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Generalizsd finite mellin integral transforms

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ABSTRACT

Aim of this paper is to see the generalized Finite Mellin Integral Transforms in [0,a] , which is mentioned in the paper , by Derek Naylor (1963) [1] and Lokenath Debnath [2]..For f(x) in 0<x< ∞ ,the Generalized Mellin Integral Transforms are denoted by $M_{-}^{\infty}[f(x),r]=F_{-}^{\infty}(r)$ *and* $M_{+}^{\infty}[f(x),r]=F_{+}^{\infty}(r)$ *.This is new idea is given by Lokenath Debonath in[2].and Derek Naylor [1].. The Generalized Finite Mellin Integral Transforms are extension of the Mellin Integral Transform in infinite region. We see for f(x) in 0<x<a,* $M_{-}^{a}[f(x),r]=F_{-}^{a}(r)$ *and* $M_{+}^{a}[f(x),r]=F_{+}^{a}(r)$ *are the generalized Finite Mellin Integral Transforms . We see the properties like linearity property , scaling property ,power property and propositions for the functions* $\frac{1}{x}f(\frac{1}{x})$ *and (logx)f(x), the theorems like inversion theorem ,convolution theorem , orthogonality and shifting theorem. for these integral transforms. The new results is obtained for Weyl Fractional Transform and we see the results for derivatives differential operators ,integrals ,integral expressions and integral equations for these integral transforms. Application of these transforms are for the summation of the series and solution of the Cauchy's linear differential equation.*

Keywords: Integral transforms, Mellin integral transform, Finite Mellin integral transform.
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INTRODUCTION

In the theory of Integral Transform , Mellin Integral Transform has presented a direct and systematic technique, for resolution of certain types of classical boundary and initial value problems .To be successful the transform must be adopted to the form of differential operators to be eliminated as well as to the range of interest and associated boundary conditions. These transforms are the extension of the Mellin Integral Transform and have a similar inversion formulae. These transforms are suited to regions bounded by the natural coordinate surfaces of a cylindrical or spherical coordinate system and apply to finite or infinite regions bounded internally.

Historically ,Riemann (1876) first recognized the Mellin transform in the famous memoir on prime numbers, Its explicit formulation was given by Cahen (1894) . Almost simultaneously Mellin (1896, 1902) gave an elaborate discussion of the Mellin transform and its inversion formula

A method is prescribed for generating such transforms . .The method is adopted for the GMIT in $[0,\infty]$ for the infinite interval x>0 and the result is modified for a finite interval 0<x<a. We see the properties for GFMIT in [0,a] like linearity property , scaling property ,power property and propositions for the functions $\frac{1}{x}f(\frac{1}{x})$ and (logx)f(x) .Also we see the theorems for this integral transforms like inversion theorem ,convolution theorem , orthogonality and shifting theorem. The new result is obtained for Weyl transform. and other results are obtained for derivatives , differential operators ,integrals ,integral expressions and integral equations by using the GFMIT in [0,a]. Application of this transforms is for the summation of the series and solution of the Cauchy's linear differential equation.

2.2.2. Basic Results

The generalized Mellin integral transform of a function f(x) in $0<x<\infty$, introduced by the integral

$$M_-^{\infty}[f(x),r] = F_-^{\infty}(r) = \int_0^{\infty} (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x)dx \text{ , r>0}$$

The inverse of this transform is

$$M_-^{\infty^1}[f(x),r] = F_-^{\infty-1}(r) = f(x) = \frac{1}{2\pi i}\int_L x^{-r} M_-^{\infty}(r)dr$$

Where L is the line Re p=c and $M_-^{\infty}[f(x),r] = F_-^{\infty}(r)$ is regular function on the strip $|\mathrm{Re}(p)| < \gamma$ with c< γ.

On the other hand , if the derivative of f(x) is prescribed at r=a , it is convenient define the associated integral transform by

$$M_+^{\infty}[f(x),r] = F_+^{\infty}(r) = \int_0^{\infty} (x^{r-1} + \frac{a^{2r}}{x^{r+1}}) f(x)dx \text{ , r>0}$$

The inverse of this transform is

$$M_+^{\infty^{-1}}[f(x),r] = F_+^{\infty^{-1}}(r) = f(x) = \frac{1}{2\pi i}\int_L x^{-r} M_+^{\infty}(r)dr$$

Where L is the line Re p=-c and $M_+^{\infty^{-1}}[f(x),r] = F_+^{\infty^{-1}}(r)$ is regular function on the strip $|\mathrm{Re}(p)| < \gamma$ with c< γ.

If the range of the integral is finite, then we define the generalized Mellin integral transform by

$$M_-^{a}[f(x),r] = F_-^{a}(r) = \int_0^{a} (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x)dx \text{ , where Re(p)<}\gamma \quad (1)$$

The corresponding inverse transform is

$$M_-^{a^{-1}}[f(x),r] = F_-^{a^{-1}}(r) = f(x) = \frac{1}{2\pi i}\int_L x^{-r} M_-^{a}(r)dr \quad (2)$$

Similarly we define the generalized finite Mellin integral transform pair by

$$M_+^{a}[f(x),r] = F_+^{a}(r) = \int_0^{a} (x^{r-1} + \frac{a^{2r}}{x^{r+1}}) f(x)dx \text{ ,r>0} \quad (3)$$

The inverse of the corresponding transform is given by

$$M_{+}^{a^{-1}}[f(x),r]=F_{+}^{a^{-1}}(r)=f(x)=\frac{1}{2\pi i}\int_{L} x^{-r}M_{-}^{\infty}(r)dr \qquad (4)$$

where line L is from c-ia to c +ia

2.2.3.Lemma

2.2.3.1. Linearity Property

The GFMIT of a function f(x) in 0<x<a , introduced by the integral

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx,$$

then for the constants α and β ,we have Linearity Property

$$M_{-}^{a}[\alpha f(x)+\beta g(x),r]=\alpha\ M_{-}^{a}[f(x),r],+\beta\ M_{-}^{a}[g(x),r] \qquad (5a)$$

