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# Bistability, instability and chaos in nonlinear optical systems

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## ABSTRACT

*The main schemes of optical bistability and phase conjugation is review with particular emphasis on the instabilities and routes to chaos that occur there. Their impact on the performances of nonlinear optical devices is discussed.*

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## INTRODUCTION

An optical beam propagating in a medium can modify its own optical path by means of the optical nonlinearity that it induces in the medium and under certain conditions profoundly affect its own propagation characteristics. Similar effects may be also occur when a beam crosses the optical path of a second beam. Up to certain light intensities the observed effects can be simply discussed within the framework of perturbation theory of the optical response of the medium [1] as is known this amounts into assuming a series expansion of the polarization around the zero field intensity. It is however an essential features of nonlinear interactions that such a scheme breaks down if the light intensity reaches certain critical values. The feedback that experiences the light beam through the material nonlinearity then may lead to multistable behaviour, whereby several realizable states may appear same of which are unstable and this is the origin of the appearance of hysteresis cycles and optical instabilities eventually leading to chatic behaviour under different conditions the optical nonlinearity may also give rise to highly stable pulse forms the so called solitary waves or solitons.

These effects provide some of the most convincing manifestations of the general laws that govern nonlinear phenomena and are intensively studied. Along with this much of the present interest in these effects has been spured by their potential use in important and far reaching application such as all optical treatment of information, real time adaptive optics, reshaping and compression of optical pulses e.t.c. In the process of optimizing the performances of such devices however one stumbles on same limitations intrinsically interwoven with the nonlinear optical processes themselves and much effort is concentrated in understanding and properly describing the mechanisms underlying these limitation and to find the appropriate materials.

The propagation proprieties of an optical beam over a distance s in a material medium are characterized by the optical path.

$$\psi = \frac{2\Pi}{\lambda} \int nds \tag{1}$$

where $\lambda$ is the wavelength of the light wave in vacuum and n is the effective refractive index of the medium. For sufficiently large light intensities the later also depends on the electric field intensity E(z, t). Thus an optical beam can act on itself and modify its time and space characteristics. [2] Under certain conditions this self-action has striking influence. For instance, as will be discussed later, by inserting the material inside an Fabry-Perot cavity are obtains [3] a multistable transmission characteristic for the later, furthermore as some of the states are unstable a hysteresis cycle will appear and the system can function as a real time memory element. Such a behaviour, which goes under the name of optical bistability can be encountered [4] and has been observed in a multitude of other configurations whereby a cavity is not the sole agent of the feedback of the optical beam on itself. The conditions for this phenomenon to occur essentially depend on the dynamic and spatial dependence of $\psi$ as a consequence of the action of the optical beam and other external factors: interfaces, external fields, temperature e.t.c. Although a description of the optical bistability can be built up with equation (1) as a starting point.

The physical origin of the electrical field dependence of the index of refraction is the nonlinear polarization set up by the electric field in the material. To simplify things we momentarily consider the stationary regime and monochromatic beams. Furthermore we concentrate on media that possess inversion symmetry. Then the induced polarization can be written

$$\rho = \xi_o X\,(I)\,E \tag{2}$$

where the effective susceptibility X(I) is solely a function of the light intensity I= $\xi_o\, c|E|^2/_2$. Below certain intensities one may use a series expansion and write [4, 5]

$$\rho = \rho^{(1)} + \rho^{(3)} + \rho^{(5)} + \text{--------} = \xi_o(X^{(1)} + X^{(3}|E|^2 + X^{(5)}|E|^4)E \tag{3}$$

where $X^{(n)}$(w, -w, w---) and $\rho^{(n)}$ are the n –th order susceptibility and polarization (at frequency w) respectively, because of the assumed symmetry only odd orders will be present in equation (3). The effective index of refraction n on equation (1) is defined by

$$n^2Ec^2/\xi_o = 1 + X(I) \tag{4}$$

only a function of I. It is in general a complex quantity, since

$$X(I) = X(I) + iX''(I) \tag{5}$$

Contains a real and an imaginary part related to dispersive and absorptive mechanism. Optical bistability has been observed with both types of process. Broadly speaking the physical mechanisms that are response for the intensity dependent X' and X'' are the optical Kerr effect and the intensity dependent absorption respective.

