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Charmonium spectrum and its leptonic decay widths in the frame work of semi relativistic quark model.

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ABSTRACT

The mass spectrum of the S-wave charmonium mesons and its leptonic decay widths is considered in the frame work of semi-relativistic quark model. A good agreement is obtained with the experimental masses. The role of one-gluon-exchange potential is discussed.

Keywords: Quark Model; One-Gluon-Exchange Potential; S-wave charmonium spectra, leptonic decay width.
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INTRODUCTION

Though QCD is accepted as the fundamental theory of strong interactions, there exist no exact solutions to the theory in the non-perturbative low energy regime. The QCD is not exactly solvable in the non-perturbative regime which is required to obtain the physical properties of the hadrons. Hence various approximation methods have been employed to solve QCD in the non-perturbative regime. The most promising of these is through lattice gauge theories [1-3]. The lattice gauge theories involve gigantic computation hence the progress has been slow and detailed predictions of the hadron properties have not been made. As a consequence, our understanding of hadrons continues to rely on insights obtained from the experiments and QCD motivated models in addition to lattice QCD results. The phenomenological models developed to explain observed properties of hadrons are either non-relativistic quark models (NRQM) with suitably chosen potential or relativistic quark models (RQM) [4-9] where the interaction is treated perturbatively. There are successful NRQM and RQM to explain the meson spectra. The NRQM usually contain three main ingredients: the kinetic energy, confinement potential and a hyperfine interaction term which has often been taken as an effective one-gluon-exchange potential (OGEP) [10]. There are models both non-relativistic and relativistic employed to explain meson spectra with OGEP.

In the present work an attempt has been made to obtain the ground state mass of the heavy mesons in the frame work of semi relativistic quark model. Hence, to study the S wave spectra of heavy mesons we have developed a semi-relativistic model, we have made use of the successful relativistic harmonic model (RHM) [11-19] in which the confinement potential is a Lorentz scalar plus vector potential. Both scalar and vector potential are harmonic oscillator potentials. The total Hamiltonian has Lorentz scalar plus vector potential along with OGEP. The full discussion of the Hamiltonian of is given in section 2. A brief discussion of the parameters used in our model follows in section 3. The results of the calculation are presented in section 4 and the conclusions are given in section 5.

2: THEORETICAL MODEL

2.1 Semi-relativistic harmonic model

In RHM [11], quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic-oscillator potential

$$V_{conf}(r) = \frac{1}{2}(1+\gamma_0)A^2r^2 + M \tag{1}$$

where γ_0 is the Dirac matrix:

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

M is the quark mass and A^2 is the confinement strength. They have a different value for each quark flavour. In RHM, the confined single quark wave function (ψ) is given by:

$$\psi = N \begin{pmatrix} \phi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M}\phi \end{pmatrix} \tag{3}$$

with the normalization

$$N = \left(\frac{2(E+M)}{3E+M}\right)^{1/2} \tag{4}$$

Where E is an eigenvalue of the single particle Dirac equation with the interaction potential given in (1). The lower component is eliminated by performing the similarity transformation,

$$U\psi = \phi \tag{5}$$

Where U is given by,

$$\frac{1}{N\left[1+\dfrac{\mathbf{P}^2}{(E+M)^2}\right]}\begin{pmatrix} \mathbf{1} & \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} \\ -\dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} & \mathbf{1} \end{pmatrix} \tag{6}$$

Here, U is a momentum and state (E) dependent transformation operator. With this transformation, the upper component ϕ satisfies the harmonic oscillator wave equation.

$$\left[\frac{\mathbf{P}^2}{E+M} + A^2r^2\right]\phi = (E-M)\phi, \tag{7}$$

which is like the three dimensional harmonic oscillator equation with an energy-dependent parameter Ω_n^2:

$$\Omega_n = A\left(E_n + M\right)^{1/2} \tag{8}$$

The eigenvalue of (7) is given by,

$$E_n^2 = M^2 + (2n+1)\Omega_n^2 . \tag{9}$$

Note that eqn. (7) can also be derived by eliminating the lower component of the wave function using the Foldy-Wouthuysen transformation as it has been done in [19].

Adding the individual contributions of the quarks we obtain the total mass of the hadron. The spurious centre of mass (CM) is corrected [20] by using intrinsic operators for the $\sum_i r_i^{\,2}$ and $\sum_i \nabla_i^2$ terms appearing in the Hamiltonian. This amounts to just subtracting the CM motion zero point contribution from the E^2 expression. It should be noted that this method is exact for the 0S-state quarks as the CM motion is also in the 0S state.

We come now to the description of the quark-anti-quark potential; it is given by the V_{OGEP}. Among the several versions of the VOGEP, we have used the following one, first derived in [10] from the QCD Lagrangian in the non-relativistic limit.

$$V_{OGEP}(r_{ij}) = \frac{\alpha_s}{4}\lambda_i.\lambda_j\,[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j}(1+\frac{2}{3}\,\sigma_i.\sigma_j)\,\delta(r_{ij})] \qquad (10)$$

where the first term is the residual Coulomb energy and the second term the chromo-magnetic interaction leading to the hyperfine splittings. The α_s is the quark-gluon coupling constant.

2.1: Leptonic decays

Mesons exhibit several different decay modes such as leptonic, semileptonic, hadronic, radiative, two photon decays etc. Depending on the decay products and the force involved, the various models are classified. It is to be noted that the description of the meson spectra is a necessary but not a sufficient condition for aiming at a good explanation of non perturbative QCD, since the strong, electromagnetic and weak couplings of mesons can also give important clues to the nature of an observed state. Hence one needs other observables which rely essentially on the same dynamical operators like transitions between various states, in order to test the model under study. So, one of the tests for the success of any theoretical model for mesons is the correct prediction of their decay rates. The leptonic partial widths are a probe of the compactness of the quarkonium system and provide important information complementary to level spacings. The quark-antiquark assignments for the vector mesons, as well as the fractional values for the quark charges, may be tested from the values of their leptonic decay widths. The decay of vector meson into charged leptons proceeds through the virtual photon ($q\bar{q} \rightarrow l^+l^-$). The $^3S_1$ and $^3D_1$ states have quantum numbers of a virtual photon, $J^{PC} = 1^{--}$ and can annihilate into lepton pairs through one photon.