Similarly $$M_{+}^{a}[\alpha f(x)+\beta g(x),r]=\alpha\ M_{+}^{a}[f(x),r],+\beta\ M_{+}^{a}[g(x),r] \qquad (5b)$$

2.2.3.2. Scaling Property

The GFMIT of a function f(x) in 0<x<a , introduced by the integral

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx,\ \text{then}$$

$$M_{-}^{a}[f(x),r]=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(bx)dx$$

substitute bx=t , x=$\frac{t}{b}$,dx=$\frac{dt}{b}$, if x=0 then t=0 and if x=1 then t=ab

$$M_{-}^{a}[f(bx),r]=\int_{0}^{ab}((\frac{t}{b})^{r-1}-\frac{a^{2r}}{(\frac{t}{b})^{r+1}})f(t)\frac{dt}{b}$$

$$=\frac{1}{b^{r}}\int_{0}^{ab}(t^{r-1}-\frac{(ab)^{2r}}{t^{r+1}})f(t)dt$$

$$M_{-}^{a}[f(bx),r]=M_{-}^{ab}[f(t),r] \qquad (6a)$$

Similarly $$M_{+}^{a}[f(bx),r]=M_{+}^{ab}[f(t),r] \qquad (6b)$$

2.2.3.3. Power Property

The GFMIT of a function f(x) in 0<x<a , introduced by the integral

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx,\ \text{then}$$

$$M_{-}^{a}[f(x^{n}),r]=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x^{n})dx$$

substitute $x^{n}=t$, $x=t^{\frac{1}{n}}$,dx=$\frac{1}{n}t^{\frac{1}{n}-1}dt$, if x=0 the t=0 and if x=a then t=a^{n}

$$M_{-}^{a}[f(x^{n}),r]=\int_{0}^{a^{n}}((t^{\frac{1}{n}})^{r-1}-\frac{a^{2r}}{(t^{\frac{1}{n}})^{r+1}})f(t)\frac{1}{n}t^{\frac{1}{n}-1}dt$$

$$=\frac{1}{n}\int_{0}^{a^{n}}(t^{\frac{r}{n}-1}-\frac{a^{2r}}{t^{\frac{r}{n}+1}})f(t)dt$$

$$=\frac{1}{n}M_{-}^{a^{n}}[f(t),\frac{r}{n}]$$

$$M_{-}^{a}[f(x^{n}),r]=\frac{1}{n}M_{-}^{a^{n}}[f(t),\frac{r}{n}] \quad (7a)$$

Similarly $$M_{+}^{a}[f(x^{n}),r]=\frac{1}{n}M_{+}^{a^{n}}[f(t),\frac{r}{n}] \quad (7b)$$

2.2.4. Theorems

2.2.4.1. Convolution Theorem

The GFMIT of a function f(x) in 0<x<a , introduced by the integral

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx,$$ then

$$M_{-}^{a}[f(x)g(t-x),r]=\frac{1}{2\pi i}\int_{c-ia}^{c+ia}x^{-r}M_{-}^{a}[f(x),r]M_{-}^{a}[g(t-x),r]dr \quad (8a)$$

Similarly $$M_{+}^{a}[f(x)g(t-x),r]=\frac{1}{2\pi i}\int_{c-ia}^{c+ia}x^{-r}M_{+}^{a}[f(x),r]M_{+}^{a}[g(t-x),r]dr \quad (8b)$$

2.2.4.2. Orthogonality (Parsevals Theorem)

The GFMIT of a function f(x) in 0<x<a , introduced by the integral

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx,$$ then

$$M_{-}^{a}[f(x)g(x),r]=\frac{1}{2\pi i}\int_{c-ia}^{c+ia}x^{-r}M_{-}^{a}[f(x),r]M_{-}^{a}[g(x),r]dr \quad (9a)$$

Similarly $$M_{+}^{a}[f(x)g(x),r]=\frac{1}{2\pi i}\int_{c-ia}^{c+ia}x^{-r}M_{+}^{a}[f(x),r]M_{+}^{a}[g(x),r]dr \quad (9b)$$

2.2.4.3 .Shifting Theorem

The GFMIT of a function f(x) in 0<x<a , introduced by the integral

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx,$$ then

$$M_{-}^{a}[x^{n}f(x),r]=M_{-}^{a}[f(x),r+n]$$

Proof **If** $M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx$, then

$$M_{-}^{a}[x^{n} f(x), r] = \int_{0}^{a} (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) x^{n} f(x) dx$$

$$= \int_{0}^{a} (x^{r+n-1} - \frac{a^{2r}}{x^{r-n+1}}) f(x) dx$$

$$= M_{-}^{a}[f(x), r+n]$$

$$M_{-}^{a}[x^{n} f(x), r] = M_{-}^{a}[f(x), r+n] \quad (10a)$$

Similarly $M_{+}^{a}[x^{n} f(x), r] = M_{+}^{a}[f(x), r+n]$ (10b)

2.2.5. Propositions

2.2.5.1. Proposition for the function $\frac{1}{x} f(\frac{1}{x})$

The GFMIT in [0,a] is

$$M_{-}^{a}[f(x), r] = F_{-}^{a}(r) = \int_{0}^{a} (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x) dx$$, where Re(p)< γ, then

$$M_{-}^{a}[\frac{1}{x} f(\frac{1}{x}), r] = \int_{0}^{a} (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) \frac{1}{x} f(\frac{1}{x}) dx$$

substitute $\frac{1}{x}$=t ,x=$\frac{1}{t}$, dx=$\frac{-dt}{t^{2}}$. if x=0 the t=∞ and if x=a then t=$\frac{1}{a}$

$$M_{-}^{a}[\frac{1}{x} f(\frac{1}{x}), r] = \int_{\infty}^{\frac{1}{a}} ((\frac{1}{t})^{r-1} - \frac{a^{2r}}{(\frac{1}{t})^{r+1}}) t f(t) (\frac{-dt}{t^{2}})$$

$$= \int_{\frac{1}{a}}^{\infty} (t^{-r+1} - \frac{a^{2r}}{t^{-r-1}}) f(t) (\frac{dt}{t})$$

