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## **Application of Fractal Geometry to Lakes**

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## Abstract

In this paper we are presenting study of characterization of irregular shapes using the concept of fractal dimension. It is demonstrated that the lakes originating as the result of natural process have ramified boundaries and have detailed structure, where as those arising from artificial conditions like human interference do not exhibit ramified boundaries and details of structure. Lonar Lake (India) originating from striking of meteorite is like a circular cup and does not have structural details. Similarly, Kankaria Lake from Ahmedabad in the heart of the city is surrounded by population from all the side and has a circular shape with a small Island in the middle and a passage leading to the Island. Fractal analysis of the lakes studied, using box counting technique, reveals that all the lakes that came into existence as a consequence of competing natural processes are having self similarity and scale invariance over a larger range length scale and exhibit fractal character with higher fractal dimensions. Where as the Lonar Lake and the Kankaria Lake have very limited degree of self similarity over a shorter range of length scale and thus have a smaller fractal dimension. Richardson Plot technique is also known to be a powerful tool in study of structural and textural details of irregular objects, thus few lake boundaries / contours are also analyzed using this technique. It is clearly seen that the natural lakes show more structural complexity as compare to the lakes resulting from human intervention / interference. However at shorter length scale some textural details are seen that represent textural complexity at shorter length scale.

**Key words:** Fractal, Fractal dimension, Fractal Geometry, Self-similarity, Richardson plot, Box counting.

## **INTRODUCTION**

B B Mandelbrot [1, 2] shown relevance of many naturally occurring phenomena and objects which have there own characteristic appearance in terms of shape. Pattern formation is one of the important in nature diffusion controlled pattern formation [3] have been recent center of interest and may processes have been identified as DLA or DLA like processes. Amongst them the dendritic crystal growth, viscous fingering, electrodeposition and DLA (diffusion limited aggregation) [4, 5, 6] have received the most of the attention. Recently we have studied [7] the growth velocities of Viscous Fingering pattern in Hele Shaw cell.DLA and patterns developed as a result of DLA are often found in various processes in physical sciences, chemical sciences, and engineering. Benoit Mandelbrot, the founder of fractals [8] has first noticed the properties of fractals on the coast of Britain.

Natural growth processes very often lead to self-similar systems with fractal dimensions [9]. The concept of fractal is derived from the Latin word 'fractus' ("fragmented") and non fractal aggregation is applicable in physics especially in turbulence [10,11], polymerization,[12,13]. Flocculation, coagulation, dendritic growth, crystallization. Gelation process also exhibit self-similarity and fractal character in many cases. The fundamental and practical importance of Diffusion limited growth processes has motivated extensive numerical and theoretical studies in the past few years. Electro-deposition processes [14, 15] are well suited for experimental studies of growth of fractals and dendritic patterns. The fractal boom began in the 1980s and Physicists took keen interest in this area. Fractal models were later identified practically in every field of science. Fractal model is being used these days for the forecasting in the trends of the share market also [16, 17].One such application has been studied by us [18] in which the time series was studied applied to Sun spots Data and a Hurst Exponent was calculated.

The patterns created by water as it seeps into the soil or flows through coffee grinds or any powder for that matter, are described by fractals known as percolation clusters. Electrical discharges and the growth of some crystals generate fractal shapes. Some solidification processes also lead to self-similar patterns and fractals [19].Gas particle hitting a solid aggregate to form a crystal by releasing energy, also exhibit fractal character [20].

Formation of lakes is also governed by innumerable competing random processes that include rains, flow of water, seasonal changes, properties of soil and land, flora and fauna of that region. The net resulting effect of all these competing processes results very much like DLA and thus the shapes of many mountain reefs and lakes exhibit resemblance to the DLA patterns.

## MATERIAL AND METHODS

## Geological aspect of Lake formation:

Matter related to the geological aspect as to how the natural processes govern the formation of mountains, rivers and lakes and comparison with artificial lakes. Many naturally occurring processes and plants show fractal structures: for example trees, the roots of the trees to the cauliflower and broccoli. The patterns of blood vessels in the human body, kidney, the lung, and some types of nerve cells have fractal feature. In material sciences, fractals appear in polymers, gels, ionic glasses, aggregates, electro- deposition, rough interfaces and surfaces [21], as well as in fine particle systems. The fractal dimension is a measure of roughness of features.

Geophysical data like topography, magnetics, surface gravity exhibit power law and scale invariance. Although concept of fractals can be used to quantify the distribution, it is now

considered to be a tool that gives deeper insight into the processes responsible to giving rise to such shapes and distributions.

Topography of earth is a result of innumerable competing process having different levels of influences of various kinds. This includes tectonic processes, faulting, folding and flexure. The topography is modified and destroyed by processes like erosion and sedimentation [22]. River trees are typical example and exhibit fractal character. Topography often appears to be complex and chaotic [23], yet there is order in complexity. More the effect of random processes affecting the topography, greater is the complexity and details to the resulting shapes.

