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HALF-Step symmetric continous hybrid block method for the numerical solutions of fourth order ordinary differential equations

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ABSTRACT

This paper proposed a half-step uniform order symmetric continuous hybrid block method for the numerical solution of fourth order ordinary differential equations. The interval of integration is taken within half-step for the approximation of fourth order ordinary differential equations. The approach of collocation and interpolation techniques was used to obtain the schemes and the additional schemes. The combination of power series and exponential function was used as basis function for the approximate solution. An order six symmetric discrete schemes was obtained and the properties of the methods show that it is zero-stable, consistence. Numerical examples are presented to show the accuracy and efficiency of the method.

AMS Subject Classification: 65L05, 65L06, 65L12
Keywords: Continuous, Hybrid, Block method, Power series, Exponential function, approximate solutions, zero stability, symmetric

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INTRODUCTION

In this paper, we considered the method of approximate solution of the general fourth order initial value problem of the form

$$y'''' = f\left(x, y, y', y'', y'''\right), y\left(x_0\right) = \tau_0, y'\left(x_0\right) = \tau_1, y''\left(x_0\right) = \tau_2, y'''\left(x_0\right) = \tau_3 \qquad (1)$$

where x_n , is the initial point, y_n is the solution at x_n, f is continuous within the interval of integration. Equation (1) is of interest to researchers because of its wide application in engineering, control theory and other real life problem, hence the study of the methods of its solution.

Most of the modeled problems in ordinary differential equations do not have analytical solutions; one must therefore consider approximation to the solutions of the problems, hence the use of numerical method is very imperative.

Conventionally, (1) is usually reduced to system of first order ordinary differential equations before an approximate method is applied to solve it. This reduction approach has been extensively discussed by several authors see Lambert (1973), Fatunla (1988) and Jator (2001). It was reported that due to the dimension of the resulting system of first order ordinary differential equations, the approach involves large human efforts. According to Awoyemi (1992), the major drawback of this approach of solution is that writing computer programs for these methods is often complicated especially when subroutines are incorporated to supply starting values required for the methods. The consequences of this are that longer time and human efforts are involved. Many prominent authors including Fatunla (1981), Taiwo and Onumanyi (1991), Yusuph and Onumanyi (2005), Jator (2007, 2009), Ogunware etal (2015), Olabode and Omole (2015) just to mention few, have developed methods for the direct solution of (1) without reducing it to systems of first order ordinary differential equations. Olabode (2007), Udoh et al (2007), independently proposed multi-derivative linear multistep methods and implemented in a predictor-corrector mode using Taylor series algorithm to supply the starting values. Although this kind of implementation yielded good accuracy, the procedure is more costly to implement because predictor-corrector subroutines involved are very complicated, since they require special techniques to supply the starting values for varying the step-size which leads to longer computer time and more human effort. This is extensively discussed by Jator (2007) and Olabode (2009). Olabode (2007) developed linear multi step method for the solution of general and special third order ordinary differential equations. Also, Muhammed (2010) developed a six step block method for the solution of fourth order ordinary differential equations.

Recently James et al (2015) developed a half-Step Continuous Block Method for the Solutions of Modeled Problems of Ordinary Differential Equations. Power series was used as the approximate solution via the collocation and interpolation approach. It was reported that the half-method perform reasonably well with the existing methods in the literature.

The need to improve on the predictor-corrector method so as to obtain another better method became important to researchers in this area. Thus, a continuous linear multi step method that computes the discrete methods at more than one point simultaneously was developed. This is what is now referred to as Block method.

This paper is motivated by the need to address some setbacks associated with the existing methods, apply the methods developed to solve higher order ordinary differential equation which is more accurate and efficient. This is because much work has not been done on the use of combination of power series and exponential function as basics function. The paper is organized as followed: Section 2 considers the mathematical formulation for developing our scheme. Section 3 considers the analysis of the basic properties of the method. Section 4, considers the implementation, application of the derived method to solve some fourth order ODEs and conclusion.