$$= \int_{\frac{1}{a}}^{\infty} (t^{-r} - \frac{a^{2r}}{t^{-r}}) f(t) dt$$

$$= \int_{\frac{1}{a}}^{\infty} (t^{-r+1-1} - \frac{a^{2r}}{t^{-r-1+1}}) f(t) dt$$

$$M_{-}^{a}[\frac{1}{x} f(\frac{1}{x}), r] = \int_{\frac{1}{a}}^{\infty} (t^{-(r-1)-1} - \frac{a^{2r}}{t^{-(r+1)+1}}) f(t) dt$$

$$M_{-}^{a}[\frac{1}{x} f(\frac{1}{x}), r] = M_{-,\frac{1}{a}}^{\infty}[f(t), -r+1] = \int_{\frac{1}{a}}^{\infty} (t^{-(r-1)-1} - \frac{a^{2r}}{t^{-(r+1)+1}}) f(t) dt \quad (11a)$$

Similarly $M_{+}^{a}[\frac{1}{x}f(\frac{1}{x}),r]=M_{+,\frac{1}{a}}^{\infty}[f(t),-r+1]=\int_{\frac{1}{a}}^{\infty}(t^{-(r-1)-1}+\frac{a^{2r}}{t^{-(r+1)+1}})f(t)dt$ (11b)

Here, 11a and 11b are the new integral transforms in $\frac{1}{a}$ to ∞

2.2.5.2. Proposition for the function (logx)f(x)

The GFMIT in [0, a] is

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx\ ,\quad \text{where Re(p)}<\gamma,$$

For a=1 , we have the particular case of (1)

$$M_{-}^{1}[f(x),r]=F_{-}^{1}(r)=\int_{0}^{1}(x^{r-1}-\frac{1}{x^{r+1}})f(x)dx,\ \text{then}$$

$$M_{-}^{1}[(\log x)f(x),r]=\int_{0}^{1}(x^{r-1}-\frac{1}{x^{r+1}})(\log x)f(x)dx$$

$$=\int_{0}^{1}[(\log x)x^{r-1}-(\log x)\frac{a^{2r}}{x^{r+1}})f(x)dx\ ,$$

$$=\int_{0}^{1}[\frac{d}{dr}(x^{r-1})+\frac{d}{dr}(x^{-r-1})f(x)dx$$

$$=\frac{d}{dr}\int_{0}^{1}(x^{r-1}+\frac{1}{x^{r+1}})f(x)dx$$

$$=\frac{d}{dr}M_{+}^{1}[f(x),r]=\frac{d}{dr}F_{+}^{1}(r)$$

$$M_{-}^{1}[(\log x)f(x),r]=\frac{d}{dr}M_{+}^{1}[f(x),r]=\frac{d}{dr}F_{+}^{1}(r) \qquad (12a)$$

Similarly $M_{+}^{1}[(\log x)f(x),r]=M_{+}^{1}[f(x),r]=\frac{d}{dr}F_{+}^{1}(r)\ M_{-}^{1}[f(x),r]=\frac{d}{dr}F_{-}^{1}(r)$ (12b)

2.2.6. GFMIT of Differential Operators

The GFMIT in [0, a] is

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx\ ,\quad \text{where Re(p)}<\gamma,\text{then}$$

$$M_{-}^{a}[(x\frac{d}{dx})f(x),r]=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})(x\frac{d}{dx})f(x)dx$$

$$=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})xf'(x)dx$$

$$=\int_{0}^{a}(x^{r}-\frac{a^{2r}}{x^{r}})f'(x)dx$$

$$=[(x^r - \frac{a^{2r}}{x^r}) f(x)]_0^a - \int_0^a (rx^{r-1} - \frac{a^{2r}}{x^{r+1}}(-r)) f(x)dx$$

$$=[(a^r - \frac{a^{2r}}{a}) f(a)] - r\int_0^a (x^{r-1} + \frac{a^{2r}}{x^{r+1}}) f(x)dx$$

$$=0 - \mathrm{r}\, M_+^a[f(x), r]$$

$$M_-^a[(x\frac{d}{dx}) f(x), r] = - \mathrm{r}\, M_+^a[f(x), r] \qquad (13a)$$

Similarly $M_+^a[(x\frac{d}{dx}) f(x), r] = 2a^r f(a) - rM_-^a[f(x), r]$ (13B)

$$M_-^a[(x\frac{d}{dx})^2 f(x), r] = \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}})(x\frac{d}{dx})^2 f(x)dx$$

$$= \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}})(x^2 f''(x) + xf'(x))dx$$

$$= \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}})x^2 f''(x)dx + \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}})xf'(x)dx$$

$$= \int_0^a (x^{r+1} - \frac{a^{2r}}{x^{r-1}}) f''(x)dx - \mathrm{r}\, M_+^a[f(x), r] \qquad (\text{from } 13a)$$

$$=[(x^{r+1} - \frac{a^{2r}}{x^{r-1}}) f'(x)]_0^a - \int_0^a ((r+1)x^r - \frac{a^{2r}}{x^r}(-r+1)) f'(x)dx - \mathrm{r}\, M_+^a[f(x), r]$$

$$=(a^{r+1} - \frac{a^{2r}}{a^{r-1}}) f'(a)] - \int_0^a (rx^r + x^r + r\frac{a^{2r}}{x^r} - \frac{a^{2r}}{x^r}) f'(x)dx - \mathrm{r}\, M_+^a[f(x), r]$$

$$=(a^{r+1} - a^{r+1}) f'(a)] - r\int_0^a (x^r + \frac{a^{2r}}{x^r}) f'(x)dx + \int_0^a (x^r - \frac{a^{2r}}{x^r}) f'(x)dx - \mathrm{r}\, M_+^a[f(x), r]$$

$$=0 - r[(x^r + \frac{a^{2r}}{x^r}) f(x)]_0^a + r\int_0^a (rx^{r-1} + \frac{a^{2r}}{x^{r+1}}(-r)) f(x)dx + \mathrm{r}\, M_+^a[xf'(x), r]$$

$$-\mathrm{r}\, M_+^a[f(x), r]$$

$$=0 - r[(x^r + a^r) f(a) + r^2\int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x)dx$$

$$=-2ra^r \mathrm{f(a)} + r^2 M_-^a[f(x), r]$$

$$= r^2 M_-^a[f(x), r] - 2ra^r \mathrm{f(a)}$$

$$M_-^a[(x\frac{d}{dx})^2 f(x), r] = r^2 M_-^a[f(x), r] - 2ra^r \mathrm{f(a)} \qquad (14a)$$

