



Development of State Duration and Expectation Model for Evaluating Remaining Life of Manufacturing Systems

Nwadinobi Chibundo Princewill^{1*}, Nwankwojike Bethrand Nduka², Abam Fidelis Ibiang²

¹Department of Mechanical Engineering, Abia State Polytechnic, Aba, Abia State

²Department of Mechanical Engineering, Micheal Okpara University of Agriculture, Umudike, Abia State, Nigeria

ABSTRACT

The key goal of predicting the state expectation of an engineering system is to predict the remaining life of the system so as to aid in maintenance decision-making activities. In this paper, a maintenance model for predicting the state expectation of industrial machines has been developed. It incorporates various stages of deterioration and maintenance states. Given that a current state has been attained, from inspection and diagnosis, this model is capable of computing the predicted average time before a system failure occurs. This study focuses on using real data of an industrial Bottle filler machine to test the effectiveness of the State Expectation model and its effect on the reliability and maintainability of the machine. The model is tested for various scenarios by changing one of the main parameters during each calculation while others are kept constant. For the state duration sensitivity analysis, as the failure rate continuously increases from 0.0178 to 0.7060, the expected mean sojourning time for each degradation state decreased from 220.97 h to 1.59 h. Subsequently at uniform incrementally varied repair/maintenance rate (0.0861 to 0.7663), the state expectation of the equipment increases from 168.11 h to 1189.98 h. This allowed us to determine the most suitable decision to improve the reliability of the Bottle filler machine. The prediction result identifies the effectiveness of the proposed method in predicting RUL of manufacturing systems.

Keywords: Remaining useful life, State expectation model, Maintenance, Decision making, Prediction

INTRODUCTION

In traditional maintenance technique, equipment is either repaired after failure (reactive maintenance) or scheduled for time-based preventive maintenance (planned maintenance). In recent years, a more cost-effective strategy called condition-based-maintenance (CBM) [1,2] has been implemented. Condition-Based Maintenance is a method that recommends maintenance decisions based on the actual health status of a machine or its components [3,4]. The application of CBM technique brings down maintenance costs and increases efficiency by taking maintenance interventions only when the system exhibits abnormality in its functions [5]. To be able to schedule maintenance based on the condition, Condition Monitoring (CM) to collect condition data; and Remaining Useful Life (RUL) prediction based on condition data are required [6]. Estimation of RUL (or time to failure) of a component or system which can be obtained based on their use and performance can be performed and is known as prognostics/forecasting [7].

As deterioration in equipment sets in, certain performance parameters in the system tend to change, thereby characterizing degradation. Degradation measures consist of sensed measurements, such as vibration analysis, oil analysis, infrared thermography, ultrasonic test and others, or inferred measurements, such as model based predictions. [8] defined RUL as the duration from current time to end of useful life for an equipment/component. It is the time left for a component to perform its functional capabilities before failure. There are several prediction methods used for determining the RUL of systems. RUL prediction can be achieved through degradation measures collected. The degradation signal should be strongly linked to the failure of the system and contains important information about its health status. In general, Remaining Useful Life (RUL) predictions are typically undertaken using model-based, analytical-based, knowledge-based, and hybrid-based approaches and tools [9,10]. In Model-Based, Remaining Useful Life prediction done using statistical and probabilistic approaches [11]. These models are

derived from operational, failure and historical data and utilised in maintenance decision making. Markov Models is used in this case where the time frequency features allow more precise results [12]. Also, no foreknowledge of the physics of the formation of a component is required [8]. The Analytical-based RUL prediction approach represents the physical failure technique attributing to Physics-of-Failure (PoF). It requires the combination of experiment, observation, geometry, and physical changes condition parameters. The limitation of this method is that there is a need for full study of the system under consideration and intensive model development computation, making the approach difficult or even impossible to implement in many real life systems [11]. Knowledge-Based model is a combination of Computational Intelligence and experience. It relates to the collection of stored information from experts and interpretation [13]. A hybrid model is a collection of methodology and technique using several techniques for RUL estimation to improve accuracy.