2.0 Mathematical Formulation

We consider the combination of power series and exponential function as a basis function for approximation:

$$Y(x) = \sum_{j=0}^{r+s-1} a_j x^j + a_{r+s} \sum_{j=0}^{r+s} \frac{\alpha^j x^j}{j!} \tag{2}$$

Interpolation and collocation procedures are used by choosing s at off grid points and collocates point r at all points.

$$Y^{iv}(x) = \sum_{j=0}^{r+s-1} j(j-1)(j-2)(j-3) a_j x^{j-4} + a_{r+s} \sum_{j=0}^{r+s} \frac{\alpha^j x^{j-4}}{(j-4)!} \tag{3}$$

where a_j , $\alpha^j \varepsilon$□ And $x \in [a,b]$, a_j's are coefficients to be determined. We construct a Half-step Collocation Method (HCM) by imposing the following conditions on (2)

$$Y(x_{n+j}) = y_{n+j},\ j = 0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}. \tag{4}$$

$$DY(x_{n+j}) = f_{n+j},\ j = 0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2} \tag{5}$$

Substituting (1) into (3) gives

$$f(x, y, y', y'', y''') = \sum_{j=0}^{r+s-1} j(j-1)(j-2)(j-3) a_j x^{j-4} + a_{r+s} \sum_{j=0}^{r+s} \frac{\alpha^j x^{j-4}}{(j-4)!} \tag{6}$$

We shall consider the off grid point of half with constant step-size (h) , Interpolating (2) at

$x = x_n, x_{n+1/8}, x_{n+1/4}, x_{n+3/8}$ and collocating (3) at $x_n, x_{n+1/8}, x_{n+1/4}, x_{n+3/8}, x_{n+1/2}$ to give a system of non-linear equation of the form

$$AX = B \tag{7}$$

where $A = [a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]^T$, $B = \left[y_n, y_{n+1/8}, y_{n+1/4}, y_{n+3/8}, f_n, f_{n+1/8}, f_{n+1/4}, f_{n+3/8}, f_{n+1/2} \right]^T$

$$X = \begin{bmatrix}
1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & \left[1+\alpha x_n + \frac{\alpha^2 x_n^2}{2!} + \frac{\alpha^3 x_n^3}{3!} + \frac{\alpha^4 x_n^4}{4!} + \frac{\alpha^5 x_n^5}{5!} + \frac{\alpha^6 x_n^6}{6!} + \frac{\alpha^7 x_n^7}{7!} + \frac{\alpha^8 x_n^8}{8!}\right] \\
1 & x_{n+1/8} & x_{n+1/8}^2 & x_{n+1/8}^3 & x_{n+1/8}^4 & x_{n+1/8}^5 & x_{n+1/8}^6 & x_{n+1/8}^7 & \left[1+\alpha x_n + \frac{\alpha^2 x_n^2}{2!} + \frac{\alpha^3 x_n^3}{3!} + \frac{\alpha^4 x_n^4}{4!} + \frac{\alpha^5 x_n^5}{5!} + \frac{\alpha^6 x_n^6}{6!} + \frac{\alpha^7 x_n^7}{7!} + \frac{\alpha^8 x_n^8}{8!}\right] \\
1 & x_{n+1/4} & x_{n+1/4}^2 & x_{n+1/4}^3 & x_{n+1/4}^4 & x_{n+1/4}^5 & x_{n+1/4}^6 & x_{n+1/4}^7 & \left[1+\alpha x_n + \frac{\alpha^2 x_n^2}{2!} + \frac{\alpha^3 x_n^3}{3!} + \frac{\alpha^4 x_n^4}{4!} + \frac{\alpha^5 x_n^5}{5!} + \frac{\alpha^6 x_n^6}{6!} + \frac{\alpha^7 x_n^7}{7!} + \frac{\alpha^8 x_n^8}{8!}\right] \\
1 & x_{n+3/8} & x_{n+3/8}^2 & x_{n+3/8}^3 & x_{n+3/8}^4 & x_{n+3/8}^5 & x_{n+3/8}^6 & x_{n+3/8}^7 & \left[1+\alpha x_n + \frac{\alpha^2 x_n^2}{2!} + \frac{\alpha^3 x_n^3}{3!} + \frac{\alpha^{34} x_{n+3/8}^4}{4!} + \frac{\alpha^5 x_n^5}{5!} + \frac{\alpha^6 x_n^6}{6!} + \frac{\alpha^7 x_n^7}{7!} + \frac{\alpha^8 x_n^8}{8!}\right] \\
0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & \left[\frac{\alpha^4 x_n^4}{4!} + \frac{\alpha^5 x_n^5}{5!} + \frac{\alpha^6 x_n^6}{6!} + \frac{\alpha^7 x_n^7}{7!} + \frac{\alpha^8 x_n^8}{8!}\right] \\
0 & 0 & 0 & 0 & 24 & 120x_{n+1/8} & 360x_{n+1/8}^2 & 840x_{n+1/8}^3 & \left[\frac{\alpha^4 x_{n+1/8}^4}{4!} + \frac{\alpha^5 x_{n+1/8}^5}{5!} + \frac{\alpha^6 x_{n+1/8}^5}{6!} + \frac{\alpha^7 x_{n+1/8}^8}{7!} + \frac{\alpha^8 x_{n+1/8}^8}{8!}\right] \\
0 & 0 & 0 & 0 & 24 & 120x_{n+1/4} & 360x_{n+1/4}^2 & 840x_{n+1/4}^3 & \left[\frac{\alpha^4 x_{n+1/4}^4}{4!} + \frac{\alpha^5 x_{n+1/4}^5}{5!} + \frac{\alpha^6 x_{n+1/4}^5}{6!} + \frac{\alpha^7 x_{n+1/4}^8}{7!} + \frac{\alpha^8 x_{n+1/4}^8}{8!}\right] \\
0 & 0 & 0 & 0 & 24 & 120x_{n+3/8} & 360x_{n+3/8}^2 & 840x_{n+3/8}^3 & \left[\frac{\alpha^4 x_{n+3/8}^4}{4!} + \frac{\alpha^5 x_{n+3/8}^5}{5!} + \frac{\alpha^6 x_{n+3/8}^5}{6!} + \frac{\alpha^7 x_{n+3/8}^8}{7!} + \frac{\alpha^8 x_{n+3/8}^8}{8!}\right] \\
0 & 0 & 0 & 0 & 24 & 120x_{n+1/2}^6 & 360x_{n+1/2}^6 & 840x_{n+1/2}^6 & \left[\frac{\alpha^4 x_{n+1/2}^4}{4!} + \frac{\alpha^5 x_{n+1/2}^5}{5!} + \frac{\alpha^6 x_{n+1/2}^5}{6!} + \frac{\alpha^7 x_{n+1/2}^8}{7!} + \frac{\alpha^8 x_{n+1/2}^8}{8!}\right]
\end{bmatrix} \tag{8}$$