Similarly $M_+^a[(x\frac{d}{dx})^2 f(x), r] = r^2 M_+^a[f(x), r] + 2a^{r-1} f'(a)$ (14b)

2.2.7. GFMIT of Integral Operators

2.2.7.1. The Integral Expression is $\int_0^a a^{-r} f(xt)g(t)dt$

The GFMIT in [0, a] is

$$M_-^a[f(x),r]=\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx \text{ , where Re(p)< } \gamma \text{ ,then}$$

$$M_-^a[\int_0^a a^{-r} f(xt),g(t)dt,r]=\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})[\int_0^a a^{-r} f(xt)g(t)dt]dx$$

$$=\int_0^a a^{-r}g(t)dt\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(xt)dx$$

substitute xt=z then x= $\frac{z}{t}$, dx=$\frac{dz}{t}$,if x=0 then z=0 and x=a then z=at

$$=\int_0^a a^{-r}g(t)dt\int_0^{at} ((\frac{z}{t})^{r-1}-\frac{a^{2r}}{(\frac{z}{t})^{r+1}})f(z)\frac{dz}{t}$$

$$=\int_0^a a^{-r}t^{-r}g(t)dt\int_0^{at} (z^{r-1}-\frac{(at)^{2r}}{z^{r+1}})f(z)dz$$

$$=\int_0^a a^{-r}t^{1-r-1}g(t)dt\int_0^{at} (z^{r-1}-\frac{(at)^{2r}}{z^{r+1}})f(z)dz$$

$$=M_0^a[g(t),1-r]\,M_-^{at}[f(z),r]$$

$$M_-^a[\int_0^a a^{-r} f(xt),g(t)dt,r]=M_0^a[g(t),1-r]\,M_-^{at}[f(z),r] \quad \text{(15a)}$$

Similarly $M_+^a[\int_0^a a^{-r} f(xt),g(t)dt,r]=M_0^a[g(t),1-r]\,M_+^{at}[f(z),r]$ (15b)

$$M_-^a[\int_0^a a^{-r}t^{\mu} f(xt),g(t)dt,r]=M_0^a[g(t),1-r+\mu]\,M_-^{at}[f(z),r] \quad \text{(15c)}$$

$$M_+^a[\int_0^a a^{-r}t^{\mu} f(xt),g(t)dt,r]=M_0^a[g(t),1-r+\mu]\,M_+^{at}[f(z),r] \quad \text{(15d)}$$

$$M_-^a[x^{\lambda}\int_0^a a^{-r}t^{\mu}x^{\lambda} f(xt),g(t)dt,r]=M_0^a[g(t),1-r+\mu]\,M_-^{at}[f(z),r+\lambda] \quad \text{(15e)}$$

$$M_+^a[x^{\lambda}\int_0^a a^{-r}t^{\mu}x^{\lambda} f(xt),g(t)dt,r]=M_0^a[g(t),1-r+\mu]\,M_+^{at}[f(z),r+\lambda] \quad \text{(15e)}$$

2.2.7.2. The Integral Expression is $\int_0^a a^{-r} f(xt)g(t)dt$

The GFMIT in [0, a] is

$$M_{+}^{a}[f(x),r]=\int_{0}^{a}(x^{r-1}+\frac{a^{2r}}{x^{r+1}})f(x)dx$$, where Re(p)< γ ,then

$$M_{+}^{a}[\int_{0}^{a}a^{-r}f(xt),g(t)dt,r]=\int_{0}^{a}(x^{r-1}+\frac{a^{2r}}{x^{r+1}})[\int_{0}^{a}a^{-r}f(xt)g(t)dt]dx$$

$$=\int_{0}^{a}a^{-r}g(t)dt\int_{0}^{a}(x^{r-1}+\frac{a^{2r}}{x^{r+1}})f(xr)dx$$

substitute xt=z then x= $\frac{z}{t}$, dx= $\frac{dz}{t}$,if x=0 then z=0 and x=a then z=at

$$=\int_{0}^{a}a^{-r}g(t)dt$$

$$=\int_{0}^{a}a^{-r}t^{-r}g(t)dt\int_{0}^{at}(z^{r-1}+\frac{(ar)^{2r}}{z^{r+1}})f(z)dz$$

$$=\int_{0}^{a}a^{-r}t^{1-r-1}g(t)dt\int_{0}^{at}(z^{r-1}+\frac{(ar)^{2r}}{z^{r+1}})f(z)dz$$

$$=M_{0}^{a}[g(t),1-r]\,M_{+}^{ar}[f(z),r]$$

$$M_{+}^{a}[\int_{0}^{a}a^{-r}f(xt),g(t)dt,r]=M_{0}^{a}[g(t),1-r]\,M_{+}^{ar}[f(z),r] \quad (16a)$$

$$M_{-}^{a}[\int_{0}^{a}a^{-r}f(xt),g(t)dt,r]=M_{0}^{a}[g(t),1-r]\,M_{-}^{ar}[f(z),r] \quad (16b)$$

Similarly $M_{=}^{a}[\int_{0}^{a}a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_{0}^{a}[g(t),1-r+\mu]\,M_{-}^{ar}[f(z),r]$;' (16c)

$$M_{+}^{a}[\int_{0}^{a}a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_{0}^{a}[g(t),1-r+\mu]\,M_{+}^{ar}[f(z),r] \quad (16d)$$

$$M_{=}^{a}[x^{\lambda}\int_{0}^{a}a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_{0}^{a}[g(t),1-r+\mu]\,M_{-}^{ar}[f(z),r+\lambda] \quad (16e)$$

$$M_{+}^{a}[x^{\lambda}\int_{0}^{a}a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_{0}^{a}[g(t),1-r+\mu]\,M_{+}^{ar}[f(z),r+\lambda] \quad (16f)$$