Consequent to the limitations that most of the condition monitoring processes depend only on physical changes data form that is not always available in real industrial cases due to operational logistics and high costs of monitoring sensors, data acquisition, analysis, expert training, and so on. Also, these methods often have poor prediction accuracy due to undesired values in the data set. Notwithstanding, there is the need to curtail equipment failure and a timely awareness of failure mechanism for maintenance decisions regards to real life engineering practice. To this end, this work is implementing a model-based method for predicting system degradation states duration and RUL of equipment with a minimal computational requirement. This is achievable through extracting information from the operational and breakdown data of systems to produce reliable information about the state of the particular system and its remaining useful life. These variables include the total operational period, the no of breakdowns, the total downtime, mean time between failures of the component, and mean time to repair of the component that is directly obtained in the field and have a direct influence on the equipment under consideration. The benefits to be gained by using this effective process of predicting the future state of a system is that it enhances the effective management of equipment and infrastructure.

METHODOLOGY

To predict the remaining useful life, it is important to sufficiently understand the functioning of the engineering system under consideration, estimate the state-of-health and identify the possible future period at which system continuous operation becomes inefficient [14]. The time-instant as mentioned earlier is known as the end-of-life (EOL) and it is possible to identify it by evaluating the expectations of the states. The computation of the RUL may be computationally intensive since it runs until EOL is reached. The proposed method applies to all cases where the state-of-health of the system is steadily decreasing as a result of deterioration.

The first step of estimating the present state serves as the prelude to predicting the future state of a system and RUL computation. The health status of the system uniquely informs the level of damage in the system. Furthermore after estimated the state at time t_p , the next step is to predict the future states of the equipment. From the proposed formulation, equation (11) is used for this purpose. This equation can be discretized and used to predict the states at any future time instant $t > t_i$. Lastly, the machine remaining useful life estimation is done by calculating the period of the immediate time to the death status time. Some machine parameters such as failure and repair rates are used in this evaluation. Machine operating regions are divided into four states. This model can predict the time trajectory of the machine states using some collected reliability data.

Model description

The failure-maintenance model as shown in Figure 1 is a state space model. Three types of maintenance action will be adopted on the basis of the requirement of the system. The three types of maintenance actions are Minor maintenance (mM), Intermediate maintenance (IM) and Major maintenance (MM). In state D_1 , the system is new, therefore require minimal/minor maintenance to remain in state D_1 . Also, in state D_2 for an operational system, there is deterioration compared to the system in state D_1 . Due to this reason, it is presumed that the system would go back to stages D_1 or D_2 based on the type of maintenance action carried out. Again, for a system degrading from state D_2 to D_3 , there is the probability that the system requires any of the maintenance actions or no maintenance to go back to the desired state. This series of action applies to state D_4 before failure due to degradation F_1 , though random failure F_0 can occur within the life of the system.

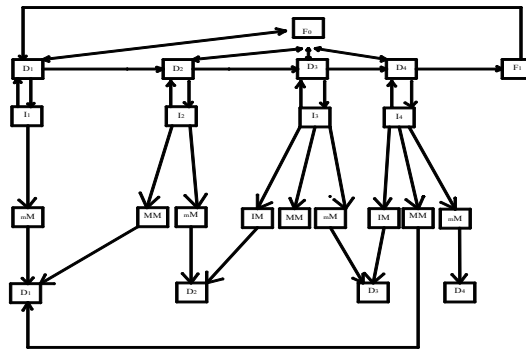


Figure 1: System/equipment maintenance model

State duration model development

Considering a system that is repairable and maintainable having N components, each subject to deterioration. Each component i is assumed to have rate of failure occurrence, $\lambda_i(t)$, where t is the actual time, ($t > 0$). The rate of failure occurrence given as:

$$\mu_i(t) = \left(\frac{B_i}{F_i}\right)^{-1} = \frac{1}{MTTR_i} \tag{1}$$

Where P_i , F_i and $MTBF_i$ indicates the total operational period (duration), the no of breakdowns (frequency) and mean time between failures of component i , respectively.

At a degradation stage or failed state, repairs are undertaken at a repair rate. The repair rate is given as:

$$\mu_i(t) = \left(\frac{B_i}{F_i}\right)^{-1} = \frac{1}{MTTR_i} \tag{2}$$

Where B_i , F_i and $MTTR_i$ indicates the total downtime (duration), the no of breakdowns (frequency) and mean time to repair of component i respectively.

