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A Study on Fractional Calculus and its Application in Electronics

Sridhar Pattanaik

Electronics (EIS), Berhampur University, India

ABSTRACT

The capability of fractional calculus for tackling the problems of physics, signal processing, fluid mechanics, control engineering, etc. had made it a new mathematical tool for the solution of different problems in the field of science and engineering. Basically, it deals with operators having non-integer and/or complex order, e.g., fractional derivative and fractional integral. The concept has a tremendous potential to change the way we see, model, and control the nature around us. In this paper, the application of fractional calculus is discussed particularly for electronics.

Keywords: *Fractional calculus, Radiation Problem, System Identification*

INTRODUCTION

The theory of Fractional Order Calculus (FOC) [1-6] is 300 years old, but researchers could able to use it in the last two decades on account of available of computational recourses. The major constraint of fractional calculus is the rigorous mathematics that has come into the literature is almost difficult for the engineers to understand. Some of the researchers have shown interest in the fractional calculus in their respective fields, but the intensity of research is very small as compared to the latent potential of FOC.

MATERIALS AND METHODS

Some of the common definitions and properties of fractional differ-integrals are briefly presented below.
Cauchy's repeated-integration formula is depicted in (1).

$$ {}_aD_x^{\alpha} f(x) \equiv \frac{1}{\Gamma(-\alpha)} \int_a^x (x-u)^{-\alpha-1} f(u)du \qquad (1)$$

for $\alpha < 0$ and $x > a$

Where ${}_aD_x^{\alpha} f(x)$ denotes the (fractional) α th-order integration of the function $f(x)$, with the lower limit of integration being 'a' and $\Gamma(.)$ is the Gamma function. For $\alpha = -n$ the above definition turns to (2).

$$ {}_aD_x^{-n} f(x) \equiv \frac{1}{(n-1)!} \int_a^x (x-u)^{n-1} f(u)du \qquad (2)$$

The above equation, according to Cauchy's repeated-integration formula, is equivalent to (3).

$$ {}_aD_x^{-n} f(x) \equiv \int_a^x dx_{n-1} \int_a^{x_{n-1}} dx_{n-2} \ldots \int_a^{x_1} f(x) dx_0 \tag{3} $$

For fractional derivatives with α > 0, the above Riemann-Liouville fractional-integration definition can still be applied, if used in conjunction with the following additional step:

${}_aD_x^{\alpha} f(x) = \frac{d^m}{dx^m}\, {}_aD_x^{\alpha-m} f(x)$ for α > 0, where *m* is chosen so that *(*α *- m)* < 0, and thus the Riemann-Liouville integration can be applied for ${}_aD_x^{\alpha-m} f(x)$ [1]. Then, $\frac{d^m}{dx^m}$ is the ordinary mth-order differential operator [1-3].

There are several other equivalent definitions for fractional derivatives and integrals, which can be found in various references, [1-8].

III. Some Peculiar Characteristics of Fractional Calculus

Fractional operators are linear operators (like their integer order counterparts), and thus they follow the usual properties of linearity, homogeneity, and scaling. The interesting features of the fractional derivative of a function evaluated at a point *x* depend on the value of the function *f (x)* at the point *x* and its neighboring points, but it also depends on values of the function between *x* and the lower-limit point. This leads to think that, the fractional integration/differentiation [4] has some "mixed" properties of differentiation and integration.

It is important to note that the FOC has also been applied in describing several special functions in mathematical physics in terms of fractional differintegrals of more elementary functions.

RESULTS AND DISCUSSION

Application of Fractional Calculus in Electronics

(a) Fractional Calculus in Radiation Engineering

Electromagnetics is the basis of wireless communication and deals with calculus of integer-order. It is of interest for the researchers to see how fractional calculus can be explored in this field to get the physical significance of such non-integer based differential or integral operators. Researchers have shown that these mathematical operators can be interesting and useful mathematical tools in electromagnetic/wireless theory [9-12]. The major work includes the novel concept of fractional multi-poles in electromagnetism, electrostatic fractional image methods for perfectly conducting wedges and cones, and the mathematical link between the electrostatic image methods for the conducting sphere and the dielectric sphere.

It is an established fact that the scalar Helmholtz equation the canonical solutions are identified as plane, cylindrical, and spherical waves for the one, two and three-dimensional cases respectively. The corresponding sources being one, two and three-dimensional Dirac delta functions. Researchers expected an intermediate wave between two canonical cases. That means thinking the transition to be continuous instead of distinct. This type of solution cannot be expected from ordinary calculus, which is the positive integer based. It has been shown that fractional integration, differentiations, which are mathematical tools studied in the field of fractional calculus, can be utilized to find the "intermediate" sources. The waves that satisfy the conventional scalar Helmholtz equation [9-11] can help the electromagnetic/wireless researchers. A fractional parameter '*ν*' which is used as a determining factor for calculating the intermediate values, and takes the fractional value between zero and unity. The solution is such that when ν = 0 represents the case of the cylindrical wave propagation and ν = 1 denotes the plane-wave propagation.

