Bloembergen’s three level maser in the light of quantum measurement theory

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ABSTRACT

In this work we have discussed the condition of maser in light of quantum zeno effect and measurement theory. In this paper we discussed the limitations of Bloembergen’s three level maser theory and shown that \( \nu_{31} > 2 \nu_{32} \) is valid for the entire situation.

Key words: three level maser, quantum zeno effect

INTRODUCTION

The three level masers require sources of saturating or pumping power at frequencies approximately doubles the signal frequency. Further it is observed that in an assembly, other factors are being fixed, an increase in numbers of atom brings the system closer together which increases intersystem interactions and so the above condition is no longer valid. In this work we have discussed the condition of maser in light of quantum zeno effect and measurement theory. In this paper we discussed the limitations of Bloembergen’s three level maser theory and shown that \( \nu_{31} > 2 \nu_{32} \) is valid for the entire situation. The analysis present in this paper is simple but it opens a new path for the actual mechanism involved in the Bloembergen’s maser.

Bloembergen’s three level maser

The three level method of population inversion was initially proposed by Basov and Prokhorov [1] who suggested it for application in a molecular beam apparatus. Bloembergen [2] subsequently suggested that the method could be readily applicable to diamagnetic solids containing weak concentration of paramagnetic ions presented the theoretical treatment of the three level maser. A three level system shown in Fig.1, the stationary states are characterized by energy \( E \) and resonant frequencies \( \nu_{ij} \).
At thermal equilibrium the populations decrease with increasing energy of state and the assembly will absorb energy from an incident radiation field. Suppose two radiation fields are incident on an assembly, one a very intense one, with frequency near the resonance $\nu_{31}$ and a very weak one near $\nu_{32}$. If the field at $\nu_{31}$ is sufficiently intense $(1, 3)$ transition may be saturated with the result that $n_1 \equiv n_2 \equiv \frac{1}{2}(n_1^e + n_2^e)$ where $n_1^e, n_2^e$ and $n_3^e$ are the populations in the state 1, 2 and 3 respectively. Also the total number of atoms $N$ is given by $N = n_1^e + n_2^e + n_3^e$. Assuming that this saturation process does not disturb the system in state 2, it is reasonable to believe that a condition may be realized in which $n_3^e > n_2^e$, in which case the assembly is in a emissive state relative to field at frequency $\nu_{32}$. Further if it is assumed that relaxation processes are operative in the assembly and that they are characterized by transition probabilities $\Gamma_{ij} = \frac{\Gamma_{ij} e^{-\hbar \nu_{ij}/kT}}{\nu_{ij}}$, and $i, j = 1, 2, 3$. The transition $(1, 3)$ is assumed to be saturated due an intense field at frequency $\nu_{31}$ while a weak field frequency $\nu_{32}$ is assumed to induce transitions between states 2 and 3. The induced transitions probabilities for the system per unit time are designated as $B_{13} = B_{31}$, and $B_{23} = B_{32}$. From the rate of equation it can be obtained that the assembly will emit power at a rate

$$P = \frac{Nh^2\nu_{32}}{3kT} \left( \frac{\Gamma_{12}\nu_{21} - \Gamma_{23}\nu_{32}}{\Gamma_{12} + \Gamma_{23} + B_{23}} \right) B_{23}$$  \hspace{1cm} (2)$$

For $B_{23}$ far from the value required for saturation, and for all $\Gamma_{ij}$ equal, where $i, j = 1, 2, 3$ so $\Gamma_{12} = \Gamma_{13} = \Gamma_{23}$ and by neglecting $B_{23}$ in the denominator of the equation (2) then the equation becomes

$$P = \frac{1}{2} \frac{N}{3} \left( \frac{\hbar \nu_{21}}{kT} - \frac{\hbar \nu_{32}}{kT} \right) B_{23}h\nu_{32}$$  \hspace{1cm} (3)$$

The net power is positive only when $\Gamma_{23}\nu_{32} (\Gamma_{12}\nu_{21}$ and $\nu_{21} = \nu_{31} - \nu_{32}$
So \[ \frac{\Gamma_{23}}{\Gamma_{12}}(\nu_{31} - \nu_{32}) \]
\[ \nu_{31}\nu_{32}(1 + \frac{\Gamma_{23}}{\Gamma_{12}}) \]

(4)

For \( \Gamma_{23} = \Gamma_{21} \) it is obtained that \( \nu_{31} > 2\nu_{32} \). Thus the three level masers requires sources of saturating or pumping power at frequencies approximately doubles the signal frequency. The three level masers require sources of saturating or pumping power at frequencies approximately double the signal frequency. Further it is observed that in an assembly, other factors are being fixed, an increase in \( N \) yields an increase in \( P \). The operation will be improved in this process but it has some limitations. The increase in \( N \) results in bringing the system closer together which increases intersystem interactions and so the analysis given above is no longer valid.