Consider a repairable system with failure rates λ_i and repair rates μ_i and let T_i denote starting time from state i , it takes for the process to enter state $i+1$, $i \geq 0$. We can compute $E(T_i)$, $i \geq 0$, by starting with $i=0$. The transition rate from state i to state $i+1$ (failure rates) is denoted λ_i . Therefore, the expectation in state i is $E(T_i)$ and it can be represented as

$$\lambda_i = \frac{1}{E(T_i)} \tag{3}$$

Also, the transition rate of the i th state is given as the reciprocal of the expectation of state i .

That is

$$\lambda_i = \frac{1}{E(T_i)} \tag{4}$$

Since T_0 is exponential with rate λ_0 , we have: $E(T_0) = \frac{1}{\lambda_0}$

For $i > 0$, a condition whether the first transition can probably take the progression into state $i-1$ or $i+1$. That is, let

$$I_i = \begin{cases} 1, & \text{if the first transition from } i \text{ is to } i+1 \\ 0, & \text{if the first transition from } i \text{ is to } i-1 \end{cases}$$

and note that $E(T_i | I_i = 0) = \frac{1}{\lambda_i + \mu_i} + (E[T_{i-1}] + E[T_i])$, (5)

$$E(T_i | I_i = 0) = \frac{1}{\lambda_i + \mu_i} + (E[T_{i-1}] + E[T_i]) \tag{6}$$

This follows since, independent of whether the first transition is into state $i-1$ or $i+1$, the time until it occurs is exponential with rate $\lambda_i + \mu_i$ if this first transition is a $i+1$ state, then no additional time is needed; whereas if it is $i-1$ state, then the added time required to reach $i+1$ is equivalent to the time needed to return to state i (this has mean/expectation $E[T_{i-1}]$) plus the additional time it then takes to reach $i+1$ (this has mean/expectation $E(T_i)$).

Hence, since the probability that the first transition of $i+1$ is $E(T_i) = \frac{1}{\lambda_i + \mu_i} + \frac{\mu_i}{\lambda_i + \mu_i} (E[T_{i-1}] + E[T_i])$, we see that

$$E(T_i) = \frac{1}{\lambda_i + \mu_i} + \frac{\mu_i}{\lambda_i + \mu_i} (E[T_{i-1}] + E[T_i]), \tag{7}$$

$$E(T_i) = \frac{1}{\lambda_i} (1 + \mu E[T_{i-1}]), \quad i \geq 1 \tag{8}$$

$$E(T_i) = \frac{1}{\lambda_i} (1 + \mu E[T_{i-1}]), \quad i \geq 1 \tag{9}$$

$$E(T_i) = \frac{1}{\lambda_i} \left(1 + \left[\frac{\mu_i}{\lambda_i} \right] + \dots + \left[\frac{\mu_i}{\lambda_i} \right]^i \right), \quad i \geq 1 \tag{10}$$

$$E(T_i) = \frac{1 - \left[\frac{\mu_i}{\lambda_i} \right]^{i+1}}{\lambda_i - \mu_i}, \quad i \geq 1 \tag{11}$$

Starting with $E(T_0) = \frac{1}{\lambda_0}$, as the last state (as the end-of-life (EOL)), we can successively compute efficiently $E(T_1)$, $E(T_2)$ and so on. This procedure is used to determine the expected time to go from state i to state j where $i < j$, as this will give quantities that will equal $E(T_i) + E(T_{i+1}) + \dots + E(T_{j-1})$. Applying this to Figure 1, we have:

$$E(T_2) = \frac{1}{\lambda_2} \left(1 + \left[\frac{\mu_3}{\lambda_3} \right] + \left[\frac{\mu_3}{\lambda_3} \right]^2 + \left[\frac{\mu_3}{\lambda_3} \right]^3 \right) \tag{12}$$

$$E(T_2) = \frac{1}{\lambda_2} \left(1 + \left[\frac{\mu_3}{\lambda_3} \right] + \left[\frac{\mu_3}{\lambda_3} \right]^2 + \left[\frac{\mu_3}{\lambda_3} \right]^3 \right) \tag{13}$$

$$E(T_3) = \frac{1}{\lambda_3} \left(1 + \left[\frac{\mu_2}{\lambda_2} \right] + \left[\frac{\mu_2}{\lambda_2} \right]^2 \right) \tag{14}$$

$$E(T_4) = \frac{1}{\lambda_4} \left(1 + \left[\frac{\mu_1}{\lambda_1} \right] \right) \tag{15}$$