Solving (8) for the a_j 's using Gaussians elimination method and substituting back into (2) and after much algebraic simplification, we obtained the continuous hybrid half-step method of the form

$$y(x) = \alpha_0 y_n + \alpha_{1/8} y_{n+1/8} + \alpha_{1/4} y_{n+1/4} + \alpha_{3/8} y_{n+3/8} + h^4 \left[\sum_{j=0}^{0(1/4)1/2} \beta_{1/8} f_{n+1/8} + \beta_{1/4} f_{n+1/4} + \beta_{3/8} f_{n+3/8} + \beta_{1/2} f_{n+1/2} \right] \tag{9}$$

is of the form

$$y(x) = \sum_{j=0}^{0(1/4)3/8} \alpha_j(x) y_{n+j} + h^4 \left(\sum_{j=0}^{k} \beta_j(x) f_{n+j} \right) \tag{10}$$

where α_j's and β_j's are continuous coefficients expressed as functions of t, where

$$t=\frac{x-x_n}{h},\frac{dt}{dx}=\frac{1}{h} \tag{11}$$

The coefficients of y_{n+j} and f_{n+j} are obtained as:

$$\alpha_0(t)=\frac{256}{3}(-t^3+\frac{44}{3}t^3-64t^2-1)$$

$$\alpha_{1/8}(t)=(256t^3-160t^2+24t)$$

$$\alpha_{1/4}(t)=-4t(64t^2-32t+3)$$

$$\alpha_{3/8}(t)=\frac{8}{3}t(32t^2-12t+1)$$

$$\beta_0(t)=\frac{h^4}{30965760}\left[55t+11t^2-364t^3+672t^5-224t^6-384t^7+192t^8\right]$$

$$\beta_{1/8}(t)=\frac{-h^4}{120960}\left[493t+53t^2-2296t^3+1344t^5-896t^6-192t^7+192t^8\right] \tag{12}$$

$$\beta_{1/4}(t)=\frac{-h^4}{120960}\left[493t+53t^2-2296t^3+1344t^5-896t^6-192t^7+192t^8\right]$$

$$\beta_{3/8}(t)=\frac{-h^4}{120960}\left[493t+53t^2-2296t^3+1344t^5-896t^6-192t^7+192t^8\right]$$

$$\beta_{1/2}(t)=\frac{-h^4}{120960}\left[493t+53t^2-2296t^3+1344t^5-896t^6-192t^7+192t^8\right]$$

