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Analyticity of distributional generalized Laplace-Finite Mellin Transform

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ABSTRACT

Mellin transform is one kind of non lincar transformations which is widely used its scale invariance property. It is important in vision & Image Processing. In this paper Laplace-finite Mellin transform is extended in the distribution generalized sense. Testing function space is defined Analyticity theorem for the generalized Leplace-finite Mellin transform is proved.

Keywords: Laplace transforms, finite Mellin transform, Laplace–finite Mellin transform, generalized function.

INTRODUCTION

Integral transforms play an important role in signal processing Image construction, pattern recognition, acoustic signal processing. Finite Mellin transforms are the extension of the Mellin integral transform. These transforms are suited to regions bounded by the natural Co-ordinate surfaces of cylindrical or spherical Co-ordinate and apply to finite or infinite regions bounded by internally.

We attempt to understand how Laplace operator can be used to find the relations with the finite Mellin Integral transform in distributions. We aim to study the generalization of Laplace–finite Mellin transform and give a technique from Laplace transform that turns out to be valid for the finite Mellin transform in distributions.

Khairnar S.M., Pise R.M [2] had discussed relation of finite Mellin transform with Laplace and Fourier transform. V.D. Sharma, P.D. Dolas [3] had proved the theorem of Analyticity of Distribution Generalized Fourier – Stieltjes transforms Khairnar S.M. & Pise R.M; [4] had discussed Finite Mellin integral transform.

In this paper the notation & terminology will follow that Zemanian [1].

In section 2 & 3 we define testing function space & distributional Laplace–finite Mellin transform. Section 4 proves the analyticity of distributional Laplace finite Mellin transform section 5 concludes the result.

2. Testing function space $LM_{f,u,b,c,\alpha}$

The space $LM_{f,u,b,c,\alpha,}$ $a \geq o$ consists of all infinitely differentiable functions $\phi(t, x)$, where $0 < t < \infty$, $0 < x < a$.

$$\gamma_{\ell,q}\,\phi(t,x) = \sup_{\substack{o<t<\infty \\ o<x<a}} \left| k_{u,v}(t) l_{b,c}{}^{(x)}\, x^{q+1} D_t^1 D_x^q \phi(t,x) \right|$$

$$< \infty \quad \text{for each } l, q = 0, 1, 2 \,.....$$

$$\text{Where } k_{u,v}(t) = e^{ut} \qquad o < t < \infty$$

$$= e^{vt} \qquad -\infty < t < 0$$

and $\lambda_{b,c}(x) = \begin{cases} x+b & 0 < x < 1 \\ x+c & 0 < x < a \end{cases}$

3. Distributional Generalized Laplace – finite Mellin transform.

For $(t, x) \in LM^{*}_{f,u,b,c\alpha}$; where

$LM^{*}_{f,u,b,c\alpha}$ is the dual of $LM_{f,u,b,c,\alpha}$ and $-b < \text{Re } p \leq b$, real number c, s, >, 0, the distributional Laplace – finite Mellin transform is defined as

$$LM_f \{f(t,x)\} = F(S,P) = < f(t,x), e^{-st}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right) > \text{ ------- } (3.1)$$

Where for each fixed x (o < x < a, o < t < ∞), the right hand side of (3.1) has a sense as an application of $f, \in LM^{*}_{f,u,b,c,\alpha}$ to $e^{-st}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right) \in LM_{f,u,b,c,\alpha}$

4. Analyticity of Distributional Generalized Laplace – Finite Mellin Transfom.
4.1. Theorem

Let $f(t,x) \in LM^{*}_{f,u,b,c,\alpha}$ and its Laplace – finite Mellin transform F (S, P) be defined as (3.1). The F (S, P) in analytic for some fixed p > 0 and $s \in \Omega_f$ where $\Omega_f = \{S : \sigma_1 < \text{Re } s < \sigma_2\}$ and

$$D_s F(S,P) = LM_f \{f(t,x)\} = F(S,P) = < f(t,x), e^{-st}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right) >$$

let S be an arbitrary but fixed point in Ω_f . Choose the real positive no. a, b, r such that $\sigma_1 < a < s - r < s + r < b < \sigma_2$. Also let Δs be complex increment such that $0 < |\Delta s| < r$

Consider

$$\frac{F(s+\Delta s, p) - F(s,p)}{\Delta s} - \left\langle f(t,x), \frac{\partial}{\partial s} e^{-st}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$$= \left\langle f(t,x), \frac{\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)}{\Delta s}\left[e^{-(s+\Delta s)t} - e^{-st}\right]\right\rangle - \left\langle f(t,x), \frac{\partial}{\partial s} e^{-st}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$$= \left\langle f(t,x), \frac{1}{\Delta s}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\left[e^{-(s+\Delta s)} - e^{-st}\right] - \frac{\partial}{\partial s} e^{-st}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$$= \langle f(t,x), \psi_{\Delta s}(t,x)\rangle,$$

where $\psi_{\Delta s}(t,x) = \frac{1}{\Delta s}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\left[e^{-(s+\Delta s)t} - e^{-st}\right] - \frac{\partial}{\partial s} e^{-st}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)$