RESULTS AND DISCUSSION

The developed model was implemented and the obtained results analyzed using reliability data for a Bottle filler machine from line 1 of 7 up Bottling Company, Aba plant. The rates utilized are as obtained from the machine’s operational data. Three scenarios are involved in the implementation (to test the validity/effectiveness of the model) of the mathematical model to obtain effective results for the machine under consideration. The first scenario involves calculating the values of Expected time of Normal State ($E(T_{normal})$) by keeping the constant values repair rate or failure rate and varying the value of repair rate or failure rate as the case may be. Table 1 presents the calculated values, and the graph is plotted in Figure 2. This procedure is repeated for Expected time of PM State and Expected time of CM State.

Table 1: Sensitivity analysis of repair rate and failure rate on expected time of normal state

| CASE 1 | | | CASE 2 | | |
|----------------|--------------------|-----------------|----------------|--------------------|-----------------|
| μ_{normal} | λ_{normal} | $E(T_{normal})$ | μ_{normal} | λ_{normal} | $E(T_{normal})$ |
| 0.0947 | 0.0178 | 10405.22 | 0.0947 | 0.0178 | |
| 0.1305 | 0.0178 | 25626.41 | 0.0947 | 0.0536 | 212.75 |
| 0.1663 | 0.0178 | 51298.6 | 0.0947 | 0.0894 | 48.88 |
| 0.2021 | 0.0178 | 90164.13 | 0.0947 | 0.1252 | 22.05 |
| 0.2379 | 0.0178 | 144965.3 | 0.0947 | 0.161 | 13.28 |
| 0.2737 | 0.0178 | 218444.5 | 0.0947 | 0.1968 | 9.27 |
| 0.3095 | 0.0178 | 313344 | 0.0947 | 0.2326 | 7.05 |
| 0.3453 | 0.0178 | 432406.2 | 0.0947 | 0.2684 | 5.67 |
| 0.3811 | 0.0178 | 578373.4 | 0.0947 | 0.3042 | 4.73 |
| 0.4169 | 0.0178 | 753987.9 | 0.0947 | 0.34 | 4.05 |
| 0.4527 | 0.0178 | 961992.1 | 0.0947 | 0.3758 | 3.54 |
| 0.4885 | 0.0178 | 1205128 | 0.0947 | 0.4116 | 3.15 |
| 0.5243 | 0.0178 | 1486139 | 0.0947 | 0.4474 | 2.83 |
| 0.5601 | 0.0178 | 1807766 | 0.0947 | 0.4832 | 2.57 |

| | | | | | |
|--------|--------|---------|--------|--------|------|
| 0.5959 | 0.0178 | 2172752 | 0.0947 | 0.519 | 2.35 |
| 0.6317 | 0.0178 | 2583840 | 0.0947 | 0.5548 | 2.17 |
| 0.6675 | 0.0178 | 3043771 | 0.0947 | 0.5906 | 2.02 |
| 0.7033 | 0.0178 | 3555288 | 0.0947 | 0.6264 | 1.88 |
| 0.7391 | 0.0178 | 4121134 | 0.0947 | 0.6622 | 1.76 |
| 0.7749 | 0.0178 | 4744050 | 0.0947 | 0.698 | 1.66 |