Evaluating (10) at $t=1/2$ gives the main method below

$$y_{n+\frac{1}{2}}-4y_{n+\frac{3}{8}}+6y_{n+\frac{1}{4}}-4y_{n+\frac{1}{8}}+y_n=\frac{-h^4}{2949120}[f_n-124f_{n+\frac{1}{8}}-474f_{n+\frac{1}{4}}-124f_{n+\frac{3}{8}}+f_{n+\frac{1}{2}}] \tag{13}$$

To obtain additional methods, we use (12) and formula for the derivatives which are expressed as follows:

$$y'(x)=\frac{1}{h}\left[\sum_{j=0}^{(\frac{3}{8})}\alpha'_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta'_j(x)f_{n+j}\right)\right] \tag{14}$$

$$y''(x)=\frac{1}{h^2}\left[\sum_{j=0}^{(\frac{3}{8})}\alpha''_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta''_j(x)f_{n+j}\right)\right] \tag{15}$$

$$y'''(x)=\frac{1}{h^3}\left[\sum_{j=0}^{(\frac{3}{8})}\alpha'''_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta'''_j(x)f_{n+j}\right)\right] \qquad (16)$$

2.1 FORMATION OF THE BLOCK FOR HALF-STEP HYBRID METHOD

Combining equations (13) and (14-16) yields the block of the form

$$\mathbf{A}^{(0)}\mathbf{Y}_m^{(i)}=\sum_{i=0}^{3}\frac{(jh)}{i}e_i y_n^{(i)}+h^{(4-i)}\left[\mathbf{d}_i f(y_n)+\mathbf{b}_i\mathbf{F}(\mathbf{Y}_m)\right], \qquad (17)$$

$\mathbf{Y}_m^{(i)}=\left[y_{n+1/8}^{(i)}\ \ y_{n+\frac{1}{4}}^{(i)}\ \ y_{n+3/8}^{(i)}\ \ y_{n+\frac{1}{2}}^{(i)}\right]^T$, $\mathbf{F}(\mathbf{Y}_m)=\left[f_{n+1/8}\ \ f_{n+1/4}\ \ f_{n+3/8}\ \ f_{n+1/2}\right]^T$,

$\mathbf{Y}_n^{(i)}=\left[y_{n-1/8}^{(i)}\ \ y_{n-\frac{1}{4}}^{(i)}\ \ y_{n-3/8}^{(i)}\ \ y_{n-\frac{1}{2}}^{(i)}\right]^T$, And $A^{(0)}=4\times4$ identity matrix, $i=0(1/8)\,1/2$

when $i=0$;

$$e_0=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix},e_1=\begin{bmatrix}0&0&0&1/8\\0&0&0&1/4\\0&0&0&3/8\\0&0&0&1/2\end{bmatrix},e_2=\begin{bmatrix}0&0&0&1/128\\0&0&0&1/32\\0&0&0&9/128\\0&0&0&1/8\end{bmatrix},e_3=\begin{bmatrix}0&0&0&1/3072\\0&0&0&1/384\\0&0&0&9/1024\\0&0&0&1/48\end{bmatrix}$$

$$d_0=\begin{bmatrix}0&0&0&\frac{3373}{495452160}\\0&0&0&\frac{37}{483840}\\0&0&0&\frac{5319}{18350080}\\0&0&0&\frac{11}{15120}\end{bmatrix},b_0=\begin{bmatrix}\frac{139}{24772608}&\frac{-283}{82575360}&\frac{179}{123863040}&\frac{-131}{495452160}\\\frac{59}{483840}&\frac{-1}{18432}&\frac{11}{483840}&\frac{-1}{241920}\\\frac{2889}{4587520}&\frac{-1539}{917504}&\frac{81}{917504}&\frac{-297}{18350080}\\\frac{1}{540}&\frac{-1}{5040}&\frac{1}{3780}&\frac{-1}{24192}\end{bmatrix}$$