2.2.7.3. The Integral Expression is $\int_{0}^{a}a^{-r}f(\frac{x}{t})g(t)dt$

The GFMIT in [0, a] is

$$M_{-}^{a}[f(x),r]=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx$$, where Re(p)< γ ,then

$$M_{-}^{a}[\int_{0}^{a}a^{-r}f(\frac{x}{t}),g(t)dt,r]=\int_{0}^{a}(x^{r-1}-\frac{a^{2r}}{x^{r+1}})[\int_{0}^{a}a^{-r}f(\frac{x}{t})g(t)dt]dx$$

$$=\int_0^a a^{-r}g(t)dt\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(\frac{x}{t})dx$$

substitute $\frac{x}{t}$=z then x= tz , dx=tdz ,if x=0 then z=0 and x=a then z=$\frac{a}{t}$

$$=\int_0^a a^{-r}g(t)dt\int_0^{\frac{a}{t}} ((tz)^{r-1}-\frac{a^{2r}}{(tz)^{r+1}})tf(z)tdz$$

$$=\int_0^a a^{-r}t^{r}g(t)dt\int_0^{\frac{a}{t}} (z^{r-1}-\frac{(\frac{a}{t})^{2r}}{z^{r+1}})f(z)dz$$

$$=\int_0^a a^{-r}t^{r+1-1}g(t)dt\int_0^{\frac{a}{t}} (z^{r-1}-\frac{(\frac{a}{t})^{2r}}{z^{r+1}})f(z)dz$$

$$=M_0^a[g(t),r+1]\ M_-^{\frac{a}{t}}[f(z),r]$$

$$M_-^a[\int_0^a a^{-r}f(\frac{x}{t}),g(t)dt,r]=M_0^a[g(t),r+1]\ M_-^{\frac{a}{t}}[f(z),r] \quad (17a)$$

Similarly $M_+^a[\int_0^a a^{-r}f(\frac{x}{t}),g(t)dt,r]=M_0^a[g(t),r+1]\ M_+^{\frac{a}{t}}[f(z),r]$ (17b)

$$M_-^a[\int_0^a a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_0^a[g(t),r+\mu+1]\ M_-^{at}[f(z),r] \quad (17c)$$

$$M_+^a[\int_0^a a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_0^a[g(t),r+\mu+1]\ M_+^{at}[f(z),r] \quad (17d)$$

$$M_-^a[x^{\lambda}\int_0^a a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_0^a[g(t),1+r+\mu]\ M_-^{at}[f(z),r+\lambda] \quad (17e)$$

$$M_+^a[x^{\lambda}\int_0^a a^{-r}t^{\mu}f(xt),g(t)dt,r]=M_0^a[g(t),1+r+\mu]\ M_+^{at}[f(z),r+\lambda] \quad (17f)$$

2.2.7.4. The GFMIT Of Integral Expression is $\int_0^a a^{-r}f(\frac{x}{t})g(t)dt$

The GFMIT in [0, a] is

$$M_+^a[f(x),r]=\int_0^a (x^{r-1}+\frac{a^{2r}}{x^{r+1}})f(x)dx$$, where Re(p)< γ ,then

$$M_+^a[\int_0^a a^{-r}f(\frac{x}{t}),g(t)dt,r]=\int_0^a (x^{r-1}+\frac{a^{2r}}{x^{r+1}})[\int_0^a a^{-r}f(\frac{x}{t})g(t)dt]dx$$

$$=\int_0^a a^{-r} g(t)dt \int_0^a (x^{r-1}+\frac{a^{2r}}{x^{r+1}}) f(\frac{x}{t})dx$$

substitute $\frac{x}{t}$ =z then x= tz , dx= tdz ,if x=0 then z=0 and x=a then z= $\frac{a}{t}$

$$=\int_0^a a^{-r} g(t)dt \int_0^{\frac{a}{t}} ((tz)^{r-1}+\frac{a^{2r}}{(tz)^{r+1}}) f(z)tdz$$

$$=\int_0^a a^{-r} t^{r} g(t)dt \int_0^{\frac{a}{t}} (z^{r-1}+\frac{(\frac{a}{t})^{2r}}{z^{r+1}}) f(z)dz$$

$$=\int_0^a a^{-r} t^{r+1-1} g(t)dt \int_0^{\frac{a}{t}} (z^{r-1}+\frac{(\frac{a}{t})^{2r}}{z^{r+1}}) f(z)dz$$

$$=M_0^a[g(t),r+1]\ M_-^{\frac{a}{t}}[f(z),r]$$

$$M_-^a[\int_0^a a^{-r} f(\frac{x}{t}), g(t)dt, r] = M_0^a[g(t),r+1]\ M_+^{\frac{a}{t}}[f(z),r] \qquad (18a)$$

Similarly $M_+^a[\int_0^a a^{-r} f(\frac{x}{t}), g(t)dt, r] = M_0^a[g(t),r+1]\ M_-^{\frac{a}{t}}[f(z),r]$ (18b)

$$M_+^a[\int_0^a a^{-r} t^{\mu} f(xt), g(t)dt, r] = M_0^a[g(t),r+\mu+1]\ M_-^{\frac{a}{t}}[f(z),r] \qquad (18c)$$

$$M_-^a[\int_0^a a^{-r} t^{\mu} f(xt), g(t)dt, r] = M_0^a[g(t),r+\mu+1]\ M_+^{\frac{a}{t}}[f(z),r] \qquad (18d)$$

$$M_+^a[x^{\lambda}\int_0^a a^{-r} t^{\mu} x^{\lambda} f(xt), g(t)dt, r] = M_0^a[g(t),1+r+\mu]\ M_-^{\frac{a}{t}}[f(z),r+\lambda] \qquad (18e)$$

$$M_-^a[x^{\lambda}\int_0^a a^{-r} t^{\mu} x^{\lambda} f(xt), g(t)dt, r] = M_0^a[g(t),1+r+\mu]\ M_+^{\frac{a}{t}}[f(z),r+\lambda] \qquad (18f)$$

