy with the spread of mutations in a population [28]. The main problem in applying a GA to constrained optimization problems is how to deal with the constraints. Constraints can be dealt with strategies such as reject, repairing and penalty strategies, and the strategy of modifying genetic operators [29]. The reject strategy excludes infeasible solutions immediately on generation, resulting in an efficient GA. The repairing strategy transforms an infeasible solution into a feasible one through a repairing process. The difficulty in designing a repairing process to comply with the problem weakens the repairing strategy. The penalty strategy uses a penalty function to penalize all infeasible solutions, hoping that infeasible solutions might evolve toward feasible. Finally, the strategy of modifying genetic operators aims to devise problem-specific representations and specialized genetic operators to maintain feasibility. Comparatively, the strategies of penalty and modifying genetic operators appear more suitable for this study [30]. Therefore, the strategies of penalty and modifying genetic operators were used in this study to deal with supply constraints and repairing strategy for demand constraints.

3.6.1. Repairing strategy for demand constraints

$$x'_{ij} = \frac{x_{ij}}{\sum_j x_{ij}} x^*_{ij} = x'_{ij} \times D_j$$

The penalty function that impels the solutions to satisfy supply constraint is formulated as follows.

$$\xi = \begin{cases} \sum_i x_{ij} - S_i & \text{if } \sum_i x_{ij} > S_i \\ 0 & \text{other wise} \end{cases}$$

Fitness evaluation

Incorporating the objective function (1) and the penalty function (2), the target function for model can be defined as

$$\varphi = \sum_i (a_i b_i p_i - p_i^2 + \theta a_i b_i p_i - \theta p_i^2) - \sum_i \sum_j h_{ij} x_{ij} - \sum_i p_i \sum_j x_{ij} - M \times \xi$$

Where M a large positive number. Then we compensate fitness for best effect for roulette wheel.

$$f^* = \begin{cases} \frac{1}{|\varphi|} & f \leq -1 \\ 0.1|\varphi| & -1 < f \leq 0 \\ \varphi & \text{other wise} \end{cases}$$

Empirical study

The parameter setting of the proposed GAs is as follows. The number of population (Pop Size), the chromosome dimension for transportation quantity matrix, for price vector dimensions, the crossover and mutation rates and the number of iterations for two sizes of problem are shown in table 1. The values of the M were set 100, respectively. Also, Fig 1 and Fig 2 show the GA convergence plot for two problems.

Table 1. Algorithm parameter for two problem .

Problem size	Percentage of crossover	Percentage of mutation	Pop Size
Small	0.85	0.05	300
Big	0.85	0.05	300
Problem size	chromosome dimension X	chromosome dimension P	number of iterations
Small	2*3	1*3	500
Big	7*10	1*10	5000

The experiments were conducted on a PC with a Intel ® Core™ 2 Duo E7500 @ 2.93GH CPU, 2 GB of RAM and Windows 7 Ultimate and implemented in MATLAB 7.10.0.499 (R2010a)

Table 2. The results of two problem

Problem size	Average profit	CPU Time
Small (2*3)	151	30
Big (7*10)	312	347

Fig 1. The GA convergence plot for small problems

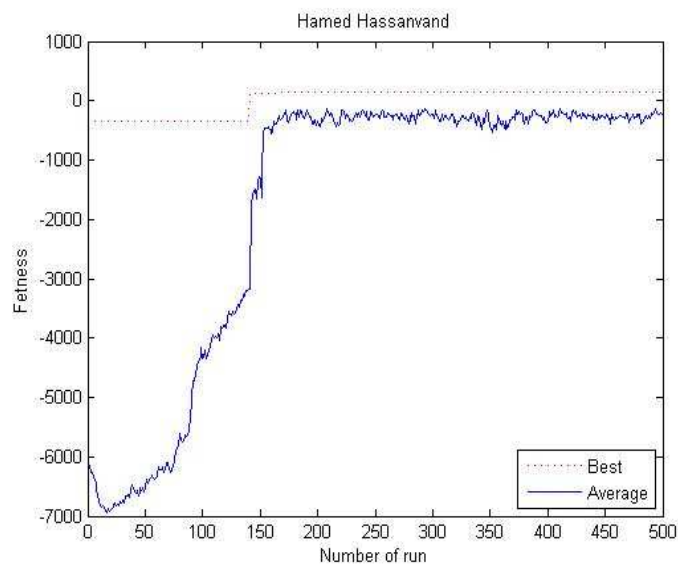
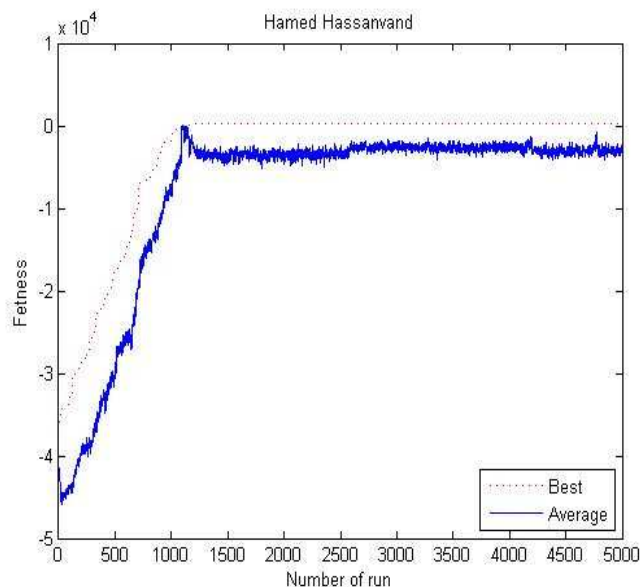


Fig 2 . The GA convergence plot for big problem.



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