when $i=1$

$$d_1=\begin{bmatrix}0&0&0&\frac{113}{573440}\\0&0&0&\frac{331}{322560}\\0&0&0&\frac{1431}{573440}\\0&0&0&\frac{31}{6720}\end{bmatrix},b_1=\begin{bmatrix}\frac{107}{516096}&\frac{-103}{860160}&\frac{43}{860160}&\frac{-47}{5160960}\\\frac{83}{40320}&\frac{-1}{1344}&\frac{13}{40320}&\frac{-19}{322560}\\\frac{1863}{286720}&\frac{-243}{286720}&\frac{45}{57344}&\frac{-81}{573440}\\\frac{17}{1260}&\frac{1}{1680}&\frac{1}{420}&\frac{-1}{4032}\end{bmatrix},$$

when $i=2$

$$d_2 = \begin{bmatrix} 0 & 0 & 0 & \frac{367}{92160} \\ 0 & 0 & 0 & \frac{53}{5760} \\ 0 & 0 & 0 & \frac{147}{10240} \\ 0 & 0 & 0 & \frac{7}{360} \end{bmatrix}, b_2 = \begin{bmatrix} \frac{3}{512} & \frac{-47}{15360} & \frac{29}{23040} & \frac{-7}{30720} \\ \frac{1}{40} & \frac{-1}{192} & \frac{1}{360} & \frac{-1}{1920} \\ \frac{117}{2560} & \frac{27}{5120} & \frac{3}{512} & \frac{-9}{10240} \\ \frac{1}{15} & \frac{1}{60} & \frac{1}{45} & 0 \end{bmatrix}$$

when $i = 3$

$$d_3 = \begin{bmatrix} 0 & 0 & 0 & \frac{251}{5760} \\ 0 & 0 & 0 & \frac{-29}{720} \\ 0 & 0 & 0 & \frac{27}{640} \\ 0 & 0 & 0 & \frac{7}{180} \end{bmatrix}, b_3 = \begin{bmatrix} \frac{323}{2880} & \frac{-11}{240} & \frac{53}{2880} & \frac{-19}{5760} \\ \frac{-31}{180} & \frac{-1}{30} & \frac{-1}{180} & \frac{1}{720} \\ \frac{51}{320} & \frac{9}{80} & \frac{21}{320} & \frac{-3}{640} \\ \frac{8}{45} & \frac{1}{15} & \frac{8}{45} & \frac{7}{180} \end{bmatrix}$$

3.0 Basic Properties of Half-step Method

3.1 Order and Error constant of the Block

Let the linear operator defined on the method be $\pounds[y(x);h]$, where,

$$\Delta[y(x);h] = \mathbf{A}^{(0)}\mathbf{Y}Y_m^{(i)} - \sum_{i=0}^{3}\frac{(jh)}{i}y_n^{(i)} - h^{(4-i)}\left[\mathbf{d}^i f(y_n) + \mathbf{b}^i \mathbf{F}(\mathbf{Y}^m)\right] \quad (18)$$

Expanding the form $\mathbf{Y}^m$ and $\mathbf{F}(\mathbf{Y}^m)$ in Taylor series and comparing coefficients of h, we obtained

$$\Delta[y(x);h] = C_0 y(x) + C_1 hy'(x) + \ldots + C_p h^p y^p(x) + C_{p+1}h^{p+1}y^{p+1}(x) + C_{p+2}h^{p+2}y^{p+2}(x) + \ldots$$

Definition: The linear operator Δ and the associated block method are said to be of order p if $C_0 = C_1 = \ldots = C_p = C_{p+1} = 0, C_{p+4} \neq 0.$ C_{p+4} is called the error constant. It implies that the local truncation error is given by $T_{n+k} = C_{p+4}h^{p+4}y^{p+4}(x) + O(h^{p+5})$