2.2.8. GFMIT of Integral Equations

2.2.8.1. The GFMIT in [0,a] is

$$M_-^a[f(x),r] = \int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}}) f(x)dx$$

The integral equation is $\int_0^a a^{-r} f(t)K(xt)dt = g(t)$,

$$M_-^a[\int_0^a a^{-r} f(t)K(xt)dt, r] = \int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})[\int_0^a a^{-r} f(t)K(xt)dt]\,dx = M_+^a[g(t),r]$$

$$\int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) \int_0^a a^{-r} f(t) K(xt) dt] \, dx = M_-^a[g(t), r]$$

$$\int_0^a a^{-r} f(t) dt] \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) K(xt) \, dx = \overline{g}(r)$$

substitute xt=z , x= $\frac{z}{t}$,dx= $\frac{dz}{t}$, if x=0 then z=0 and if x=a then z=at

$$\int_0^a a^{-r} f(t) dt] \int_0^{at} ((\frac{z}{t})^{r-1} - \frac{a^{2r}}{(\frac{z}{t})^{r+1}}) K(z) \frac{dz}{t} = \overline{g}(r)$$

$$\int_0^a a^{-r} t^{-r} f(t) dt] \int_0^{at} (z^{r-1} - \frac{(at)^{2r}}{z^{r+1}}) K(z) dz = \overline{g}(r)$$

$$\int_0^a a^{-r} t^{1-r-1} f(t) dt] \int_0^{at} (z^{r-1} - \frac{(at)^{2r}}{z^{r+1}}) K(z) dz = \overline{g}(r)$$

$\overline{f}(1-r)\ \overline{K}_-^{at}(r) = \overline{g}(r)$, similarly $\overline{f}(1-r)\ \overline{K}_+^{at}(r) = \overline{g}(r)$

replace (r) by (1-r)

$\overline{f}(r)\ \overline{K_-^{at}}(1-r) = \overline{g}(1-r)$

$\overline{f}(r) = \frac{\overline{g}(1-r)}{\overline{\overline{K_-^{at}}(1-r)}}$, where $\overline{L}(r) = \frac{1}{\overline{K_-^{at}}(1-r)}$ and $\overline{L}(r) = \frac{1}{\overline{K_+^{at}}(1-r)}$

$\overline{f}(r) = \overline{g}(1-r)\ \overline{L}(r)$, then the solution is

$$f(x) = \int_0^a a^{-r} g(t) L(xt) dt \qquad (19)$$

2.2.8.2. The GFMIT in [0,a] is

$$M_-^a[f(x), r] = \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x) dx$$

If K(x)= $x^{\frac{1}{2}} Y_v(x)$, where $Y_v(x)$ is the Bessel function of second kind of order v.

and L(x)= $x^{\frac{1}{2}} H_v(x)$, where $H_v(x)$ is the Struvi's function

The integral equation is $\int_0^a a^{-r} (xt)^{\frac{1}{2}} f(t) Y_v(xt) dt = g(t)$

The GFMIT in [0,a] of this integral equation is

$$M_-^a[\int_0^a a^{-r} (xt)^{\frac{1}{2}} f(t) Y_v, (xt) dt, r]$$

$$= \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}})[\int_0^a a^{-r} (xt)^{\frac{1}{2}} f(t) Y_v(xt) dt] dx = M_-^a[g(x), r]$$

$$\int_0^a a^{-r} f(t)dt \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}})(xt)^{\frac{1}{2}} Y_v(xt)dx = \overline{g}(r)$$

substitute xt=z then x= $\frac{z}{t}$, dx= $\frac{dz}{t}$, if x=0 then z=0 and if x=a then z=at

$$\int_0^a a^{-r} f(t)dt \int_0^{at} ((\frac{z}{t})^{r-1} - \frac{a^{2r}}{(\frac{z}{t})^{r+1}})(z)^{\frac{1}{2}} Y_v(z)\frac{dz}{t} = \overline{g}(r)$$

$$\int_0^a a^{-r} t^{-r} f(t)dt \int_0^{at} (z^{r-1} - \frac{(at)^{2r}}{z^{r+1}})(z)^{\frac{1}{2}} Y_v(z)dz = \overline{g}(r)$$

$$\int_0^a a^{-r} t^{-r} f(t)dt \int_0^{at} (z^{r-1} - \frac{(at)^{2r}}{z^{r+1}})K_v(z)dz = \overline{g}(r)$$

$\overline{f}(1-r)\ \overline{K_-^{at}}(r) = \overline{g}(r)$, similarly $\overline{f}(1-r)\ \overline{K_+^{at}}(r) = \overline{g}(r)$

replace (r) by (1-r)

$\overline{f}(r)\ \overline{K_-^{at}}(1-r) = \overline{g}(1-r)$

$\overline{f}(r) = \dfrac{\overline{g}(1-r)}{K_-^{at}(1-r)}$, where $\overline{H}(r) = \dfrac{1}{K_-^{at}(1-r)}$ and $\overline{H}(r) = \dfrac{1}{K_+^{at}(1-r)}$

$\overline{f}(r) = \overline{g}(1-r)\ \overline{H}(r)$, then its solution is

$$f(x) = \int_0^a a^{-r} (xt)^{\frac{1}{2}} g(t) H_v(xt)dt \qquad (20)$$

2.2.8.3. The GFMIT in [0,a] is

$$M_-^a[f(x), r] = \int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x)dx$$

The integral equation defined by g(x)= $\int_0^a a^{-r} (xt)^{\frac{1}{2}} f(t)Y_v(xt)dt = g(t)$ is called the Y-transform of f(x) , then we write g(t)= $Y_v[f(x), t]$

Similarly the integral equation defined by f(x)= $\int_0^a a^{-r} (xt)^{\frac{1}{2}} g(t)H_v(xt)dt$, is called the H-transform of g(t) , we write f(x)= $H_v^*[g(t), x]$. Here $H_v = -H_v^*$.

