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Bianchi type –III cosmological model electromagnetic field with cosmic strings in general theory of relativity

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ABSTRACT

Bianchi type –III Cosmological model filled with a magnetized Cosmic strings is investigated in general relativity. Here we assume that $F_{23}$is only non-vanishing component of electromagnetic field tensor F_{ij}. We obtain exact solution by assuming that the expansion θ in the model is proportional to the shear σ which leads to $C = B^n$ where B and C are functions of time only. The physical and geometrical behaviour of cosmological model are discussed.

Key words: Bianchi type –III, Cosmic strings, Magnetic field.

INTRODUCTION

Space-times admitting a three parameter group of automorphisms are important in discussing the cosmological models. The case where the group is simply transitive over the 3-dimensional, constant time subspace is particularly useful for two reasons. First, Bianchi has shown that there are only nine distinct sets of structure constants for groups of this type. Therefore, we can use algebra to classify the homogeneous Space -times. The second reason for the importance of Bianchi type Space -times is the simplicity of the field equations.

When we study the Bianchi type models, we observe that the models contain isotropic special cases and they permit arbitrarily small anisotropic levels at some instant of cosmic times.

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies.

It is still a challenging problem before us to know the exact physical situation at very early stages of the formation of our universe. The string theory is a useful concept before the creation of the particle in the universe. The string are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the universe.

The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, the galaxy formation can be explained by the density fluctuations of the vacuum strings.

Lorenz [1] has presented Tilte electromagnetic Bianchi type III cosmological solution. Tikekar and Patel [2] obtained some exact solutions of massive string of Bianchi type –III space time presence and absence of magnetic field. Bali and Jain [3] have studied Bianchi type –III non-static magnetized cosmological model for perfect fluid distribution in general relativity. Amirhaschi H. Zainuddin [4] have obtained Bianchi type III string cosmological models for perfect fluid distribution in general relativity. Pradhan [5] has presented Massive string cosmology in Bianchi type III space-time with electromagnetic field. Amirhaschi H. Zainuddin [6] studied Magnetized Bianchi type III massive string cosmological model in general relativity. Pradhan and Amirhaschi H. Zainuddin [7] have studied Dark energy model in anisotropic Bianchi type –III space time with variable EoS parameter. Adhav [8] have obtained Bianchi type –III magnetized wet dark fluid cosmological model in general relativity. Upadhaya and Dave [9] have investigated some magnetized Bianchi type –III Massive string Cosmological models in general relativity.

In this paper, we have investigated Bianchi type –III cosmological model Electromagnetic field with cosmic strings in General theory of relativity. To obtained exact solution field equations, we have considered Massive strings. The physical and geometrical behaviour of the model are also discussed.

1. THE METRIC AND FIELD EQUATIONS

We consider the spatially homogeneous and anisotropic Bianchi type –III metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2ax}dy^2 + C^2dz^2 \quad (2.1)$$

Where a is non -zero constant and A, B and C are function of t.

The energy momentum tensor for a cloud of massive string coupled with electromagnetic field of the form

$$T_i^j = \rho v_i v^j - \lambda x_i x^j + E_i^j \quad (2.2)$$

Where, ρ is the rest energy density for a cloud of strings with particles attached along the extension

Thus $\rho = \rho_p + \lambda$ (2.3)

Where, ρ_p is particle energy density and λ is the tension density of the string. v^i - are the four vectors representing the velocity of cloud of particles and x^i - the four vectors representing the direction of anisotropy, i.e. z- direction.

Where v_i and x_i satisfy condition

$$v_i v^i = -1,\; x_i x^i = 1 \text{ and } v_i x^i = 0 \quad (2.4)$$

Electromagnetic field is defined as

$$E_i^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j \quad (2.5)$$

Where, E_i^j is electromagnetic energy tensor and F_i^j is the electromagnetic field tensor.