Expanding the block in Taylor series expansion gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(\frac{1}{8}h)^j}{j!}y^j - y_n - \frac{1}{8}hy_n' - \frac{1}{128}h^2y_n'' - \frac{1}{3072}h^3y_n''' - \frac{3373}{495452160}h^4y_n^{(iv)} - \\ \sum_{j=0}^{\infty}\frac{h^{j+4}}{j!}y^{j+4}\left(\frac{139}{24772608}(\frac{1}{8})^j - \frac{283}{82575360}(\frac{1}{4})^j + \frac{179}{123863040}(\frac{3}{8})^j - \frac{131}{495452160}(\frac{1}{2})^j\right) \\ \sum_{j=0}^{\infty}\frac{(\frac{1}{4}h)^j}{j!}y^j - y_n - \frac{1}{4}hy_n' - \frac{1}{32}h^2y_n'' - \frac{1}{384}h^3y_n''' - \frac{37}{483840}h^4y_n^{(iv)} - \\ \sum_{j=0}^{\infty}\frac{h^{j+4}}{j!}y^{j+4}\left(\frac{59}{483840}(\frac{1}{8})^j - \frac{1}{18432}(\frac{1}{4})^j + \frac{11}{483840}(\frac{3}{8})^j - \frac{1}{241920}(\frac{1}{2})^j\right) \\ \sum_{j=0}^{\infty}\frac{(\frac{3}{8}h)^j}{j!}y^j - y_n - \frac{3}{8}hy_n' - \frac{9}{128}h^2y_n'' - \frac{9}{1024}h^3y_n''' - \frac{5319}{18350080}h^4y_n^{(iv)} - \\ \sum_{j=0}^{\infty}\frac{h^{j+4}}{j!}y^{j+4}\left(\frac{2889}{4587520}(\frac{1}{8})^j - \frac{1539}{9175040}(\frac{1}{4})^j + \frac{81}{917504}(\frac{3}{8})^j - \frac{297}{18350080}(\frac{1}{2})^j\right) \\ \sum_{j=0}^{\infty}\frac{(\frac{1}{2}h)^j}{j!}y^j - y_n - \frac{1}{2}hy_n' - \frac{1}{8}h^2y_n'' - \frac{1}{48}h^3y_n''' - \frac{11}{15120}h^4y_n^{(iv)} - \\ \sum_{j=0}^{\infty}\frac{h^{j+4}}{j!}y^{j+4}\left(\frac{1}{540}(\frac{1}{8})^j - \frac{1}{5040}(\frac{1}{4})^j - \frac{1}{3780}(\frac{3}{8})^j - \frac{1}{24192}(\frac{1}{2})^j\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (19)$$

Comparing the coefficients of h, the order of the block is P = 6

With error constant $C_{p+4} = \left[\frac{31}{24051816857600}, \frac{101}{5073430118400}, \frac{1107}{3006477107200}, \frac{-1}{7431782400}\right]^T$

3.2 Zero stability of the method

$$\left|\left[\lambda A^{(0)} - A^{(i)}\right]\right| = \left|\left[\lambda\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} - \begin{pmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{pmatrix}\right]\right| = 0$$

$\lambda^4 - \lambda^3 = 0, \lambda = 0,0,0,1$. Hence the block is zero stable.

3.3 Consistence

A numerical method is said to be consistent if the following conditions are satisfies

i. The order of the scheme must be greater than or equal to 1 i.e. p≥1.