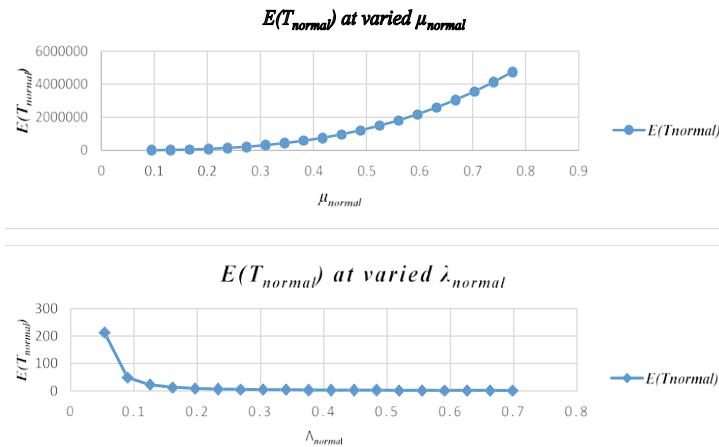


Figure 2: Sensitivity data plot for $E(T_{normal})$

The results obtained from the sensitivity analysis of the estimated filler machine lifespan in Table 1, shows the state duration of P_{normal} at varying repair and failure rates with one held constant as the case may be. The results are plotted as shown in Figures 2a and 2b. From the plots, at constant failure rate and varied repair/maintenance rate, the state expectation of the equipment was observed and the result obtained was graphically represented in Figure 2a. Here, the graph shows that the repair rate is directly proportional to the computed state expectation time of the equipment under consideration. It was also observed that at commissioning of the equipment, the rate of repair is very low. As the machine ages, rate of repair increases. On the failure rate variation, the resulting graph show that as the failure rate continuously increases, the expected mean sojourning time for the degradation state decreases. The reduction appears sharp at the early stage implying that there have been minimal maintenance actions. But over time, the effect of failure rate becomes negligible as a result of improved maintenance actions.

Table 2: Sensitivity analysis of repair rate and failure rate on expected time of PM state

| CASE 1 | | | CASE 2 | | |
|---------|-------------|----------|---------|-------------|----------|
| μ_3 | λ_3 | $E(T_3)$ | μ_3 | λ_3 | $E(T_3)$ |
| 0.0395 | 0.0241 | 220.97 | 0.0395 | 0.0241 | 220.97 |
| 0.0753 | 0.0241 | 576.22 | 0.0395 | 0.0599 | 34.96 |
| 0.1111 | 0.0241 | 1114.59 | 0.0395 | 0.0957 | 16.54 |
| 0.1469 | 0.0241 | 1836.09 | 0.0395 | 0.1315 | 10.57 |
| 0.1827 | 0.0241 | 2740.71 | 0.0395 | 0.1673 | 7.72 |
| 0.2185 | 0.0241 | 3828.46 | 0.0395 | 0.2031 | 6.07 |
| 0.2543 | 0.0241 | 5099.33 | 0.0395 | 0.2389 | 4.99 |
| 0.2901 | 0.0241 | 6553.32 | 0.0395 | 0.2747 | 4.24 |
| 0.3259 | 0.0241 | 8190.44 | 0.0395 | 0.3105 | 3.68 |
| 0.3617 | 0.0241 | 10010.68 | 0.0395 | 0.3463 | 3.25 |
| 0.3975 | 0.0241 | 12014.04 | 0.0395 | 0.3821 | 2.92 |
| 0.4333 | 0.0241 | 14200.53 | 0.0395 | 0.4179 | 2.64 |
| 0.4691 | 0.0241 | 16570.14 | 0.0395 | 0.4537 | 2.41 |
| 0.5049 | 0.0241 | 19122.88 | 0.0395 | 0.4895 | 2.22 |
| 0.5407 | 0.0241 | 21858.74 | 0.0395 | 0.5253 | 2.06 |

| | | | | | |
|--------|--------|----------|--------|--------|------|
| 0.5765 | 0.0241 | 24777.72 | 0.0395 | 0.5611 | 1.92 |
| 0.6123 | 0.0241 | 27879.83 | 0.0395 | 0.5969 | 1.79 |
| 0.6481 | 0.0241 | 31165.06 | 0.0395 | 0.6327 | 1.69 |
| 0.6839 | 0.0241 | 34633.42 | 0.0395 | 0.6685 | 1.59 |
| 0.7197 | 0.0241 | 38284.9 | 0.0395 | 0.7043 | 1.5 |

