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Different Versions of the Inverse Fourier Stieltjes Transform

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ABSTRACT

This paper deals with conventional Fourier Stieltjes transform of Lebesgue integrable function in dual space. The present paper mainly provides inversion formula for the conventional Fourier Stieltjes transform. The aim of the paper is to extend inverse Fourier Stieltjes transform and develop different versions for it.

Keywords: Fourier Stieltjes transform , inverse Fourier Stieltjes transform, Abel transform.

INTRODUCTION

Now a days, Fourier Stieltjes transform plays an important role in signal processing and many other scientific disciplines which is defined by,

$$FS\{f(t,x)\} = \int_{-\infty}^{\infty}\int_{0}^{\infty} f(t,\ x)\, e^{-ist}(x+y)^{-p}\, dx\, dt$$

for a Fourier Stieltjes transformable function $f(t,x)$.

If $F(s,\ y)$ is the distributional Fourier Stieltjes transform of $f(t,x)$ then in the sence of convergence in $D'(\Omega)$,

$$f(t,x) = \lim_{r,v\to\infty} \frac{1}{4\pi^2 i} \int_{\sigma-ir}^{\sigma+ir} \int_{0}^{\sigma'+ir'} F_2(s,\ y) e^{ist}(x+y)^{p-1}\, dy\, ds \qquad (1.1)$$

where σ and σ' are any fixed real number such that $\sigma_1 < a < \sigma < b < \sigma_2$, $\sigma_1' < \sigma < \sigma_2'$ and F_2 is partial derivative of $F(s,\ y)$ with respect to y .

The idea of the above result is to transfer the inversion formula onto a transform of $\varphi(t,x) \in D(\Omega)$ and to use the fact that the resulting expression converges to φ .

MATERIALS AND METHODS

The notation and terminology of this work will follow that of [1]. We will also proceed on similar lines [1] to develop the different formulae for inverse Fourier Stieltjes transform of a convensional function $f(t,x)$ defined in (1.1).

RESULTS AND DISCUSSION

For $p>1$, an integration by parts with respect to y brings equation (1.1) to the form,

$$\lim_{r,r'\to\infty}\frac{1}{4\pi^2 i}\int_{\sigma-ir}^{\sigma+ir}\left\{e^{ist}\left[F(s,y)(x+y)^{p-1}\right]_0^{\sigma'+ir'}-\left[\int_0^{\sigma'+ir'}F(s,y)(p-1)(x+y)^{p-2}dy\right]\right\}ds$$

As $r'\to\infty$,first term tends to zero. Hence,

$$f(t,x)=\lim_{r,r'\to\infty}\frac{-1}{4\pi^2 i}\int_{\sigma-ir}^{\sigma+ir}\int_0^{\sigma'+ir'}F(s,y)e^{ist}(p-1)(x+y)^{p-2}dyds. \quad (3.1)$$

This is less general since it only valid for $p>1$. It is convenient to have x independent contours and so the change of variable $y=xz$ allows to rewrite equation (1.1) in the alternate form,

$$f(t,x)=\lim_{r,v\to\infty}\frac{1}{4\pi^2 i}x^p\int_{\sigma-ir}^{\sigma+ir}\int_0^{\sigma'+ir'}F_2(s,xz)e^{ist}(1+z)^{p-1}dzds. \quad (3.2)$$

The integral converges for $p>1$.The counterpart of equation (3.2), obtained by integration by parts namely,

$$f(t,x)=\lim_{r,v\to\infty}\frac{-1}{4\pi^2 i}(p-1)x^{p-1}\int_{\sigma-ir}^{\sigma+ir}\int_0^{\sigma'+ir'}F(s,xz)e^{ist}(1+z)^{p-2}dzds. \quad (3.3)$$

Since this is only well defined for $p>1$, equations (1.1) and (3.2) are more general than equation (3.3).
Setting $p=1$ equation (1.1) gives an expression that can be integrated explicitly to give,

$$f(t,x)=\lim_{r,v\to\infty}\frac{1}{4\pi^2 i}x^p\int_{\sigma-ir}^{\sigma+ir}\int_0^{\sigma'+ir'}F_2(s,y)e^{ist}dyds$$

$$=\lim_{\substack{r\to\infty\\ \varepsilon\to 0}}\frac{1}{4\pi^2 i}x^p\int_{\sigma-ir}^{\sigma+ir}\int_0^{\sigma'+ir'}\left[F(s,-x+i\varepsilon)-F(s,-x-i\varepsilon)\right]e^{ist}ds. \quad (3.4)$$

For analytic and asymptotic properties of $F(s,y)$, it is a simple consequence of Cauchy's theorem that this is the correct solution of equation,

$$F(s,y)=\lim_{r,v\to\infty}\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_0^{\infty}f(t,x)e^{-ist}(x+y)^{-p}dxdt,$$

for $x>0$.
Let us define the quantity that appears on the right hand side of equation (3.4) to be,

$$(s,\Delta h)=\lim_{\substack{r\to\infty\\ \epsilon\to 0^+}}\frac{1}{4\pi^2 i}\int_{\sigma-ir}^{\sigma+ir}\left[F(s,-h+\varepsilon)-F(s,-h-\varepsilon)\right]e^{ist}ds,$$

for $h>0$.

We can prove that $f(t,x)=(s,\Delta h)$ when $p=1$, but this is not the case for other values of p. By shrinking the contour down to the cut equation (1.1) takes the form,

$$f(t,x) = \lim_{r,r'\to\infty} \frac{-1}{4\pi^2 i} \int_{\sigma-ir}^{\sigma+ir} \int_{0}^{\sigma'+ir'} F(s,y) e^{ist} (p-1)(x+y)^{p-2} dyds$$

In similar fashion equation (1.1) gives rise to ,

However these formulae are only correct if the behavior of $F(s, y)$ near the origin is such that these integrals exist. The contour integral version of these formulae is more general since they do not have this restriction.

Equations (3.5) and (3.6) have the structure of Abel transform . The inverse Abel transform is well known and can be used to give a formula for the discontinuity across the cut, $(s, \Delta h)$ in terms of the original generalized function $f(t,x)$. A version that is suitable if $p < 2$ and $F(0, 0) = 0$ is,

$$(s, \Delta h) = \frac{\sin \pi p}{\pi(p-1)} \int_{-\infty}^{\infty} \int_{0}^{h} f_2(t, x) e^{-st} (h-x)^{1-p} dxdt,$$

where $f_2(t, x)$ stands for partial derivative of $f(t, x)$ with respect to x.

CONCLUSION

A definitions for Fourier Stieltjes transform and its inverse are introduced in this work. These definitions are used to generate the concepts of other versions for the same transform which are most frequently used tools in speech processing radar and quantum physics.

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