ii. $\sum_{j=0}^{k}\alpha_j = 0$

iii. $\rho(r) = \rho'(r) = 0$

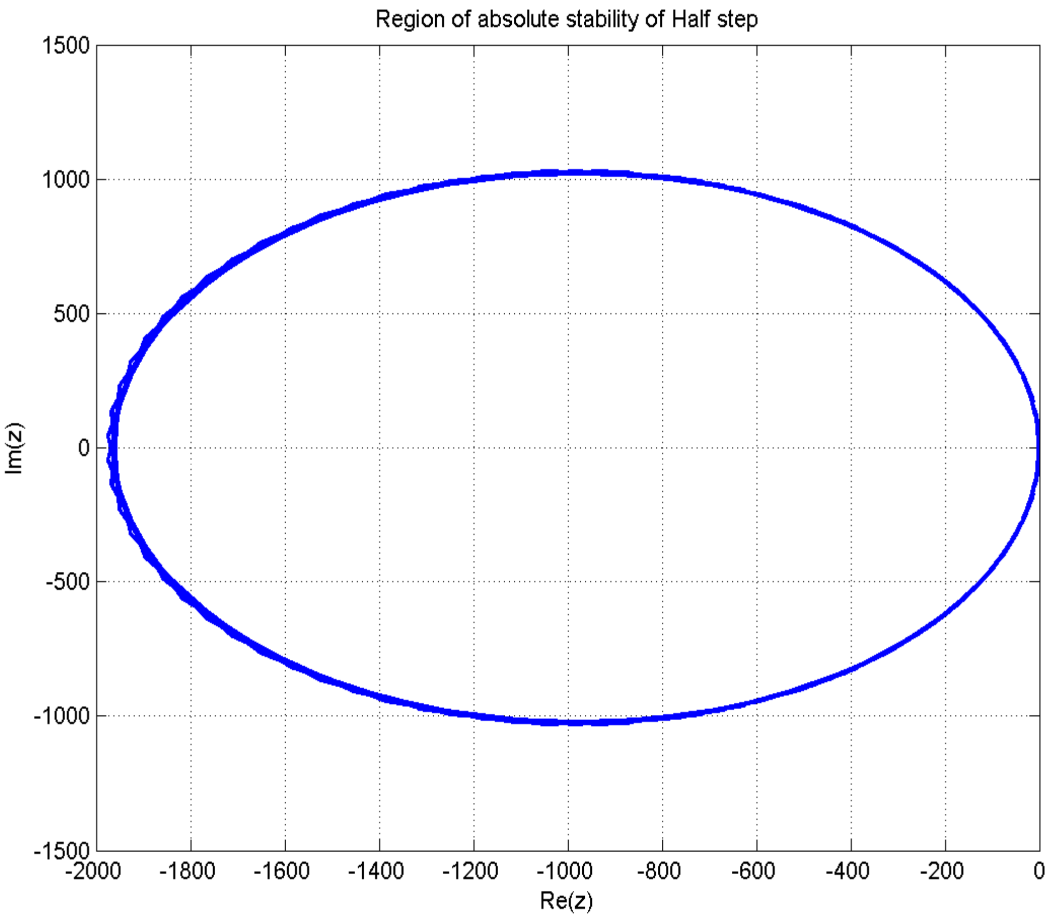
iv. $\rho^{iv}(r) = 4!\sigma(r)$

where, $\rho(r)$ and $\sigma(r)$ are the first and second characteristics polynomials of the method. According to Olabode and Omole (2015), the first condition is a sufficient condition for the associated block method to be consistent. Our method is order p = 6≥1. Hence it is consistent.

3.4 Convergence

The necessary and sufficient conditions for a numerical method are that it must be zero-stable and consistence. Dalquitz (1964)

Figure 1, showing the region of absolute stability of the Half-Step Block Method



4.1 Numerical Experiments

The method is tested on some numerical problems to examine the accuracy of the proposed methods.

Problem 1:

$$y^{(iv)} - y = 0, \quad y(0) = 1, y'(0) = 0, y''(0) = -2, y'''(0) = 0, h = \frac{1}{320}$$

Exact solution: $y(x) = \frac{-1}{4}e^{x} - \frac{1}{4}e^{-x} + \frac{3}{2}\cos(x)$

Problem 2:

$$y^{(iv)} - 4y''' + 6y'' - 4y' + y = 0,$$

$$y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1, h = \frac{1}{100}$$

Exact solution: $y(x) = xe^{x} - x^{2}e^{x} + \frac{2}{3}x^{3}e^{x}$

Problem 3:

$$y^{(iv)} + 2y'' + y = \sin(x),$$

$$y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0, h = \frac{0.1}{32}$$

Exact solution: $y(x) = \sin(x) - \sin(x)\cos(x) - \frac{1}{2}\sin(x)\cos(x)x$

The result of these problems is shown in tables 1-3. The error is defined as

Error = $\left| y(x) - y_n(x) \right|$, where y(x) is the exact solution and $y_n(x)$ is our computed result.