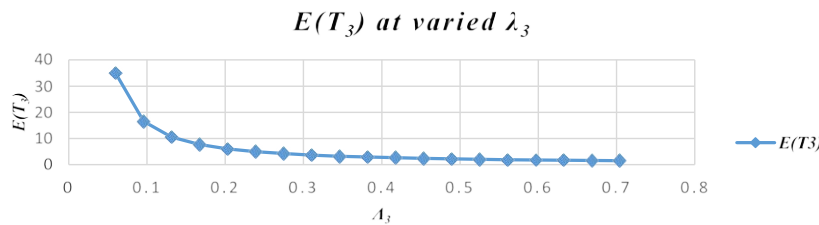
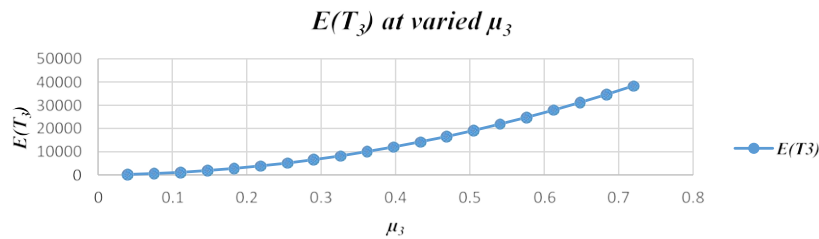


Figure 3: Sensitivity data plot for $E(T_3)$

Considering Table 2 and Figure 3, it was observed that maintenance action started immediately in this state converse to the situation in P_{normal} state. Also, it was seen that repair rate increased proportionately over the period of stay in this state. The failure rate showed a steady decline at the early life of this P_{pm} state as the state duration decreases also. This implies that proper maintenance actions should be undertaken to prolong the life of the equipment in this state.

Table 3: Sensitivity analysis of repair rate and failure rate on expected time of failure state

| CASE 1 | | | CASE 2 | | |
|---------|-------------|----------|---------|-------------|----------|
| μ_4 | λ_4 | $E(T_4)$ | μ_4 | λ_4 | $E(T_4)$ |
| 0.0861 | 0.0258 | 168.11 | 0.0861 | 0.0258 | 168.11 |
| 0.1219 | 0.0258 | 221.89 | 0.0861 | 0.0616 | 38.92 |
| 0.1577 | 0.0258 | 275.67 | 0.0861 | 0.0974 | 19.34 |
| 0.1935 | 0.0258 | 329.46 | 0.0861 | 0.1332 | 12.36 |
| 0.2293 | 0.0258 | 383.24 | 0.0861 | 0.169 | 8.93 |
| 0.2651 | 0.0258 | 437.02 | 0.0861 | 0.2048 | 6.94 |
| 0.3009 | 0.0258 | 490.81 | 0.0861 | 0.2406 | 5.64 |
| 0.3367 | 0.0258 | 544.59 | 0.0861 | 0.2764 | 4.74 |
| 0.3725 | 0.0258 | 598.37 | 0.0861 | 0.3122 | 4.09 |
| 0.4083 | 0.0258 | 652.15 | 0.0861 | 0.348 | 3.58 |
| 0.4441 | 0.0258 | 705.94 | 0.0861 | 0.3838 | 3.19 |
| 0.4799 | 0.0258 | 759.72 | 0.0861 | 0.4196 | 2.87 |
| 0.5157 | 0.0258 | 813.5 | 0.0861 | 0.4554 | 2.61 |
| 0.5515 | 0.0258 | 867.29 | 0.0861 | 0.4912 | 2.39 |
| 0.5873 | 0.0258 | 921.07 | 0.0861 | 0.527 | 2.21 |
| 0.6231 | 0.0258 | 974.85 | 0.0861 | 0.5628 | 2.05 |
| 0.6589 | 0.0258 | 1028.63 | 0.0861 | 0.5986 | 1.91 |
| 0.6947 | 0.0258 | 1082.42 | 0.0861 | 0.6344 | 1.79 |
| 0.7305 | 0.0258 | 1136.2 | 0.0861 | 0.6702 | 1.68 |
| 0.7663 | 0.0258 | 1189.98 | 0.0861 | 0.706 | 1.59 |