In all these structures there is no characteristic length scale in the system besides the physical upper and lower cutoffs.

#### **Experimental:**

For the present study several lakes from different regions such as India, America, Russia. and Finland are selected based on their importance or structural appearance or geological relevance. The images of the lakes were retrieved from Google earth from suitable height so that optimum picture clarity should be utilized. The pictures were then processed by converting them into 256 gray scale and then selecting suitable threshold, the images of the lakes were selected. The images so obtained were then converted to two color bitmaps for further analysis and stored as BMP files.

For box counting technique earlier programs written in Turbo-basic were used which later the same algorithm was implemented in C++. The programme converts the bitmap file into a matrix for counting the occupied cells using different size of boxes and keeps record of the same. Thus using different box sizes (r), the number of boxes (N) required to cover the pattern is counted and the results are saved in a file for further use and processing. A log (N) is plotted against log (r), the points shown in the plot are actual data points and the line joining the points is the least square fit applied to those points. The equation in the inset is the equation best representing the data. From the slope of the line, the fractal dimension of the shape is found.

#### Lakes studied

We studied twelve lakes from different regions, arbitrarily chosen from different regions having technical differences in terms of complexity of shape, structure and texture of the boundary. Eight of them are from India and remaining are from rest of the world. The details of lakes with approximate farets Diameter and longitude and latitudes are as follows:

Lake 1. Lake near Petrozavadsk, Russia. Farets diameter 230 Km, 61°,51' N and 34°,03' E.

Lake 2. Oulu Lake, Finland. Farets diameter 59 Km, 64°,20' N and 27°,20' E.

Lake 3. Black Sea, Europe. Farets diameter 1129 Km, 44°,57' N and 34°,38' E.

Lake 4. Bonner Lake, North America. Farets diameter 40 Km, 48°,07', N and 116°,19' W.

Lake 5. Star Dust Lake Tahoe, USA. Farets diameter 330 Km, 66°,04' N and 120°,57' W.

Lake 6. Bamni Lake near Hingoli, India. Farets diameter 18 Km, 19°,46' N and 76°, 39' E.

Lake 7. Digras Dam near Pusad, India. Farets diameter 11 Km, 20°,08' N and 77°, 45' E.

Lake 8. Kankaria Lake Ahmedabad, India. Farets diameter 0.6 Km, 23°,00' N and 72°, 36' E.

Lake 9. Lake near Rahur, India. Farets diameter 19 Km, 19°,19' N and 74°, 30' E.

Lake 10. Lonar Lake (Crater), India. Farets diameter 1.25 Km, 19°,58' N and 76°, 30' E.

Lake 11. Jaikwadi Dam, Paithan, India. Farets diameter 39 Km, 19°,29' N and 75°, 18' E.

Lake 12. Isapur Dam, near Hingoli, India. Farets diameter 19 Km, 19°,43' N and 77°, 26' E.

1.1 Details of Analysis

Lake 1. Lake near Petrozavadsk, Russia with Farets diameter of 230 Km, is shown in Fig. 1 and the extracted image in two colour bit map is shown in Fig. 2.

Lake 2. Oulu Lake, Finland. Farets diameter 59 Km, is shown in Fig.5 and the extracted image in two colour bit map is shown in Fig. 6.

Lake 3. Black Sea, Europe. Farets diameter 1129 Km, is shown in Fig. 9 and the extracted image in two colour bit map is shown in Fig. 10.

Lake 4. Bonner Lake, North America. Farets diameter 40 Km, is shown in Fig. 13 and the extracted image in two colour bit map is shown in Fig. 14.

Lake 5. Star Dust Lake Tahoe, USA. Farets diameter 330 Km, is shown in Fig. 17 and the extracted image in two colour bit map is shown in Fig. 18.

Lake 6. Bamni Lake near Hingoli, India. Farets diameter 18 Km, is shown in Fig. 21 and the extracted image in two colour bit map is shown in Fig. 22.

Lake 7. Digras Dam near Pusad, India. Farets diameter 11 Km, is shown in Fig. 25 and the extracted image in two colour bit map is shown in Fig. 26.

Lake 8. Kankaria Lake Ahmedabad, India. Farets diameter 0.6 Km, is shown in Fig. 29 and the extracted image in two colour bit map is shown in Fig. 30.

Lake 10. Lonar Lake (Crater), India. Farets diameter 1.25 Km, is shown in Fig. 33 and the extracted image in two colour bit map is shown in Fig 34

Lake 9. Lake near Rahur, India. Farets diameter 19 Km, is shown in Fig. 37 and the extracted image in two colour bit map is shown in Fig. 38.

Lake 11. Jaikwadi Dam, Paithan, India. Farets diameter 39 Km, is shown in Fig. 41 and the extracted image in two colour bit map is shown in Fig. 42.

Lake 12. Isapur Dam, near Hingoli, India Farets diameter 19 Km, is shown in Fig. 45 and the extracted image in two colour bit map is shown in Fig. 46.

