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Construction Schemes for Fully Anti-Symmetric Four-Body Wave Functions in (3+1) and (2+2) Configurations

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ABSTRACT

One of the important problems that arise when investigating dynamics of Few-Body Nuclear systems within the framework of the Hyperspherical Function (HF) Method, is the problem of constructing wave functions that are anti-symmetric under particle interchange. Parentage scheme of symmetrization (PSS) allows to construct N-body symmetrized HF from functions with arbitrary quantum numbers by the use of the transformation coefficients related with the permutations of last two particles. N-body HF corresponding to the representation of the N-particle permutation group were obtained for N=3,4,5,6 by finding parentage coefficients and constructing linear combinations of the N-particle functions corresponding to the irreducible representations of N-1 particle permutation group . However, construction schemes for the fully anti-symmetric wave functions, consisting of spin and isospin parts along with the hyperspherical parts, has not been systematically addressed in the literature. Solution of this problem becomes sufficiently complex as number of particles increases. This article develops construction schemes for four particle wave functions that are anti-symmetric under particle interchange by building all possible combinations of spin, isospin, and hyperspherical parts.

Key words: Wave function, symmetry, N-particle

INTRODUCTION

One of the important problems that arise when investigating dynamics of Few-Body Nuclear systems within the framework of the Hyperspherical Function (HF) Method, is the problem of constructing wave functions that are anti-symmetric under particle interchange. Parentage scheme of symmetrization (PSS) allows to construct N-body symmetrized HF from functions with arbitrary quantum numbers by the use of the transformation coefficients related with the permutations of last two particles. N-body HF corresponding to the representation of the N-particle permutation group were obtained for N=3,4,5,6 by finding parentage coefficients and constructing linear combinations of the N-particle functions corresponding to the irreducible representations of N-1 particle permutation group. However, construction schemes for the fully anti-symmetric wave functions, consisting of spin and isospin parts along with the hyperspherical parts, has not been systematically addressed in the literature. Solution of this problem becomes sufficiently complex as number of particles increases. This article develops construction schemes for four particle wave functions that are anti-symmetric under particle interchange by building all possible combinations of spin, isospin, and hyperspherical parts. It is demonstrated that there are sixteen possible ways to construct fully anti-symmetric four-body wave functions when spin and isospin parts are represented by representations of four-particle permutation group . A complete set of fully anti-symmetric four-body wave functions is obtained for both (3+1) an (2+2) configurations. Proposed construction schemes can easily be generalized for the systems with any number of particles.

The fractional quantum Hall effect (FQHE)—despite its long history —is still one of the most intriguing and widely examined phenomena in the field of condensed matter physics. Although basic requirements for the FQHE development are commonly known and understood (e.g. a two-dimensional topology of a sample, a low but non-zero disorder, etc.), new prerequisites are constantly being discovered. For example, it was recently shown that a high carrier mobility is also indissolubly bounded to this quantum effect—below a selected μmin insulating phases are favourable. Even within the widely accepted model, based on a composite-fermion (CF) creation, further work is needed to provide the explanation of certain incompressible fractions, also in the lowest Landau level.

As an attempt to grasp the entire physics of Hall systems, the topological approach has been formulated. In this paper, we briefly recall its basic assumptions (first section). We also demonstrate that it can be used to identify all Hall-like states with odd denominators within the LLL (second section).It should be emphasized that quantum particles do not travel the braid trajectories. However, the relationship between topological properties, reflected in π1(Ω), and quantum properties of an arbitrary system cannot be ignored. For example, in the standard quantization method, multi-particle state vectors, ΨN, are selected as functions from Ω into complex numbers.

Hence, when classical particles—as arguments of a multi-particle wave function—are encircling a closed path in their configuration space, ΨN acquires a phase equal to a one-dimensional unitary representation (1DUR) of the corresponding element from the system's braid group. Furthermore, the summation over classes of homotopical trajectories (with a 1DUR defining weight factors) is implemented in the Feynman path integral construction of a propagator. As a consequence, the braid group and its scalar representations settle all types of the quantum statistics available for an investigated system (e.g. anyons may be considered only in selected two dimensions, where π1(Ω)≠SN; anyons cannot be realized on a torus; the quasi-particles proposed by Einarsson are connected to multi-dimensional state vectors from non-scalar quantum theories, which only mimic the scalar description. It is easy to note that in great magnetic fields the single-loop cyclotron path of a classical particle (as a representative of a braid group element) can be too small to reach an arbitrary neighbour. This applies if Coulomb repulsion forces protect a uniform distribution (thus, their appearance is a crucial requirement for our approach). In the presented conditions (e.g. a fractional filling of the LLL) simple exchanges, σi, are unenforceable and need to be excluded from the braid group describing the system. Only if the restricted version of the full group, π1(Ω), is a subgroup (the appropriate axioms are satisfied), the quantum statistics of particles is determined and the creation of a collective state can be considered. Since the FQHE is actually observed in experiments, the reduction of π1(Ω) has to result in the emergence of cyclotron subgroups for selected LLL filling factors, ν=N/N0. Generators, bi, of these systems' braid groups stand for novel, multi-loop exchanges of classical particles (loopless, σi, cannot be defined). Finally, the recognition of all possible subgroups, together with appropriate commensurability conditions that determine magnetic field strengths for which they describe the many-body system, allows identification of all possible collective Hall-like states in the lowest Landau band. Several assumptions concerning the theory and wave functions proposed in the following sections of this paper are worth commenting, since they may be confusing or unclear.

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