The optical Kerr effect is conventionally defined as [5]

$$n = n_o + n_2\, I \tag{6}$$

where $n_o$ is the real index refraction and

$$n_2 = \frac{3X^{(3)}}{n_o^{(2)}\, \xi_o\, c} \qquad (7)$$

and $X^{(3)}$ is the third order susceptibility of the medium defined by

$$\rho^{(3)}(\omega) = 3\, \xi_o\, X^{(3)}\, (\omega, -\omega, \omega)\, |E|^2 E \qquad (8)$$

we assume that ω is far from any resonance of the medium and we may then neglect the higher order terms in equation(3) hard nonlinearity. Distortion of the valence electronic densities, vibrational, rotational and translational motion of molecules are the main mechanisms that contribute to equation (8), the later ones can lead to thermal changes of the index or to electrostriction. Optical Bistable operation has been achieved with all these mechanism but first, the purely electronic one, is by far the most attractive (fast response, time, no losses, wideband operation); It is also the one where the least progress has been made because of lack of appropriate nonlinear materials.

The classical example of intensity dependent absorption is the saturable absorption where the index of absorption becomes

$$\delta = \frac{a_o}{I + I/I_s} \qquad (9)$$

where $a_o$ is the low intensity absorption coefficient (linear absorption) and $I_s$ is a material characteristic, the saturation intensity. In the absorption case one takes advantage of the occurrence of resonance to enhance the nonlinearity (soft nonlinearity). Absorptive bistability has now been seen in a number of gaseous systems (atomic or molecular) and excitons in semiconductors.

Both dispersive and absorptive mechanisms may cooperate in certain cases and at presently the most promising case is the semiconductors, there the large band density of states over a very extended frequency range allows one to combine many of the advantages of the dispersive and absorptive cases and keep the spatial dimensions small.

**THEORETICAL CONSIDERATIONS AND CALCULATIONS**

The electric field $E_1$ of the incident monochromatic beam of frequency ω sets up an electric field inside the cavity.

$$E(z, t) = \mathrm{Re}(Ee^{-iwt}) \qquad (10)$$

which induces a polarization at the same frequency.

$$\rho = \rho^{(1)} + \rho^{(3)} = \mathrm{Re}(\rho e^{ikz - iwt}) \qquad (11)$$

where we neglect higher order nonlinearities and assumes ω far away from any resonances of the medium (loss – less propagation). $\rho^{(3)}$ given by equation (8) so that

$$\rho = X^{(1)} + X^{(3)}|E|^2)\, E = X(I)\, E \qquad (12)$$

The envelopes E and P are slowly varying functions of the time and space variables, E (z, t) and p(z, t) respectively. We assume that the effective susceptibility X satisfies a Debye relaxation equation [2]

$$\tau X + X = X^{(3)}|E(t)|^2 \tag{13}$$

or in integral form

$$X = X^{(1)} + \frac{X^{(3)}}{\tau} \int_{-\infty} |E(s)|^2 \frac{e^{-(t-s)}}{\tau_{ds}} \tag{14}$$

where $\tau$ is a phenomenological relaxation time. Assuming transverse plane waves Maxwell equation gives

$$\frac{\delta^2 E}{\delta z^2} = \frac{1}{c^2} \frac{\delta^2}{\delta t^2} \left(E + \frac{\rho}{\mathcal{E}_o}\right) \tag{15}$$

$$E = \mathrm{Re}\,\{ E_+ e^{ikz} + E_- e^{-ikz}\}\, e^{-iwt} \tag{16}$$

for the field inside the cavity using the slow varying envelope [6] approximation (SVEA) in equation (15) and separating the forward (+) and backward (-) travelling components one finds [8, 6]

$$\frac{\delta \mathcal{E}}{\delta z} + \frac{n_o}{c} \frac{\delta \mathcal{E}}{\delta t} = \frac{ik}{2} \left\{ (X_{++} + X_{--})\, \mathcal{E}_+ - X_{+-}\, \mathcal{E}_- \right\} \tag{17a}$$

$$-\frac{\delta \mathcal{E}}{\delta z} + \frac{n_o}{c} \frac{\delta \mathcal{E}}{\delta t} = \frac{ik}{2} \left\{ (X_{++} + X_{--})\, \mathcal{E}_- - X_{-+}\, \mathcal{E}_+ \right\} \tag{17b}$$

where

$$X_{ij} = \frac{X^{(3)}}{\tau} \int_{-\infty}^{t} \mathcal{E}_i (z, s)\, \mathcal{E}_j (z, s)\, e^{-(t-s)}/\tau_{dt}$$

