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Dimensionless Intensity in the Semiclassical theory of Laser, Electric Field inside a Fabry-Perot Cavity and Voltage in a Triode Oscillator Circuit

[1]J. Saikia, [2]H. Saikia, [3]R.K. Dubey* and [4]G.D. Baruah

[1]Department of Physics, J.B. College, Jorhat, Assam
[2]Department of Physics, Majuli College, Majuli, Jorhat, Assam
[3]Institute of Education-Haldia,West Bengal
[4]Centre for Laser and Optical Science, New Uchamati, Doom Dooma

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ABSTRACT

The present work reports an analogy in three different topics appearing in the semiclassical theory of laser, Fabry Perot Cavity and triode oscillator circuit. We have shown that the buildup of the parameter dimensionless intensity in the semiclassical theory and the buildup of electric field inside a Fabry Perot cavity bear a striking similarity. In this connection Vander Pol's triode oscillator circuit is also discussed.

INTRODUCTION

The present work is concerned with a comparative study between the buildup of dimensionless intensity in the semiclassical theory of laser and the buildup of electric field due to multiple reflections inside a Fabry Perot cavity. We have also brought under our discussion the buildup of voltage in a triode oscillator circuit. It is worthwhile to note that this type of work has not been reported earlier. An analogy between spatial hole burning and the intensity contour of the beams multiple reflections inside Fabry Perot cavity was established in an earlier work [1]. Recently [2] the subject of spatial hole burning, multiple reflections inside a Fabry Perot cavity and squeezed states of light have been discussed and in all the cases worthwhile analogy among the three phenomena has been noted. We also note here that in many ways some phenomena of physics appearing under different contexts are quite analogous. As for example the comparison between the laser near threshold and matter near a phase transition was developed by DeGiorgio and Scully (3) long back.

2. BUILD UP OF DIMENSIONLESS INTENSITY

The dimensionless intensity is an important parameter obtained in the semiclassical theory of laser (4, 5). The parameter is defined as

$$I_n = \frac{1}{2}\frac{\wp}{\hbar^2 \gamma_a \gamma_b} E_n^2$$

$$= \frac{a_n}{\beta_n} \qquad ....(1)$$

Where $\wp = \wp^*$ is the magnitude of the electric-dipole matrix element. a_n is the term known as linear net gain, that is

$$a_n = \mathcal{L}(\omega - \upsilon_n) F_1 - \frac{1}{2}\frac{\upsilon}{Q_n}$$

$$\mathcal{L}(\omega - \upsilon_n) = \gamma^2 \left[\gamma^2 + (\omega - \upsilon_n)^2 \right]^{-1}$$

$$F_1 = \frac{1}{2} \upsilon \wp^2 (\in_0 \hbar \gamma)^{-1} \bar{N}$$

$$\beta_n = \mathcal{L}^2 (\omega - \upsilon_n) F_3$$

$$F_3 = (3/2)(\gamma_{ab}/\gamma) F_1$$

All these terms may be deduced from the semiclassical theory of laser (4). We are particularly interested in the parameter I_n. The equation of motion for the dimensionless intensity I_n is given by [4] as

$$\dot{I}_n = 2I_n (a_n - \beta_n I_n) \qquad ....(2)$$

This intensity equation of motion (2) can be integrated by the method of partial fraction with the solution.

$$I_n(t) = \frac{a_n [I_o/(a_n - \beta_n I_o)] \exp(2a_n t)}{1 + \beta_n [I_o (a_n - \beta_n I_o)] \exp(2a_n t)} \qquad ....(3)$$

Where $I_o \equiv I_n$ (o). μ is expected from this equation that initially this yields exponential gain $I_n(t) = I_o \exp(2a_n t)$ for small I_o and it yields the steady state result $I_n = a_n/\beta_n$. The time development is illustrated in Fig.1.

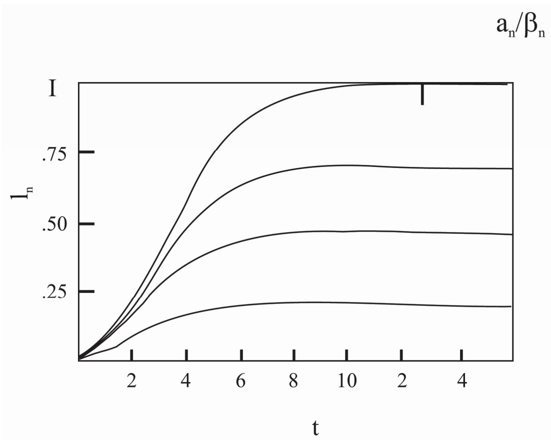

Fig.1 : Build up of dimensionless intensity for small value of $I_n(0) = .01$.