The integral equation is $\int_0^a a^{-r} K_1(\frac{x}{y}) f_1(y)\frac{dy}{y} = g_1(x)$.\Using GFMIT in [0,a] is

$$M_-^a[\int_0^a a^{-r} K_1 f(\frac{x}{y}() f_1 x(y)\frac{dy}{y}), r] = M_-^a[g_1(x), r]$$

$$\int_0^a (x^{r-1} - \frac{a^{2r}}{x^{r+1}})dx \int_0^a a^{-r} K_1(\frac{x}{y}) f_1(y)\frac{dy}{y} = \overline{g_1}(r)$$

$$\int_0^a a^{-r} f_1(y)\frac{dy}{y}\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})K_1(\frac{x}{y})dx=\overline{g_1}(r)$$

substitute $\frac{x}{y}=z$, x=yz , dx=ydz , if x=0 then z=0 and if x=a then z=$\frac{a}{y}$

$$\int_0^a a^{-r} f_1(y)\frac{dy}{y}\int_0^{\frac{a}{y}} ((yz)^{r-1}-\frac{a^{2r}}{(yz)^{r+1}})K_1(z)ydz=\overline{g_1}(r)$$

$$\int_0^a a^{-r} y^{r-1} f_1(y)\frac{dy}{y}\int_0^{\frac{a}{y}} (z^{r-1}-\frac{(ay0^{2r}}{z^{r+1}})K_1(z)dz=\overline{g_1}(r)$$

$$\overline{f_1}(r)\ \overline{K_1}(r)_{=}^{\frac{a}{t}}=\overline{g_1}(r),$$

$$\overline{f_1}(r)=\overline{g_1}(r)\frac{1}{\overline{\overline{K_{1\frac{a}{t}}}}(r)},\quad \text{where}\quad \overline{L_1}(r)=\frac{1}{\overline{\overline{K_{1\frac{a}{t}}}}(r)}$$

$\overline{f_1}(r)=\overline{g_1}(r)\ \overline{L_1}(r)$, its solution is

$$f_1(x)=\int_0^a a^{-r} g_1(t)L_1(\frac{x}{y})dt \qquad (21)$$

2.2.9. GFMIT of Weyl Fractional Transform

The Weyl fractional transform of the function f(t) is denoted by

$W^{-\alpha}[f(t)]=F(x,\alpha)$ and defined as

$$W^{-\alpha}[f(t)]=F(x,\alpha)=\frac{1}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} f(t)dt \quad ,0<\text{Re}(\alpha)<1 \ ,x>0,$$

The GFMIT in [0, a] is

$$M_{-}^{a}[f(x),r]=F_{-}^{a}(r)=\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx, \quad \text{where Re(p)}<\gamma, \text{ then}$$

$$M_{-}^{a}[F(x,\alpha),r]=\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})[\int_x^\infty \frac{1}{\Gamma(\alpha)}(t-x)^{\alpha-1} f(t)dt]dx$$

$$=\frac{1}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} f(t)dt\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})dx$$

$$=\frac{1}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} f(t)dt\,[\frac{x^r}{r}-\frac{a^{2r}}{(-r)x^r}]_0^a$$

$$=\frac{1}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} f(t)dt\,[\frac{a^r}{r}-\frac{a^{2r}}{(-r)a^r}]$$

$$=\frac{1}{r\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} f(t)dt\,[a^r+a^r]$$

$$=2a^r\,\frac{1}{r}\,\frac{1}{\Gamma(\alpha)}\int_x^\infty (t-x)^{\alpha-1} f(t)dt$$

$$=\frac{2a^r}{r}\,F_-^a\{x,\alpha)$$

$$M_-^a[F(x,\alpha),r]=\frac{2a^r}{r}\,F_-^a\{x,\alpha) \qquad (22)$$

2.2.10. Application of The GFMIT to the Summation of the Series

2.2.10.1. The GFMIT in [0, a] is

$$M_-^a[f(x),r]=F_-^a(r)=\int_0^a (x^{r-1}-\frac{a^{2r}}{x^{r+1}})f(x)dx\,,\quad \text{where Re(p)} < \gamma,$$

then its inversion is

$$\text{f(x)}=\frac{1}{2\pi i}\int_{c-ia}^{c+ia} x^{-r}M_-^a[f(x),r]dr\,,\ \text{exists for r>0}$$

$$\text{f(nx)}=\frac{1}{2\pi i}\int_{c-ia}^{c+ia} (nx)^{-r}M_-^a[f(x),r]dr\,,$$

$$=\frac{1}{2\pi i}\int_{c-ia}^{c+ia} n^{-r}x^{-r}M_-^a[f(x),r]dr$$

$$\sum_{n=0}^{\infty} f(nx)=\frac{1}{2\pi i}\sum_{n=0}^{\infty}\int_{c-ia}^{c+ia} n^{-r}x^{-r}M_-^a[f(x),r]dr$$

$$=\frac{1}{2\pi i}\int_{c-ia}^{c+ia}(\sum_{n=0}^{\infty} n^{-r})x^{-r}M_-^a[f(x),r]dr$$

$$=\frac{1}{2\pi i}\int_{c-ia}^{c+ia} x^{-r}M_-^a[f(x),r]\xi(r)dr$$

$$\sum_{n=0}^{\infty} f(nx)=\frac{1}{2\pi i}\int_{c-ia}^{c+ia} x^{-r}M_-^a[f(x),r]\xi(r)dr \qquad (23a)$$

where $\sum_{n=0}^{\infty}\frac{1}{n^r}$ is the Riemann Zeta function is denoted by $\xi(r)$ and defined as

$$\xi(r)=\sum_{n=0}^{\infty}\frac{1}{n^r}\ ,\quad \text{Re(r)>1}$$

Similarly $$\sum_{n=0}^{\infty} f(nx)=\frac{1}{2\pi i}\int_{c-ia}^{c+ia} x^{-r}M_+^a[f(x),r]\xi(r)dr \qquad (23b)$$