To prove $\psi_{\Delta s}(t,x) \in LM_{f,u,b,c,\alpha}$ we shall show that as $|\Delta s| \to 0$, $\psi_{\Delta s}$ (t, u) converges in $LM_{f,u,b,c,\alpha}$ to zero. Let C denote the circle with centre at s and radius r_1, where $0 < r < r_1 < \min(s-a, b-s)$

We may interchange differentiation on s with differentiation on t & by using Cauchy's integral formula.

$$\left(-D_t^l\right)\psi_{\Delta s}(t,x)=\frac{1}{\Delta s}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\left[(-1)^{\ell}(s+\Delta s)^{\ell}e^{-(s+\Delta s)t}-(-1)^{\ell}s^{\ell}e^{-st}\right]-\frac{\partial}{\partial s}(-s)^{\ell}e^{-st}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)$$

$$=\frac{1}{2\pi i\,\Delta s}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\int_c\left[\frac{1}{z-s-\Delta s}-\frac{1}{z-s}\right](-1)^{\ell}Z^{\ell}e^{-zt}dz-\frac{1}{2\pi i}\int_c\frac{(-z)^1 e^{-zt}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)}{(z-s)^2}dz$$

$$=\frac{1}{2\pi i}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\int_c\left[\frac{1}{(z-s-\Delta s)(z-s)}-\frac{1}{(z-s)^2}\right](-z)^{\ell}e^{-zt}dz$$

$$=\frac{\Delta s}{2\pi i}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\int_c\frac{(-z)^{\ell}e^{-zt}dz}{(z-s-\Delta s)(z-s)^2},$$

$$D_t^1.D_x^q\,\psi_{\Delta s}(t,x)=\frac{\Delta s}{2\pi i}\left[a^{2p}P(-p-q)\,x^{-p-1-q}-P(p-q)\,x^{p-1-q}\right]\int_c\frac{(-z)^{\ell}e^{-zt}dz}{(z-s-\Delta s)(z-s)^2}dz$$

where P (-p –q) is a polynomial in (-p –q) etc.

Now for all $z\in c$ and $0<t<\infty$,

$$\sup_I|k_{u,v}(t)\lambda_{b,c}(x)x^{q+1}D_x^q\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)|\le N\text{, for some constant } N,$$

where N is a constant independent of z and t.

Moreover $|z-s-\Delta s|>r_1-r>0$ and $|z-s|=r_1$

$C_1=\text{Max}\{|z|^{\ell},\ z\in c\}$ consequently

$$\sup|k_{u,v}(t)\lambda_{b,c}(x)x^{q+1}D_t^{\ell}D_x^q\psi_{\Delta s}(t,x)|$$

$$=\sup|k_{u,v}(t)\lambda_{b,c}(x)x^{q+1}\frac{\Delta s}{2\pi i}\left[a^{2s}P(-p-q)x^{-p-1-q}-P(p-q)x^{p-1-q}\right]\int_c\frac{(-z)^1e^{-zt}}{(z-s-\Delta s)(z-s)^2}dz|$$

$$\le\sup|k_{u,v}(t)\lambda_{b,c}(x)x^{q+1}\left[a^{2s}P(-p-q)x^{-p-1-q}-P(p-q)x^{p-1-q}\right]\frac{|\Delta s|}{2\pi}\int_c\frac{|z|^{\ell}|e^{-zt}|}{|z-s-\Delta s||z-s|^2}|dz|$$

$$\le\frac{|\Delta s|}{2\pi}\int_c\frac{N.C_1}{(r_1-r)r_1^2}|dz|$$

$$\le\frac{|\Delta s|C_2}{2\pi(r_1-r)r_1^2}\int_c dz\qquad,\{C_2=NC_1\}$$

$$\leq \frac{|\Delta s| C_2}{(r_1 - r) r_1}$$

$\therefore$ The R.H.S. is independent of t and Converges to zero of $|\Delta s| \to 0$

This shows that $\psi_{\Delta s}(t, x)$ converges to zero in $LM_{f,u,b,c,\alpha}$ as $|\Delta s| \to 0$

CONCLUSION

In this paper Laplace-finite mellin transform is extended in the distributional Generalized sense. Analyticity theorem for distributional generalized Laplace-finite mellintransform is proved.

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