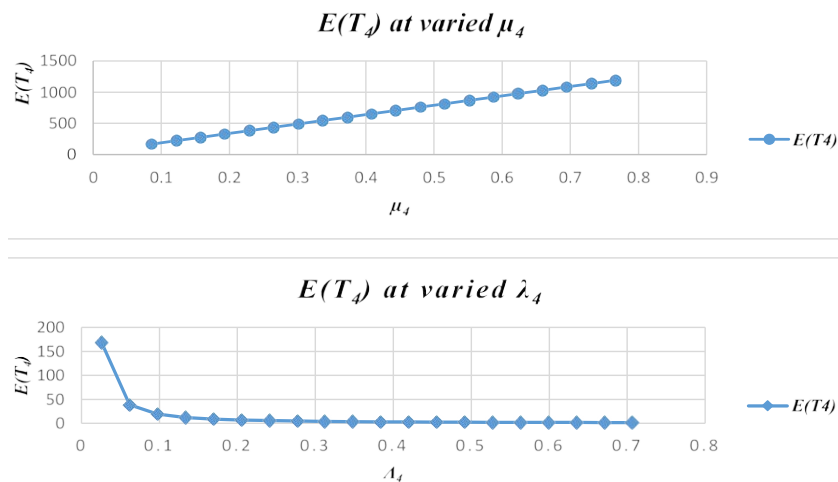


Figure 4: Sensitivity data plot for $E(T_d)$

From results obtained in Table 3, at varied repair rate for the deterioration state P_{cm} in the developed network model, it was observed that expected duration of stay is least. Also, that the repair rate is directly proportional to the expected duration given that maintenance actions are sustained else, failure occurs. This results are plotted as shown in Figures 4a and 4b.

CONCLUSION

A general methodology was developed for estimation of machine remaining useful life using reliability data. This mathematical formulation is capable of calculating the expected transition time from any deterioration state to the failure state (expected remaining life) of the equipment in consideration. The program was applied to a Bottle Filler machine in one of the bottling lines utilized for the study. The model is tested for various scenarios by changing one of the main parameters during each calculation while others are kept constant. For the state duration sensitivity analysis, as the failure rate continuously increases from 0.0178 to 0.7060, the expected mean sojourning time for each degradation state decreased from 220.97 h to 1.59 h. Subsequently at uniform incrementally varied repair/maintenance rate (0.0861 to 0.7663), the state expectation of the equipment increases from 168.11 h to 1189.98 h. This allowed us to determine the most suitable decision to improve the reliability of the Bottle filler machine. This model will be of benefit to industries because the ability to predict the future state of a system is a proactive practice.

REFERENCES

- [1] Jardine, A., Lin, D. and Banjevic, D., *Mech Syst Signal Process*, **2006**. 20(7): p. 1483-1510.
- [2] Ye, Z.S., Shen, Y. and Xie, M., *Eur J Oper Res*, **2012**. 221(2): p. 360-367.
- [3] Suprasad, V.A., Relex Software Corporation, Leland McLaughlin, Hoang Pham, Ph.D., Rutgers University, **2006**. p. 1-6.
- [4] Byron A., *The Jethro Project*, **2008**. p. 1-5.
- [5] Medjaher, K., Tobon-Mejia, D.A. and Zerhouni, N., *IEEE Journal of Transactions on Reliability*, **2012**. 61(2): p. 292-302.
- [6] Dorst, N.F.G., et al., Manufacturing Networks Group, **2014**. p. 1-13.
- [7] Coble, J.B., *An Automated Method to Identify Prognostic Parameters*. **2010**.
- [8] Xiongzi, C., et al., 10th IEEE International Conference on Electronic Measurement & Instruments (ICEMI), **2011**. p. 2-9.
- [9] Schwabacher, M., *AIAA InfoTech Aerospace*, 2005. p. 26-29
- [10] Okoh, C., *Proceedings of the 6th CIRP Conference on Industrial Product-Service Systems*, **2014**. p. 158-163.

- [11] Pecht, M.G., *Prognostics and Health Management of Electronics*, New Jersey. **2008**.
- [12] Gasvik, K.J., et al., *Int J Adv Manufact Technol*, **2014**. 70(1): p. 321-326.
- [13] Chen, C., Vachtsevanos, G. and Orchard, M.E., *Mech Syst Signal Proces*, **2012**. 28(4): p. 597-607.
- [14] Engel, S.J., et al., *Aerospace Conference Proceedings*, **2000**. 21(6): p. 457–469.