2.2.10.2. The GFMIT in[0, a] is

$$M_{-}^{a}[f(x),r] = F_{-}^{a}(r) = \int_{0}^{a} (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x)dx \,,\quad \text{where } \mathrm{Re}(p) < \gamma,$$

then its inversion is

$$\mathrm{f(x)} = \frac{1}{2\pi i}\int_{c-ia}^{c+ia} x^{-r} M_{-}^{a}[f(x),r]dr \,,\ \text{exists for } r>0$$

The Hurvitz Zeta function is denoted by $\xi(p,a)$ and defined as

$$\xi(r,a) = \sum_{n=0}^{\infty} \frac{1}{(x+a)^{p}} \quad ,0<a<1 \text{ and } \mathrm{Re}(p)>1, \text{ then by using inversion}$$

$$\mathrm{f(x+a)} = \frac{1}{2\pi i}\int_{c-ia}^{c+ia} (x+a)^{-r} M_{-}^{a}[f(x),r]dr \,,$$

$$\sum_{n=1}^{\infty} f(x+a) = \frac{1}{2\pi i}\sum_{n=1}^{\infty}\int_{c-ia}^{c+ia} (x+a)^{-r} M_{-}^{a}[f(x),r]dr$$

$$= \frac{1}{2\pi i}\int_{c-ia}^{c+ia} [\sum_{n=1}^{\infty} (x+a)^{-r}] M_{-}^{a}[f(x),r]dr$$

$$= \frac{1}{2\pi i}\int_{c-ia}^{c+ia} M_{-}^{a}[f(x),r]\xi(x,a)dr$$

$$\sum_{n=1}^{\infty} f(x+a) = \frac{1}{2\pi i}\int_{c-ia}^{c+ia} M_{-}^{a}[f(x),r]\xi(x,a)dr \qquad (24a)$$

$$\text{Where } \xi(x,a) = \sum_{n=0}^{\infty} (x+a)^{-r}$$

Similarly

$$\sum_{n=1}^{\infty} f(x+a) = \frac{1}{2\pi i}\int_{c-ia}^{c+ia} M_{+}^{a}[f(x),r]\xi(x,a)dr \qquad (24b)$$

2.2.10.3. If $\sum_{n=1}^{\infty} (-1)^{n-1} n^{-r} = [1-2^{1-n}\xi(r)$, the for GFMIT in [0,a] is

$$M_{-}^{a}[f(x),r] = F_{-}^{a}(r) = \int_{0}^{a} (x^{r-1} - \frac{a^{2r}}{x^{r+1}}) f(x)dx \,,$$

where Re(p)< γ, then its inversion is

$$\mathrm{f(x)} = \frac{1}{2\pi i}\int_{c-ia}^{c+ia} x^{-r} M_{-}^{a}[f(x),r]dr \,,\ \text{exists for } r>0$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} f(nx) = \frac{1}{2\pi i}\int_{c-ia}^{c+ia}\sum_{n=1}^{\infty}(-1)^{n-1}(nx)^{-r} M_{-}^{a}[f(x),r]dr$$

$$= \frac{1}{2\pi i}\int_{c-ia}^{c+ia}[\sum_{n=1}^{\infty}(-1)^{n-1}n^{-r}]x^{-r} M_{-}^{a}[f(x),r]dr$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} f(nx) = \frac{1}{2\pi i} \int_{c-ia}^{c+ia} [1-2^{1-r}]\xi(r) x^{-r} M_{-}^{a}[f(x), r] dr \qquad (25a)$$

Similarly $$\sum_{n=1}^{\infty} (-1)^{n-1} f(nx) = \frac{1}{2\pi i} \int_{c-ia}^{c+ia} [1-2^{1-r}]\xi(r) x^{-r} M_{+}^{a} f(x), r] dr \qquad (25b)$$

2.2.11. Application of the GFMIT to the Cauchy's Linear Differential Equation

The Cauchy's linear differential equation is

$$x^2 f''(x) + xf'(x) + f(x) = 0$$

By using GFMIT in [0,a) , we have

$$M_{-}^{a}[\ x^2 f''(x) + xf'(x) + f(x), r] = 0$$

$$M_{-}^{a}[f(x), r] = \frac{rf(a) - af'(a)}{r^2} \qquad (26a)$$

Similarly $$M_{+}^{a}[f(x), r] = \frac{rf(a) - af'(a)}{r^2} \qquad (26b)$$

$$f(x) = \frac{1}{2\pi i} \int_{c-ia}^{c+ia} x^{-r} M_{-}^{a}[f(x), r] dr$$

$$f(x) = \frac{1}{2\pi i} \int_{c-ia}^{c+ia} x^{-r} M_{-}^{a}[\frac{rf(a) - af'(a)}{r^2}] dr \qquad (27a)$$

Similarly $$f(x) = \frac{1}{2\pi i} \int_{c-ia}^{c+ia} x^{-r} M_{+}^{a}[\frac{rf(a) - af'(a)}{r^2}] dr \qquad (27b)$$

2.2.12. Remarks

1. Properties and Theorems of Mellin Integral Transform in $[0, \infty]$ are exists for the GFMIT in{0,a]

2.GFMITin[0,a] of derivatives , differential operators ,integrals , integral expressions and integral equations are obtained

3. Propositions for the functions $\frac{1}{x} f(\frac{1}{x})$ and (logx)f(x) gives new results for the GFMIT in [0, a]

4. New result is obtained of Weyl fractional transform by using GFMIT in [0,a]

5.Application of the GFMIT in [0,a] is in summation of the series is discussed. .

6.Using GFMIT in [0,a] solution of the Cauchy's linear differential Equation is obtained

CONCLUSION

All the properties and theorems of Mellin integral transform are exists to GFMIT in [0,a]..Results for the derivatives , differential operators, integrals , integral expressions and integral equations are exists for GFMIT in [0,a].This integral transform is applies to obtain the results of summation of series and solution of the Caychy's linear differential equation. In this paper we get new result of Weyl fractional transform by using GFMIT in [0,a] and new integral transform named by Mellin type integral transform in $[\frac{1}{a}, \infty]$ or it is also named as Pise-Khairnar integral transform in $[\frac{1}{a}, \infty]$

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