In the stationary regime these equations reduce [7] to

$$\frac{\delta \mathcal{E}_+}{\delta \mathcal{E}} = -ik\,\{|E_+|^2 + 2|E_-|^2\}\, E_+ \tag{18a}$$

$$\frac{\delta \mathcal{E}}{\delta \mathcal{E}} = -ik\,\{|E_-|^2 + 2|E_+|^2\}\, E_- \tag{18b}$$

where E = wz/c and $k = 3X^{(3)}/2n_o$ with solution

$$E_+ = E_\pm^{\circ}\, e^{i\varnothing} \pm = E_\pm^{\circ}\, e^{i\varnothing\pm^{\circ} \pm ik} \{ |E_\pm|^2 + 2|E_{z\pm}|^2\}\, \xi \tag{19}$$

with $E_\pm^{\circ}$ and $\varnothing^{\circ}$ determined by the boundary conditions. Since k is real (nolosses) $|E_\pm|^2$ do not vary with $\xi$ but $\varnothing_+ - \varnothing_-$ does [8] or

$$\varnothing_+ - \varnothing_- = 2n_o\,\xi + 2n_2\,\{|E_+|^2 + 2|E_-|^2\}\,\xi - \varnothing_o \tag{20}$$

where $\varnothing_o$ is chosen so that $\varnothing_+ = \varnothing_-$ at $\xi = \omega L/c$, from this and the conditions for flux conservation at the back mirror.

$$(I - R)\, n_o|E_+|^2 = I_t \quad (21)$$

$$|E_-|^2 = R|E_+|^2 \quad (22)$$

one obtains expression for the transmission with

$$\varnothing = \delta_o + \delta^1 T_t \quad (23)$$

where $\delta_o = 4\pi\, n_o\, L/\lambda$ and $\delta^1 = 4\pi\, n_2(I+R)L/(I–R)$
Expression can also be written as

$$\frac{I_t}{I_i} = \frac{\varnothing - \varnothing_o}{\delta^i I_i} = G \quad (24)$$

so that the operation point of the nonlinear Fabrg – Perot cavity is the intersection of the Airy function and the straight line and this allows the graphical solution depicted in fig. 1 which clearly shows the Bistable operation and hysteresis. As $I_i$ further increases one obtains additional hysteresis loops and multistability.

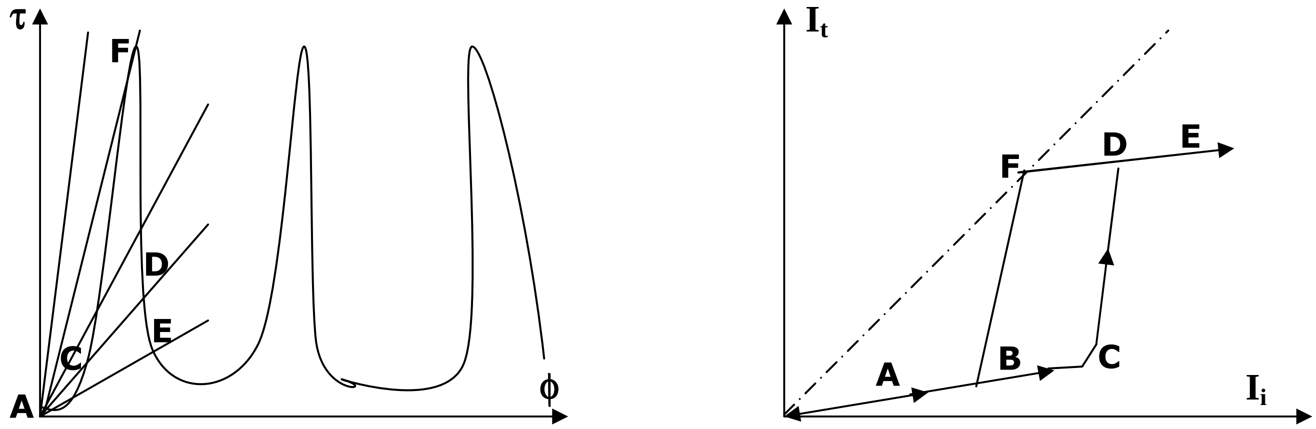


**Fig. 1 Graphical solution of the hysteresis loop in dispersive bistability**

The hysteresis is obtained because the intersection of the straight line and the dashed part of the Airy function is an unstable point or stated differently if the system is forced to operate there even the smallest deviation will grow monotonically with time. To prove this the time evolution of the field amplitude $E_+$ and phase $\varnothing$ must be taken into account. To a good approximation we may assume G and $\varnothing$ reach their stationary solutions established above according to the equations.