It is worthwhile to note that the buildup of intensity does not take place from zero value.

In this connection we consider the geometrical model based on which the semiclassical theory of laser is built. Then geometrical model is used to describe the situation inside a Fabry-Perot cavity and self consistency principle is involved. Fig.2 shows the geometrical model used by Lamb (4).

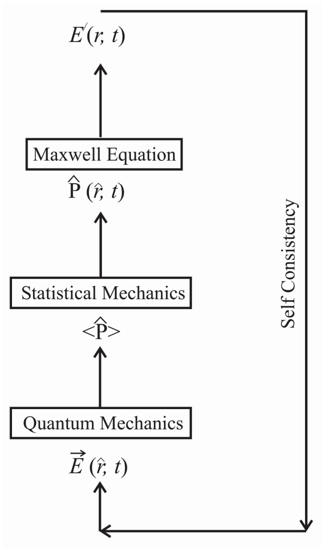

Fig.2 :Geometrical Mode of Semiclassical Theory.

Electric filed E assumed in cavity induces microscopic dipole moments (P) in the active medium according to the laws of quantum mechanics. These moments are then summed to yield the macroscopic polarization of the medium P (r, t) which acts as a source of Maxwell's equations. The condition of self-consistency then requires that the assumed field E equal the reaction field $E^{/}$.

3. FABRY-PEROT RESONATING CAVITY

We must emphasize that Fabry-Perot Cavity is a situation of resonance of an electromagnetic wave at optical frequencies which is no different than the resonance of any other system, be it mechanical or electrical. There is always an interchange of energy in such a system between the potential and kinetic forms in the case of a mechanical system, with attendant friction losses, or between electric and magnetic energy with resistive losses in an electromagnetic problem. Quite often the phenomenon of resonance gets lost in the mathematics when analyzing a low frequency system. However, fortunately a much simpler physical picture emerges when we consider systems where the wavelength is much less than the dimensions of the components. An example is the Fabry-Perot Cavity. In order to make the problem as familiar as possible we consider the excitation of the cavity as shown in Fig.3, by an external source as an electric field in the form of a tunable laser or a variable frequency oscillator. We consider all waves, incident on the cavity from the left, inside the cavity or transmitted through it to the right to be uniform plane waves of limited spatial extent transverse to the direction of propagation. Our problem is to relate the fields, running wave intensities and stored energy on the inside of the cavity to those quantities that we can measure on the outside. It may be noted here that we are dealing with a classical situation analogues to the semiclassical picture. Let us follow a wave as it bounces back and forth between the mirrors. Consider the initial field at the plane just to the right of M_1, leveled by E_0.

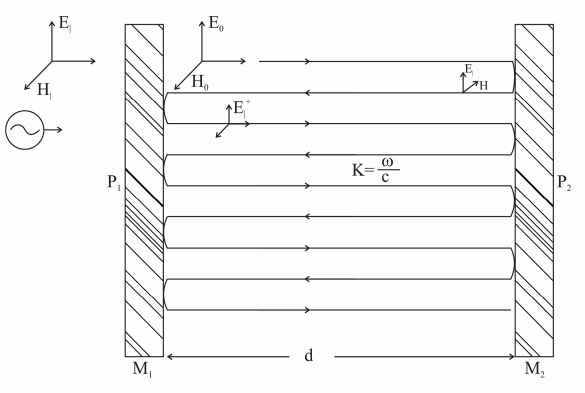

Fig.3 : Resonating Cavity

It propagates to M_2 and back to the starting plane and experiences an amplitude change of ρ_1, ρ_2 and a phase factor $\exp[-jk\,2d]$ as it travels that round trip and thus generators the field E_1^+. This new field experiences the same change as E_0 and it in turn generates E_2^+ and so on. However there shall be an time when the resultant filed E_n^+ will no longer experience another change and virtually remains constant. This is the condition of self consistency in the geometrical model of the semiclassical theory of laser. Let us consider again the electric fields prior to the self consistency condition. We note that at every point along the path from the mirrors M_1 to M_2, the fields E_1^+, E_2^+, E_3^+ and so on are to be added to E_0 to which we assign the reference phase of 0^O. This phasorial addition is shown in Fig.4. Where, because there is an assumed lagging phase angle, we have assumed that the round-trip phase shift (RTPS) 2θ = 2kd is almost but not quite an integral multiple of 2π radius. The deficiency is laveled by ϕ and is related to kd by