Table 1: Result of test problem 1

Xval	Exact Solution	Computed Solution	Error	Time
0.0031250000	0.99999023437897361000	0.99999023437897316000	4.440892e-16	0.0426
0.0062500000	0.99996093756357807000	0.99996093756355631000	2.176037e-14	0.0447
0.0093750000	0.99991210969686317000	0.99991210969668598000	1.771916e-13	0.0513
0.0125000000	0.99984375101724210000	0.99984375101647549000	7.666090e-13	0.0501
0.0156250000	0.99975586185848631000	0.99975586185611853000	2.367773e-12	0.0574
0.0187500000	0.99964844264972075000	0.99964844264378827000	5.932477e-12	0.0564
0.0218750000	0.99952149391541234000	0.99952149390253553000	1.287681e-11	0.0600
0.0250000000	0.99937501627536340000	0.99937501625018499000	2.517841e-11	0.0633
0.0281250000	0.99920901044469668000	0.99920901039922916000	4.546752e-11	0.0656
0.0312500000	0.99902347723384288000	0.99902347715671957000	7.712331e-11	0.0680

Table 2: Result of test problem 2

Xval	Exact Solution	Computed Solution	Error	Time
0.0100000000	0.01000017002091131900	0.01000016961633346300	4.045779e-10	0.0446
0.0200000000	0.02000138733833788800	0.02000138219766566700	5.140672e-09	0.0478
0.0300000000	0.03000477511965851000	0.03000475141673330000	2.370293e-08	0.0515
0.0400000000	0.04001154165535324900	0.04001146993588522400	7.171947e-08	0.0545
0.0500000000	0.05002298300255914000	0.05002281155803040000	1.714445e-07	0.0569
0.0600000000	0.06004048568786080800	0.06004013341448866100	3.522734e-07	0.0590
0.0800000000	0.08009969016664333900	0.08009857638626638500	1.113780e-06	0.0642
0.0900000000	0.09014464253733747700	0.09014284862148601300	1.793916e-06	0.0681
0.1000000000	0.10020216323885874000	0.10019940797116712000	2.755268e-06	0.0702

Table 3: Result of test problem 3

Xval	Exact Solution	Computed Solution	Error	Time
0.0035156250	0.00352800634504235600	0.00352800634703801740	1.995661e-12	0.0760
0.0070312500	0.00708086228399340540	0.00708086230735874720	2.336534e-11	0.0788
0.0105468750	0.01065869817355602500	0.01065869827838707000	1.048310e-10	0.0808
0.0140625000	0.01426164437259058700	0.01426164468312683000	3.105362e-10	0.0836
0.0175781250	0.01788983124319687400	0.01788983197107592300	7.278790e-10	0.0940
0.0210937500	0.02154338915180198400	0.02154339061915040400	1.467348e-09	0.0961
0.0246093750	0.02522244847025323400	0.02522245113259991400	2.662347e-09	0.0981
0.0281250000	0.02892713957690928900	0.02892714404592539400	4.469016e-09	0.1001
0.0316406250	0.03265759285774405600	0.03265759992380544400	7.066061e-09	0.1083
0.0351562500	0.03641393870744658000	0.03641394936203161400	1.065459e-08	0.1113
0.0386718750	0.04019630753052816400	0.04019632298843546500	1.545791e-08	0.1135
0.0421875000	0.04400482974243502500	0.04400485146381804400	2.172138e-08	0.1156
0.0457031250	0.04783963577066086000	0.04783966548288787200	2.971223e-08	0.1244
0.0492187500	0.05170085605586793900	0.05170089577520801900	3.971934e-08	0.1283
0.0527343750	0.05558862105300743700	0.05558867310613500900	5.205313e-08	0.1307
0.0562500000	0.05950306123244864000	0.05950312827776062900	6.704531e-08	0.1327
0.0597656250	0.06344430708110937500	0.06344439212986208200	8.504875e-08	0.1419
0.0632812500	0.06741248910358876900	0.06741259554086104200	1.064373e-07	0.1462
0.0667968750	0.07140773782331033500	0.07140786942877491700	1.316055e-07	0.1484
0.0703125000	0.07543018378366206400	0.07543034475217168200	1.609685e-07	0.1513
0.0738281250	0.07947995754914438700	0.07948015251113226000	1.949620e-07	0.1595

CONCLUSION

In this paper, we have presented an order six half-step symmetric continuous block method for numerical integration of fourth order ordinary differential equations. This method shows that apart from single step or linear multistep method, half-step method can also be adopted for the direct solution of initial value problems of fourth order ordinary differential equations. The accuracy of the proposed method is demonstrated via three numerical examples as presented in table 1-3. The region of absolute stability of the block method presented in figure 1 shows that it is A(alpha) stable. Future research work will be directed at developing a half-step method for direct implementation of order greater than four.

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