$$\tau_1\varnothing + \varnothing = \delta_o + \delta^1 G I_i \quad (25)$$

$$\tau_1 G + G = \frac{1}{I + F^2\sin^2\varnothing/2} \quad (26)$$

Then according to the stability criterion the solution $\{\varnothing_o, G_o\}$ is stable if

$$I > G_o\varnothing_o \quad (27)$$

otherwise it is unstable; one can easily see that the latter situation occurs for the intersection points along the dashed parts of the Airy function. We also see that there will be a critical slowing down so that this type of optical bistability shares many common features with other first order phase transitions. When the complete time evolution of equation (18) and equation (13) is taken into account new instabilities appear even for the other points according to a well defined pattern which leads to chaos.

Equation (26) can be assumed to be a consequence of equation (18) when the time evolution of the field amplitudes is taken into account. Equation (25) on the other hand follows from equation (13) which is based on a simple Debye theory of polarization relaxation. One can easily show that the nonlinear index of refraction obeys equation [2, 12]

$$n = n_o + \frac{n_2}{\tau} \int_{-\infty}^{t} I(s)\, e^{-(t-s)/\tau} ds \tag{28}$$

where $n_2$ is the stationary value defined by equation (6). In terms of equation (28) one may set up a dynamic equation for the optical path equation (1). Introducing equation (28) in equation (13) and differentiating with time one finds

$$\tau\varnothing + \varnothing = n_2 L I(t) \tag{29}$$

which is precisely the equation (25)

## RESULTS AND DISCUSSION

Optical Bistable operation in the dispersive regime has now been observed on a number of systems both in the stationary and dynamic (or transient) regime. This is the case in same atomic [9, 10] and molecular [11] vapours under near resonant conditions where the Kerr index can be substantially enhanced by the resonance condition. There have also been observations in the purely dispersive case (far from any resonance) in liquid crystal, [12] Polydiacetylene Crystals [16], LiNbo3 [17] and other hybrid configuration [18]. The use of liquid crystals [12] as the nonlinear medium among other things allowed a detailed study of the dynamic behaviour of the optical Bistable operation, there one can critically modify the Kerr coefficient and its dynamics by varying the temperature as well as the cavity and one is able to cover all cases of transient regime up to and including the stationary case, the linear round trip phase shift $\varnothing_o$, the pulse duration, the Debye relaxation time $\tau$, the stationary Kerr coefficient $n_2$, the round trip time were varied at will relative to each other and different transient and stationary regimes were studied. In fig. 2 we show some experimental hysteresis loops obtained in different cases.

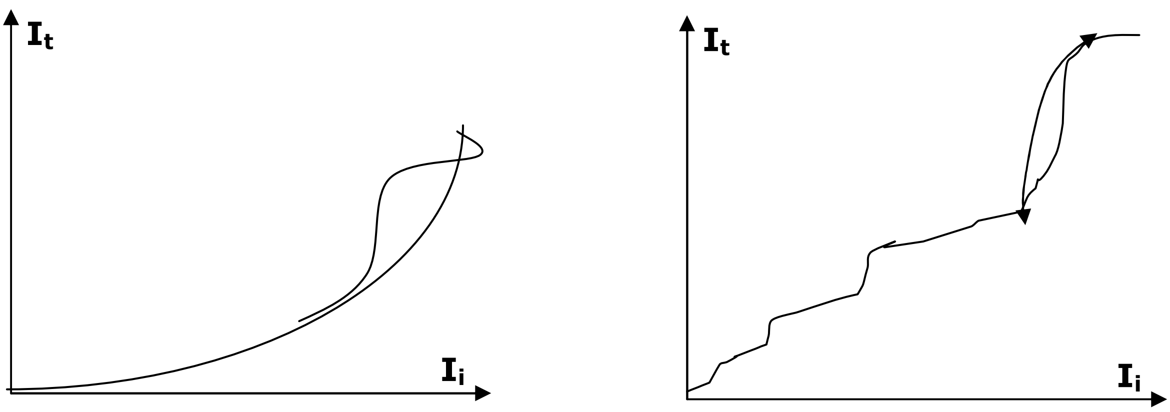


**Fig 2. Experimental hysteresis loops (a) optical fiber ring cavity (ref. 62) (b) InSb cavity (ref. 37)**

It was also hinted that not all stationary solutions are stable and this was the origin of the hysteresis. It turns out that the actual situation is much more complicated and beyond certain critical values of the external parameters even initially stable solutions become unstable eventually leading the system to be chaotic state [19] by a succession of instabilities. Instabilities are inherently built in these nonlinear systems follows from quite general [20] features of the structure of their stationary solutions.