$$2\theta = 2kd = q2\pi - \phi.$$

If the angle ϕ is significant, the total field propagating to the right inside the cavity is the difference between the origin and the spiral of the phasors—quite similar to the straight line distance between the beginning and end of a coiled rope. This distance is small, but if that rope is uncoiled, the distance is much larger. The distance will presumably depend on the total number of coils and the radius of the individual coil. In a similar way the total field E_T will be many times the initial value E_0, if $\rho_{1,2}$ are close to unity and ϕ = 0. The following quantities are all maximized by the simple equation ϕ = 0; the total field travelling to the right (and thus to the left also); the magnetic field associated with *E*, the intensities of the waves; the number of photons bouncing back and forth and the stored energy. These physical facts are characteristic of resonance which is given by

$$\text{Round Trip Phase Shift (RTPS)} = 2kd = k2\pi \qquad \ldots\ldots\ldots\ldots\ldots(4)$$

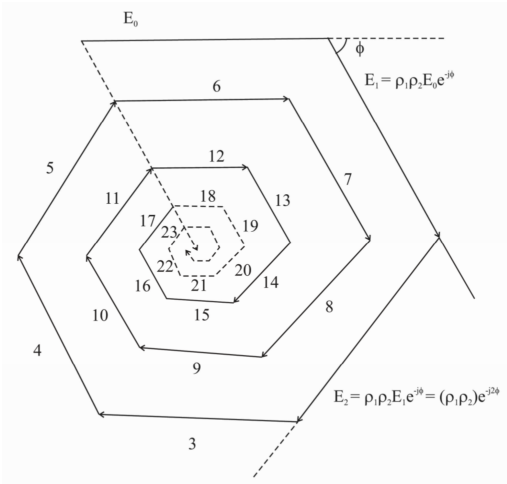

Fig.4 : Phasor diagram illustrate the field up of electric field inside the resonating cavity. This situation is similar to the filed up of a parameter known as "dimensionless intensity" in the semiclassical theory.

Since $k = \frac{\omega n}{C} = \frac{2\pi}{\lambda}$, we can use it to find the resonant wavelength.

$$k2d = \frac{\omega n - 2d}{C}$$
$$= \frac{2\pi - 2d}{\lambda}$$
$$= q - 2\pi$$
$$\text{or,} \quad d = \frac{q\lambda}{2}$$

Where $\lambda = \frac{\lambda_o}{2}$. This view of resonance states that there has to be an integral number of half wavelengths between the two mirrors. This also implies that the integral q is a very large number for optical frequencies and reasonable size cavities.

From Fig.3 one can estimate the total field E_T travelling to the right as follows :

$$E_T = E_0 + E_1 + E_2 + \ldots$$
$$= E_0 + E_0\rho_1\rho_2 e^{-jK2d} + E_0\left(\rho_1\rho_2 e^{-jK2d}\right)^2 + \ldots$$
$$= \sum_0^{\alpha} E_n^+ = E_0\left[1 + \rho_1\rho_2 e^{-jK2d} + \left(\rho_1\rho_2 e^{-jK2d}\right)^2 + \left(\rho_1\rho_2 e^{-jK2d}\right)^3 + \ldots\right]$$

$$\text{Or,} \quad E_T = \frac{E_0}{1 - \rho_1\rho_2 e^{-j2\theta}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The field returning from M_2 is just ρ_2 times the round trip phase factor $\exp[-jk2d]$, multiplying the wave going to the right,

$$\overline{E}_T = \sum \overline{E}_n = \rho_2 e^{-j2\theta} E_T^+$$
$$= E_0\left[\frac{\rho_2 e^{-j2\theta}}{1 - \rho_1\rho_2 e^{-j2\theta}}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

From equation (5) we can also have an idea of buildup of electric field in the cavity, it may be noted that under condition of equilibrium of resonance the incident field E_0should be equal to the resultant field E_T.