We assume that F_{23} is the only non-vanishing component of F_{ij} which corresponds to the presence of magnetic field along z-direction.

For the line – element (2.1), in a co-moving system, we have

$$T_1^1 = \frac{(F_{23})^2 e^{2ax}}{2B^2C^2} \quad (2.6)$$

$$T_2^2 = \frac{-(F_{23})^2 e^{2ax}}{2B^2C^2} \quad (2.7)$$

$$T_3^3 = -\lambda - \frac{(F_{23})^2 e^{2ax}}{2B^2C^2} \tag{2.8}$$

$$T_4^4 = -\rho + \frac{(F_{23})^2 e^{2ax}}{2B^2C^2} \tag{2.9}$$

Allother $T_i^{\ j} = 0$ (2.10)

The Einstein field equation in the general relativity is given by

$$R_i^j - \frac{1}{2} R g_i^{\ j} = -8\pi k T_i^{\ j} \tag{2.11}$$

Where, R_i^j is known as Ricci tensor and $R = g^{ij} R_{ij}$ is the Ricci scalar and $T_i^{\ j}$ is energy momentum tensor for matter.

The field equations (2.11) together with the line element (2.1) with equations (2.6) to (2.10) we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G \left[\frac{(F_{23})^2 e^{2ax}}{2B^2C^2} \right] \tag{2.12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G \left[\frac{(F_{23})^2 e^{2ax}}{2B^2C^2} \right] \tag{2.13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = 8\pi G \left[\lambda + \frac{(F_{23})^2 e^{2ax}}{2B^2C^2} \right] \tag{2.14}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = 8\pi G \left[\rho - \frac{(F_{23})^2 e^{2ax}}{2B^2C^2} \right] \tag{2.15}$$

$$a\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) = 0 \tag{2.16}$$

From equation (2.16) become

$$\frac{\dot{B}}{B} = \frac{\dot{A}}{A} \tag{2.17}$$

Integrating, we get

$$B = kA \tag{2.18}$$

Where k is a constant of integration

As we wish to consider space –time with Bianchi type –III, we have $B = A$, taking $k = 1$ without loss of generality. Then the field equations (2.12) to (2.15) reduce to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-8\pi G\left[\frac{(F_{23})^2 e^{2ax}}{2B^2C^2}\right] \tag{2.19}$$

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=8\pi G\left[\frac{(F_{23})^2 e^{2ax}}{2B^2C^2}\right] \tag{2.20}$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{a^2}{B^2}=8\pi G\left[\lambda+\frac{(F_{23})^2 e^{2ax}}{2B^2C^2}\right] \tag{2.21}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{B}\dot{C}}{BC}-\frac{a^2}{B^2}=8\pi G\left[\rho-\frac{(F_{23})^2 e^{2ax}}{2B^2C^2}\right] \tag{2.22}$$

From the equation (2.19) and (2.20), we have

$$F_{23}=0 \tag{2.23}$$

The field equations (2.19) to (2.22) reduce to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=0 \tag{2.24}$$

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{a^2}{B^2}=8\pi G\lambda \tag{2.25}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{B}\dot{C}}{BC}-\frac{a^2}{B^2}=8\pi G\rho \tag{2.26}$$

2. SOLUTION OF THE FIELD EQUATIONS

We assume that the expansion is proportional to the shear which is physically conditions. This condition leads to

$$C=B^n \tag{3.1}$$

Where n is a constant

I. Massive strings

In this case $\rho+\lambda=0$ (3.2)

From equation (2.25), (2.26) and (3.2), we have

$$\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}-\frac{a^2}{B^2}=0 \tag{3.3}$$