The leptonic decay width of the vector meson is given by the Van Royen-Weisskopf formula [21],

$$\Gamma_{(nS\rightarrow \ell^+\ell^-)} = 4\alpha_{em}^2 \left\langle Q^2 \right\rangle \frac{|R_{nS}(0)|^2}{m_{nS}{}^2} \qquad (11)$$

$$\Gamma_{(nD\rightarrow \ell^+\ell^-)} = 25\alpha_{em}^2 \left\langle Q^2 \right\rangle \frac{|R''_{nD}(0)|^2}{2m_{nD}{}^2 m_q^4} \qquad (12)$$

In the above equations, m_{nS} and m_{nD} are the masses of nS and nD vector meson, Q is the charge content of the quark, m_q is the mass of the quark, α_{em} is the fine structure constant, $R_{nS}(0)$ is the radial S wave function at the origin and $R''_{nD}(0)$ is the second derivative of the radial D wave function at the origin. The vector meson is described by the wave function $\psi(\vec{r})$ for quark-antiquark pair where $\vec{r} = \vec{r}_q - \vec{r}_{\bar{q}}$.The leptonic decay width is proportional to the average value of the squared charge, the squared wave function at the origin which gives the probability that the quark and antiquark will interact with the photon at the origin of their relative coordinates and the mass of the vector mesons. Several experimental quantities are sensitive to squared wave function at the origin. Of these, the one best measured and free from theoretical ambiguities is the rate for vector meson decay into lepton pairs. The average values of the squared charge for different vector mesons are calculated using their wave functions. For charmonium system there are four parameters associated with the calculation of leptonic decay width. These are the masses of the c and b quarks, the confinement strength $a_{c,}$ the harmonic oscillator size parameter b. The quark-antiquark assignments for the vector mesons, as well as the fractional values for the quark charges, may be tested from the values of their leptonic decay widths. The leptonic decay width is proportional to the average value of the squared charge, squared wave-function at the origin and the mass of the vector mesons. Using (11 &12) we have computed leptonic decay widths of light vector mesons which are listed and are compared with experimental values [4] in table IV.

3. Fitting Procedure

The parameters are the masses of charm (M_c) which is taken as parameter. The other parameters are confinement strength A2, the oscillator size parameter b and the strong coupling constant α_s. The value of b is fixed by minimizing the expectation value of the Hamiltonian for the pseudo scalar mesons. The confinement strength A2 is fixed by the stability condition for the variation of the mass of the mesons against the size parameter b. The α_s is fixed by the J/Ψ - η_c mass splitting. The mass difference arises from the colour magnetic term of OGEP. The values of the parameters used in our calculation are listed in table I.

4 Results of S wave Meson Spectra and leptonic decay widths

In the present study, the product of quark-antiquark oscillator wave functions is expressed in terms of oscillator wave functions corresponding to the relative and CM coordinates. The Hamiltonian matrix is constructed for each meson separately in the basis states of $\left| N_{CM} = 0, L_{CM} = 0;\ {}^{2S+1}L_J \right\rangle$ and the masses of the pseudo scalar mesons (PSM) and vector mesons (VM) after diagonalisation for successive values of $n_{\max}$ in M1 are listed in table II and table III respectively. We get a very good agreement with the experimental masses [24] of the ground-state of PSM and VM. The OGEP is attractive for PSM hence the diagonalisation in the space of radially excited states brings down the value of PSM to their physical mass. The calculated values of leptonic decay widths for J/ψ, $\psi(2S)$ and $\psi(4040)$ are given in table IV. The agreement is not very good in comparison with the available experimental data [22].

Table I .Values of the parameters used in our model .

Parameters	
M_c (MeV)	847
Ω(fm)	0.77
α_s	0.6
A^2 (MeV fm^{-2})	3693

Table II. The PSM masses (in MeV) for successive values of n_{max}

nmax	$\eta_c\,(c\bar{c})$
1	3215.42
2	3093.24
3	3022.42
4	2988.94
5	2983.34
Expt.	2980±1.2

Table III. The VM masses (in MeV) for successive values of n_{max}.

nmax	$J/\psi\,(c\bar{c})$
1	3186.73
2	3137.17
3	3121.69
4	3101.56
5	3099.67
Expt.	3096.916±0.011

Table IV Leptonic decay widths of charmonium states [in keV].

Meson	Exptl leptonic width [22]	Calculated leptonic width
J/ψ(1S)	5.55 ± 0.14±0.02	4.27
Ψ(2S)	2.38 ± 0.04	1.89
Ψ(4040)	0.86 ± 0.07	0.43

CONCLUSION

In our present work, we have investigated the masses of ground state charmonium mesons in the semi relativistic model. The calculation shows that the computation of mesonic masses and mass splittings using OGEP is adequate for both PSM and VM. For the attractive OGEP for PSM, the contribution from the off-diagonal elements is found to be significant. For the VM, there is no substantial change in the masses by increasing n_{max} as OGEP is repulsive and hence perturbative techniques are adequate and are justified. There is a significant contribution from the colour electric term of OGEP. The masses for both PSM and VM converge to the experimental values when the diagonalisation is carried out in a larger basis. The proposed model does not give a very good prediction of leptonic decay widths of charmonium.

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