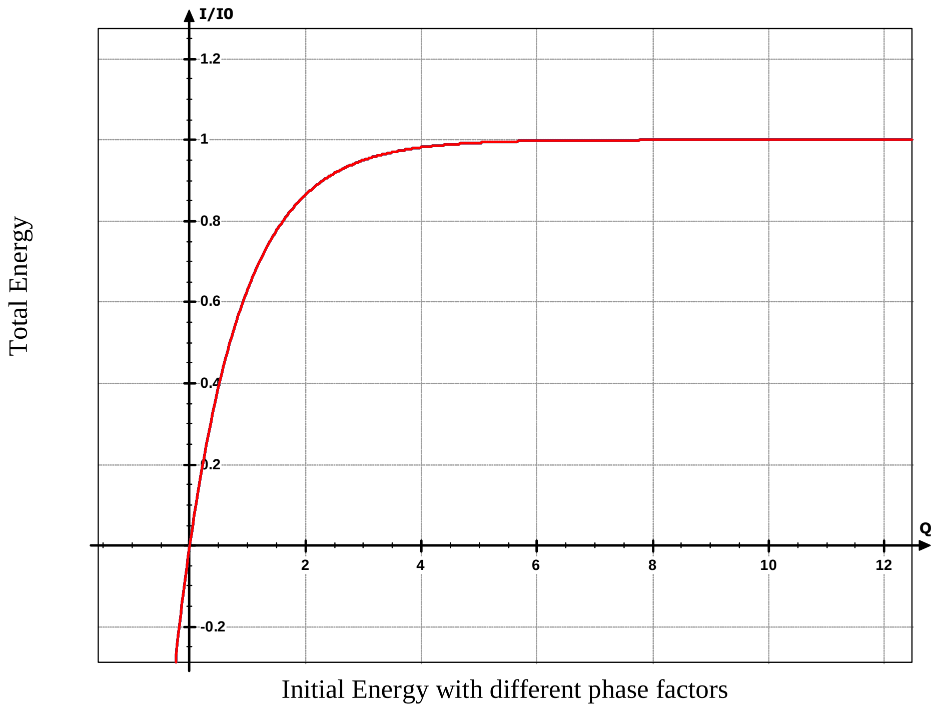

Fig.5 : The buildup of electric field.

The buildup of electrical field as given by equation (5) is shown in Fig.5. As may be inferred from it the buildup exhibits an exponential growth with a saturation at a latter stage. Thus the dimensionless intensity in the semiclassical theory of laser corresponds to the electric field in the Fabry-Perot cavity.

4. VANDER POLE'S TRIODE OSCILLATOR

Vander Pole (6) obtained an equation of motion in his treatment of the triode oscillator as

$$\ddot{V} - \frac{d}{dt}\left(av - \beta^{/} \upsilon^{3}\right) + \omega^{2}\upsilon = 0 \qquad \text{.................... (7)}$$

Here 'a' is the linear net gain (i.e. the gain in excess of losses), $\beta^{/}$ is the saturation co-efficient and ω is the resonance frequency in the absence of dissipation or gain. Equation (7) may be solved approximately by an important technique called the method of slowly varying amplitude, writing

$$\upsilon(t) = \frac{1}{2}V(t)\exp(-i\omega t) + C.C. \qquad \text{....................... (8)}$$

Where V(t) is the amplitude. The slowly varying equation of motion is obtained as

$$\dot{V}(t) = \frac{1}{2}V(t)\left(a - \beta V^{2}\right) \qquad \text{....................... (9)}$$

For sufficiently large V(t), saturation sets in and the steady state condition

$$\dot{V}(t) = 0$$

Occurs for the value

$$V^2 = \alpha/\beta \qquad \text{……………………. (10)}$$

Analytical solution of equation (9) is discussed by Lamb in the semiclassical laser theory and the solution is given by equation (3). It may be noted that equation (7) and hence (9) does not build up from the zero amplitude. In practice some fluctuations get things going.

RESULTS AND DISCUSSION

From what has been described above it is worthwhile to make a conclusion and indicate the salient features. We have briefly describe the geometrical model based on which the semiclassical theory of laser is built. The main characteristic is the self consistency nature of the incident field *E* and the resultant field $E^{/}$. This also describes the threshold condition of oscillation for laser action indicating how loss compensates gain. A parameter known as dimensionless intensity is defined and it is shown how build up of this intensity takes place inside the cavity. The entire situation is co-related with another picture of resonating cavity where it is shown how in a multiple reflection the electric filed undergoes changes. A phaser diagram has also been used to describe the situation. Under condition of equilibrium this must indicate the essential features of the geometrical model, because the self consistency nature is nothing but a condition of resonance. The Vander Pol’s oscillator circuit also describes the build up of voltage like the build up of dimensionless intensity in the semiclassical theory. In this case we have not consider a fully quantum mechanical version of laser where it is shown how build up photon density takes place from zero value.

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