Using equation (3.1), the above equation reduce to

$$\frac{\ddot{B}}{B}+(n+1)\frac{\dot{B}^2}{B^2}=\frac{a^2}{B^2} \tag{3.4}$$

Now put $\dot{B}=f(B)$ in equation (3.4), we get

$$\frac{d}{dB}\left(f^2\right)+2\frac{(n+1)}{B}f^2=2\frac{a^2}{B} \tag{3.5}$$

Integrating equation (3.5), we get

$$f^2 = \left(\frac{dB}{dt}\right)^2 = \frac{a^2}{n+1} + k_1 B^{-2n-2} \tag{3.6}$$

Where k_1 is integration constant

The Bianchi type –III model in this case reduces to the form

$$ds^2 = -\left(\frac{dt}{dB}\right)^2 dB^2 + B^2\left[dx^2 + e^{-2ax}dy^2\right] + B^{2n}dz^2 \tag{3.7}$$

Where A=T, x=X, y=Y, z=Z

$$ds^2 = -\left[\frac{dT^2}{\frac{a^2}{n+1} + k_1 B^{-2n-2}}\right] + T^2\left[dX^2 + e^{-2ax}dY^2\right] + T^{2n}dZ^2 \tag{3.8}$$

PHYSICAL AND GEOMETRICAL BEHAVIOUR OF THE MODEL

The rest energy density (ρ) and string tension density (λ) for the model (3.7), we have

$$\rho = \frac{1}{8\pi}\left[\frac{na^2}{(n+1)T^2} + \frac{(2n+1)k_1}{T^{2(n+2)}}\right] \tag{3.9}$$

$$\lambda = -\frac{1}{8\pi}\left[\frac{na^2}{(n+1)T^2} + \frac{(2n+1)k_1}{T^{2(n+2)}}\right] \tag{3.10}$$

Using (2.3) we get

$$\rho_p = \frac{1}{8\pi}\left[\frac{2na^2}{(n+1)T^2} + \frac{2(2n+1)k_1}{T^{2(n+2)}}\right] \tag{3.11}$$

$$\theta = (n+2)\left[\frac{a^2}{(n+1)T^2} + \frac{k_1}{T^{2(n+2)}}\right]^{\frac{1}{2}} \tag{3.12}$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left[\frac{a^2}{(n+1)T^2} + \frac{k_1}{T^{2(n+2)}}\right] \tag{3.13}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} = \text{constant} \tag{3.14}$$

The particle density ρ_p and tension density λ of the cloud string vanish asymptotically in general if (n+2)>0. The expansion in the model stops when n=-2. The model starts expanding with a big bang at T=0 and the expansion in the model decreases as time increases if (n+2)>0. Since $\lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of t.

II. Special case

When n=1 in equation (3.1), is given by

$$B = C \tag{3.15}$$

From equation (2.24) and (3.15), we get

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}=0 \tag{3.16}$$

Now put $\dot{B}=f(B)$ in equation (3.16), we get

$$\frac{d}{dB}\left(f^2\right)+\frac{f^2}{B}=0 \tag{3.17}$$

Integrating equation (3.17), we get

$$f^2=\left(\frac{dB}{dt}\right)^2=\frac{k_2}{B} \tag{3.18}$$

Where k_2 is integration constant

The Bianchi type –III model in this case reduces to the form

$$ds^2=-\left(\frac{dt}{dB}\right)^2 dB^2+B^2\left[dx^2+e^{-2ax}dy^2+dz^2\right] \tag{3.19}$$

Where A=T, x=X, y=Y, z=Z

$$ds^2=-\left(\frac{B}{k_2}\right)dT^2+T^2\left[dX^2+e^{-2ax}dY^2+dZ^2\right] \tag{3.20}$$

CONCLUSION

The field equation of a scalar tensor theory of gravitation proposed by Sen- Dunn is solved in the presence of magnetized cosmic string for spatially homogenous and anisotropic bianchi type –III model. In order to solve field equations we have used more general equations of state for the proper rest energy density and string tension density. Since $\frac{\sigma}{\theta}$ is constant in both cases, the models do not approach isotropy at any time. The geometrical and physical behavior of the model is also discussed.

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