Srno	Fig	Lake no	Laka / Dam	Box	<b>P</b> <sup>2</sup>	Richardson Fract dim	R <sup>2</sup>
51 110.	Fig			Fract dim	К		
1	Fig 1	Lake 1	Petrozavodsk lake Finland	1.7483	0.997	1.2435	0.9307
2	Fig 5	Lake 2	Oulu lake Finland	1.7804	0.9989	1.2865* 1.4734**	0.9597 0.9494
3	Fig 9	Lake 3	Blacksea lak Finland	1.8492	0.9986	1.1132	0.9252
4	Fig 13	Lake 4	Bonner lake	1.7195	0.997	1.2371	0.9577
5	Fig 17	Lake 5	Stardust Lake	1.7975	0.9982	1.1762* 1.5432**	0.9458 0.9044
6	Fig 21	Lake 6	Bamni Lake near Hingoli, India	1.7346	0.9976	1.354	0.9469
7	Fig 25	Lake 7	Dam Digras, India	1.7238	0.9972	1.314	0.9747
8	Fig 29	Lake 8	Kankarya 100m Ahmedabad , India	1.8474	0.9986	1.0311	0.7292
9	Fig 33	Lake 9	Lonar Crater, India	1.8285	0.9974	1.0532	0.955
10	Fig 37	Lake 10	Lake near rahur, India	1.6458	0.9983	1.3931	0.961
11	Fig 41	Lake 11	Jaikwadi Dam. Paithan, India	1.701	0.9968	1.2776	0.9726
12	Fig 45	Lake 12	Isapur Dam Near Hingoli, India	1.6538	0.998	1.5498	0.911

Table 1 shows summary of fractal Dimension of lakes / Dams

\*Values of fractal dimensions at shorter length scale, \*\*Values of fractal dimensions at larger length scale

## **RESULTS AND DISCUSSION**

The study of shapes of different lakes selected from different regions, using box-counting technique indicates that almost all the shapes exhibit self-similarity over wide range of length scale. Depending on the complexity of shape, the resulting fractal dimensions are different as shown in the table 1. Shapes having complexity to their structure exhibit higher fractal dimension as compared to those having minor details to their shapes. As seen from the table Lake in fig3, lake 8 and 9 have limited details to their shape and exhibit higher fractal dimensions because of their space filling appearance (a filled square has D = 2).

Richardson's plot technique is a better technique in quantification of the complexity of the structure from the point of view of structural and textural details. The fractal analysis using Richardson's Plot reveals that most of the boundaries of the lake exhibit self similarity over a wide rang of length scale and thus exhibit fractal character at all length scales. This is

characteristic of random shapes evolving as a result randomly occurring natural processes giving rise to those shapes. Lake 3, lake 8 and 9 give a very low value of fractal dimension, close to 1 indicting that they have very limited structure at all the length scales, lake 8 is lakes that is made round and is occupied on all sided by dense population, effectively human interference forced the shape to near round shape without structure or texture to its shape and thus has lower fractal dimension. The human waste and other sewage discharges are if grows to a lake, slowly it will turn to an eutrophic lake [24] during course oligotrophic lake turns in to an eutrophic lake .becuse of eutrophication such lakes show less crisp boundaries and hence they show lower fractal Dimsion. Lake 9 is the Lonar lake is a crater that came in to existence as a result of impact of a huge meteorite, and thus it is near circular shape with limited structure or texture to its shape as is indicated by lower fractal dimension.



Fig.1: Lake near Petrozavadsk, Russia.



Fig. 2. Two color bitmap of fig 1, Lake near Petrozavadsk, Russia.



Fig. 5: Oulu Lake, Finland.





Fig. 9 Black Sea. Lake



Fig. 10. Black Sea Lake two color bitmap.



Fig.13 Bonner Lake USA.



Fig. 14 Two color bitmap of Fig. 13.: Bonner Lake USA



Fig.17 Star Dust Lake USA.



Fig. 18 Two color bitmap of Fig. 17: Star Dust Lake USA



Fig.21 Bamni Lake near Hingoli, India.



Fig. 22 Two color bitmap of Fig. 21. Bamni Lake near Hingoli, India.



Fig.25: Digras Dam near Pusad



Fig. 26: Two colour bit map of Fig. 25



Fig. 29. Kankaria Lake Ahmedabad, India.



Fig.30: Two colour bit map of Fig. 29



Fig. 33 Lonar Lake (Crater), India.



Fig. 34 Two colour bit map of Fig.33

#### CONCLUSIONS

Lakes (Lake 2 and 5 ) have complex shape and exhibit two different fractal dimensions at different length scales. At shorter length scale the fractal dimension is low, indicating limited amount of texture to their shape, however the fractal dimension on the longer length scale is on the higher side indicating more of structure to their shapes at longer length scale as is evident from fig. 5 and 17.

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