**Measurement and Quantum Zeno Effect**

Frequent measurements inhibits the transitions between quantum states this was based on the usual quantum theory of measurement involving projection operators, showed that an unstable particle which was continuously observed to see if it decayed would never be found to do so. Later, several authors [3-6] studied the problem and generalized disturbance introduced into a quantum system by a performed measurements to inhibition of transitions or quantum jumps as the frequency of observation or measurement was increased. This dynamical behavior is widely known as Quantum Zeno effect. The Quantum Zeno effect has become a topic of great interest in the areas of polarized light[3], the physics of atoms and atomic ions[6,7-8], neutron physics[8], quantum tunneling[9-11], Quantum optics[12] and Lasing without inversion[7,13-14] etc. The quantum mechanical view of the world has compelled us to reshape and revise our ideas of reality and notions of cause, effect and measurement. The probability of finding the system in its initial state after being left to itself for a certain period of time \( t \) is termed as survival probability and for very short time limit it can be written as

\[ P(t) = (1 - \frac{t^2}{\tau^2}) \ldots \]

For \( N \) equal spaced measurements over a time period \((0, T)\). If \( \tau \) is the time interval between two measurements, then \( T = N\tau \). Let us assumed that the measurements are made at times time \( T/N, 2T/N \ldots (N-1)T/N \) and \( T \) are instantaneous. So the survival probability after \( N \) measurements, which is in the limit of continuous measurements

\[ Lt_{N \rightarrow \infty} P^N(T) = Lt_{N \rightarrow \infty} \left(1 - \frac{T^2}{N^2 \tau^2}\right)^N = 1 \]

It is seen in equation (1) that the state will survive for a time \( T \) goes to 1 in the limit \( N \) goes to infinity, means that the continuous measurements actually prevent the system from ever decaying. The seminal formulation of quantum Zeno effect deals with the probability of observing an unstable system in its initial state throughout a time interval.

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DISCUSSION AND CONCLUSION

The limitation of Bloembergen’s three level maser system is brought in the notice of measurement theory and consider each interaction as a measurement. Further we write the equation (4) without disturbing the generality as

$$\frac{\Gamma_{33}}{\Gamma_{12}} \left( 1 - \frac{\nu_{32}}{\nu_{31}} t \right)$$  \hspace{1cm} (5)

Where \( t \) represent time of operation. As increase in \( N \) equation (3) brings the system closer together which increases intersystem interactions and decrease in \( P_L \) at large value of \( N \). We would like to judge the matter in the light of quantum measurement theory and quantum Zeno effect. If we consider the intersystem interaction as measurement then each measurement will the atom to be in their initial state. If we consider the time interval \([0, T]\) and interaction takes place in the interval, the time required per interaction will be as \( T/n, T/n-1, T/n-2, \ldots, T \). Here we will write the equation (5) for \( n \) number of measurement in an approximate form as

$$\left( \frac{\Gamma_{33}}{\Gamma_{12}} \right)^n = \left( 1 - \frac{\nu_{32}}{\nu_{31}} \frac{T}{n} \right)^n$$

Here \( t=T/n \) and we also consider \( n \) increases to infinity then the equation can be written as

$$Lt \quad n \to \infty \quad \left( \frac{\Gamma_{33}}{\Gamma_{12}} \right)^n = Lt \quad n \to \infty \quad \left( 1 - \frac{\nu_{32} \frac{T}{n}}{\nu_{31}} \right)^n$$  \hspace{1cm} (6)

The right hand sides of equation will be 1 always. From this equation we find the condition \( \frac{\Gamma_{33}}{\Gamma_{12}} = \frac{\Gamma_{21}}{\nu_{32}} \), so we can conclude that whatever be the value of \( N \), if it represents more interaction then we can say the above relation \( \nu_{31} \frac{2\nu_{32}}{\nu_{31}} \) is valid for all. The equation (6) represents Quantum Zeno effect. Here in this work it was considered the intersystem interaction as measurement then each measurement will cease the atom to be in their initial state. Though our work is simple in nature but it will give rise of explanation of various phenomenon laser theory like lasing without inversion just in connection with the Bloembergen’s theory.

REFERENCE

[1.] Basov N.G. and Prokhorov.A., Soviet Physics, JETP, 1, 184 (1955)