(b) Electronic Circuit Analysis

Electrical circuits with fractance: Resistors and capacitors are described by an integer- order models in Classical electrical circuits. Electrical element with fractional-order impedance [13-15] is called fractance. Basically, there are two kinds of fractance: (i) tree fractance and (ii) chain fractance. The electrode-electrolyte interface is an example of a fractional-order process. Value of the fractional coefficient '*η*' *is* closely associated with the smoothness of the interface, as the surface is infinitely smoothed then *η*→1. The current field in the transmission line of infinite length is expressed in terms of the fractional derivative of order 1/2 of the potential *φ*(0,*t)* is also a fractional based model.

The tree fractance and chain fractance consist not only of resistors and capacitor's properties, but also exhibit electrical properties with non-integer-order impedance. One can think of the generalized voltage divider.

(c) Fractional Calculus in Control Engineering

It is the challenge of the researchers to find new and effective methods for the time-domain analysis of fractional-order dynamical systems [16-17] for tackling problems of control theory. As a new generalization of the classical *PID*-controller namely the $PI^{\lambda}D^{\mu}$-controller has been developed by researchers. The idea of $PI^{\lambda}D^{\mu}$-controller, involving fractional-order integrator and fractional-order differentiator, is a more efficient control of fractional-order dynamical systems. If $\lambda = 1$ and $\mu = 0$, the equation converts to a PI controller. Likewise, if $\lambda = 0$ and $\mu = 1$ the equation yields the PD controller. It has been successfully used the fractional-order controller to develop the so-called CRONE-controller *(Commande Robuste d'Ordre Non Entrier controller)* which is an interesting example of application of fractional derivatives in control theory. The prime advantage of the $PI^{\lambda}D^{\mu}$-controller compared to the conventional classical *PID*-controller has a better performance record.

(d) Fractional Calculus in Electronic System Designing

The response of a fractional order system [18-19] by using analogical circuits with fractional order behavior can be done in three methods briefly presented.

i. Component by component implementation: The approximation of the transfer function is done by the recursive circuit. The gain between V_o and V_i in Laplace transform is the continuous fraction approximation to the original system [18-19].

$$\frac{V_o}{V_i} = 1 + \cfrac{w_n}{s + \cfrac{w_{n-1}}{1 + \cfrac{w_{n-2}}{s + \cfrac{w_{n-3}}{\vdots}}}} \tag{4}$$

Where $$w_{n-2j} = \frac{1}{R_j C_j} \tag{5}$$

and $$w_{n-2j+1} = \frac{1}{R_{j+1} + C_j} \tag{6}$$

The method presented has two principal disadvantages: (i) the frequency band of work is limited and (ii) this is an approximation. Therefore, it requires a lot of low tolerance components, depending on the accuracy required by the designer.

ii. Field Programmable Analog Array (FPAA): The designer implements the circuit component by component into a FPAA. Which enables changing of the dynamical behavior of the fractional order system with a few simple modifications and each element has custom tolerance.

iii. Fractional order impedance component: It is a capacitor with fractional order behavior introduced. In general, it consists in a capacitor of parallel plates, where one of them presents a fractal dimension. Each branch could be modeled as a low pass resistor/capacitor (RC) circuit filter, and it is linked to the principal branch.

(e) Modeling of Speech Signal

Linear Predictive Coding (LPC) approach is basically used for the speech processing, and it is based on the integer order models [20-22]. It has been found through simulation that by using a few integrals of fractional orders as basis functions, the speech signal can be modeled accurately.

V. Limitations of Fractional Calculus for Modeling

In the traditional calculus where α discussed in (1) is a whole number; the derivative of an elementary function is an elementary function. Unfortunately, in the fractional calculus this is not true. The fractional derivative of an elementary function is usually a higher order transcendental function. The memory effect associate with the fractional calculus makes it more computational expensive. A numerical approximation [7] of a fractional differential equation requires an exponential increase in the work with increasing time. The workload increases exponentially, if multiple instances of fractional derivatives are included.

Depending upon the efficiency, accuracy of the results required different algorithms can be employed. For some algorithms, the structure of the fractional differential equation can drastically change the computational cost. A better understanding of algorithm performance would allow the user to implement the most efficient algorithm for a given problem/equation/system.

A major difficulty with fractional models is in its time-domain simulation. The analytical solution of a model's output is not simple to compute in case of fractional calculus. During the last twenty years numerical algorithms have been developed using either continuous or discrete rational models approximating fractional systems.

CONCLUSION

The present work is the general efforts in recent years to explore potential utilities and possible physical implications of the mathematical machinery of fractional derivatives and fractional integrals in electronics. New algorithm in terms of accuracy and speed needs to be developed. Soft computing technique with fractional calculus may be a suitable combination to solve some problems in the future.

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