It has been predicted [13] that optical instabilities and chaos will also occur intrinsically as a consequence of the anharmonic motion of the bound charges, the same one that gives rise to the intrinsic bistability. The starting point is the Duffing equation for a damped anharmonic oscillator with a driving sinusoidal force. For small fields E and amplitudes R the motion can be arranged [14] in terms of simple limit cycles and fixed points. As the electric field becomes large the motions becomes much more complicated and contains [15] a periodic solutions very sensitive to the initial conditions.

This behaviour is particularly easy to visualize when |R| is depicted as a function of $\omega/\omega_o$, or a given intensity of the electric field exceeding certain critical value. For $\omega/\omega_o$ >1 the solutions corresponds to limit cycles but as $\omega/\omega_o$, decreases below 1 then a set of cascading biffurcations at frequencies $\omega_n$ starts which ends at a frequency $\omega_{Th}$ where a chaotic state is established characterized by the appearance of a strange attractor in phase space. Quite revealing is also the power spectrum which shows sharp peaks for $\omega/\omega_n$ but becomes smooth as $\omega < \omega_{Th}$. fig 3 also shows the behaviour of the system as $\omega$ is increased from $\omega << \omega_n$ to $\omega >> \omega_o$.

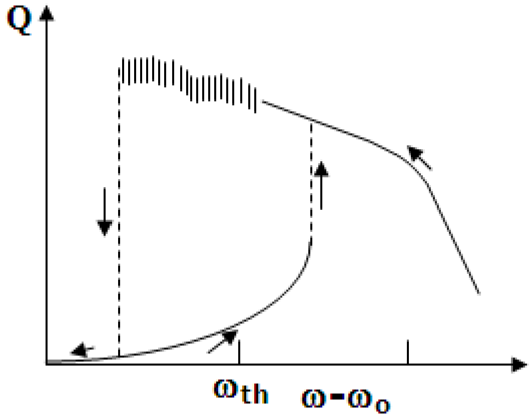


**Fig. 3 Hysteresis and Instabilities and route to chaos in the Duffing oscillator (ref. 60)**

It is also interested to note that the behaviour of the harmonics of $\omega$ depicted in fig 4 and as can be seen hysteresis and instabilities occur there as well.

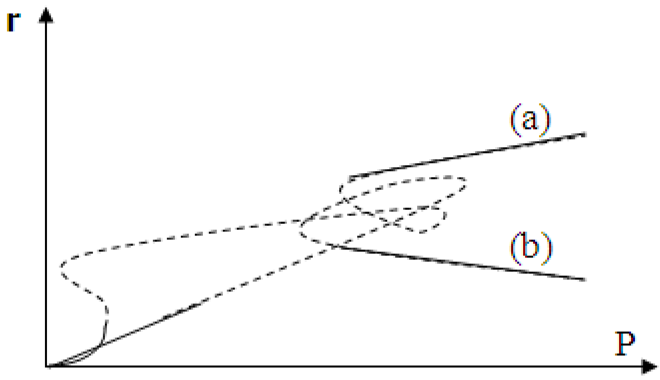


**Fig. 4 Hysteresis and Instabilities in (a) the fundamental and (b) the third of the forced Duffing oscillator**

## CONCLUSION

The accumulated results in the passive system already had a tremendous impact on the choice of some parameters for designing optical Bistable devices but still the picture is not clear. Furthermore, they reveal some intrinsic limitations in some proposed schemes of optical bistability which will not be easily overcome. Despite intensive studies the optical bistables devices based on the nonlinear fabry-Perot cavity in the dispersive regime are far from the expectations required for these devices to become competitive with the electronic ones, these are low powers (o, 1 – 1 MW) fast switching times (~ 10 – 100ps) for both states small dimensions (1 - 10μm) and room temperature operation (300$^{o}$K). No existing schemes meet all these requirements and additional ones in order to be of practical use for real time treatment of optical information. The efforts however to meet these requirements led to much deeper understanding of the nonlinear optical interactions, enriched the area of nonlinear optics with new and powerful concepts and allowed to draw analogies with other nonlinear processes in mechanics, hydrodynamics phase transitions etc and this is one of the major motivations for the continuing activity in this area.

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