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European Journal of Applied Engineering and Scientific Research, 2014, 3 (3):43-48
(http://scholarsresearchlibrary.com/archive.html)

ISSN: 2278 – 0041
CODEN(USA): EJAEAJ)

Introduction io Fourier-Finite Mellin Transforms

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ABSTRACT

The Fourier transform is simply the frequency spectrum of a signal. The Fourier Transform is optimum when dealing with boundary value problems. The Mellin Transform has special importance in scale representation of signal because it is scale invariant transform. For control systems engineering, stability of electrical networks etc., Mellin Transform is used. It is also useful for solving the Cauchy differential equation and Wave equation with the help of Matlab. When these two transforms are combined the resultant Fourier-Finite Mellin Transform may be applied in image processing, pattern recognition, speech processing, radar signal analysis etc. Some partial differential equation may be solved by using Fourier-Finite Mellin Transform. This paper discusses an extension of Fourier-Finite Mellin Transform in the distributional generalized sense. The Twelve testing function space are defined by using Gelfand-Shilove technique. In this paper the results on countable union space are also described.

Keywords: Fourier transform, Finite Mellin transform, Fourier-finite Mellin transform, generalized function.

INTRODUCTION

The classical theory of integral transformations has been extended to generalized functions by many people. But the main credit goes to Zemanian [9] who gave the way for the extension and called it the theory of generalized integral transformations. Generalized function has been studied on certain larger spaces by Gelfand, Shilov [10], Zemanian [8]. Fourier transforms is used for deriving probability densities of sums and differences of random variables is well known. The Fourier transform transforms a function from its more easily understood time or spatial domain into a function existing in frequency space. The essence, and beauty, of the transform is that it demonstrates almost any function can be broken up into a sum of known periodic sinusoidal functions, each of which is characterized by its amplitude and frequency.

The Mellin Transform has special importance in scale representation of signal because it is scale invariant transform. The scale-invariance property of the Mellin transform combined with the translation invariance property of the Fourier transform provide a way of representing signal free of Doppler distortion. It is useful for resolution of certain types of classical boundary and initial value problems. Mellin transform is also useful for the summation of the series and solution of the Cauchy's linear differential equation.

The Fourier-Finite Mellin transform may be applied in image processing, pattern recognition, speech processing, radar signal analysis etc., and some partial differential equation may be solved by using Fourier-Finite Mellin Transform. The Fourier-Finite Mellin Transform may be used for image recognition and processing, movement detection [2] and derivation of densities for algebraic combinations of random variables and many more. The Fourier-Mellin Transform should provide a truly translation, rotation and scale-invariant measure of an image.

In the present paper, Fourier-Finite Mellin transform is extended in the distributional sense. The plan of the paper is as follows. The definitions are given in section 2. In section 3, testing function spaces are defined by Gelfand-Shilov technique. Section 4 describes the Distributional Generalized Fourier-Finite Mellin Transform (FM_fT). In section 5. Some results on countable union spaces are proved. The notations and terminologies are as per Zemanian [8,9].

2. Definitions

The Fourier Transform with parameter s of $f(t)$ is denoted by $F[f(t)] = F(s)$ and is given by

$$F[f(t)] = F(s) = \int_{-\infty}^{\infty} e^{-ist} f(t)dt \tag{2.1}$$

Whenever this integral is exists, for parameter $s > 0$.

The Finite-Mellin integral transform with parameter p of $f(x)$ is denoted by $M_f[f(x)] = F(p)$ and is given by

$$M_f[f(x)] = \int_0^a \left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right) f(x)dx \tag{2.2}$$

Whenever this integral is exists for $p > 0$ is the parameter.
The Fourier-Finite Mellin Transform is defined as,

$$FM_f\{f(t,x)\} = F(s,p) = \int_0^{\infty} \int_0^a f(t,x)K(t,x)dtdx, \tag{2.3}$$

Where $K(t,x) = e^{-ist}(\frac{a^{2p}}{x^{p+1}} - x^{p-1})$

3. Various Testing Function Spaces

3.1. The space $FM_{f,b,c,\alpha}$

It is given by

$$FM_{f,b,c,\alpha} = \{\phi : \phi \in E_+ / \gamma_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x)\right| \le C_{lq} A^k k^{k\alpha}\} \tag{3.1}$$

for each $k, l, q = 0,1,2, \dots \dots ....$

Where $\lambda_{b,c}(x) = \begin{cases} x^{+b} & 0 < x < 1 \\ x^{+c} & 1 < x < a \end{cases}$

Where the constants A and C_{lq} depend on the testing function ϕ.

3.2. The Space $FM_{f,b,c}^{\beta}$

This space is given by $FM_{f,b,c}^{\beta} = \{\phi : \phi \in E_+ / \sigma_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x)\right| \le C_{kq} B^l l^{l\beta}\}$ (3.2)

The constants C_{kq} and B depend on ϕ.

3.3. The Space $FM_{f,b,c,\alpha}^{\beta}$

This space is formed by combining the conditions (3.1) and (3.2)

$$FM_{f,b,c,\alpha}^{\beta} = \{\phi : \phi \in E_+ / \rho_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x)\right| \le C A^k k^{k\alpha} B^l l^{l\beta}\} \tag{3.3}$$

3.4. The Space $FM_{f,b,c,\gamma}$

It is given by

$$FM_{f,b,c,\gamma} = \{\phi : \phi \in E_+ / \xi_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right| \leq C_{lk} A^q q^{q\gamma}\} \quad (3.4)$$

3.5. The Space $FM_{f,b,c,\alpha,m}$

It is defined as

$$FM_{f,b,c,\alpha,m} = \{\phi : \phi \in E_+ / \gamma_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right| \leq C_{lq\delta}(m+\delta)^k k^{k\alpha}\} \quad (3.5)$$

For any $\delta > 0$, where m is the constant depending on the function ϕ.

3.6. The Space $FM_{f,b,c,}^{\beta n}$

It is defined as,

$$FM_{f,b,c,}^{\beta n} = \{\phi : \phi \in E_+ / \sigma_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right| \leq C_{kq\epsilon}(n+\epsilon)^l l^{l\beta}\} \quad (3.6)$$

For any $\epsilon > 0$, where n is the constant depending on the function ϕ.

3.7. The Space $FM_{f,b,c,\alpha,m}^{\beta n}$

This space is defined by combining (3.5) and (3.6)

$$FM_{f,b,c,\alpha,m}^{\beta n} = \{\phi : \phi \in E_+ / \rho_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right| \leq C_{\delta\epsilon}(m+\delta)^k (n+\epsilon)^l k^{k\alpha} l^{l\beta}\} \quad (3.7)$$

For any $\delta > 0$, $\epsilon > 0$ and for given $m > 0$ and $n > 0$.

3.8. The Space $FM_{f,b,c,\alpha,\gamma,p}$

This space is given by

$$FM_{f,b,c,\gamma,p} = \{\phi : \phi \in E_+ / \xi_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{0<t<\infty \\ 0<x<a}} \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right| \leq C_{lkr}(p+\gamma)^q q^{q\gamma}\} \quad (3.8)$$

For any $\gamma > 0$, where p is the constant depending on the function ϕ.

3.9. The Space $FM_{f,b,c,\alpha}^{\nu}$

It is given by

$$FM_{f,b,c,\alpha}^{\nu} = \{\phi : \phi \in E_- / \gamma_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{-\infty<t<0 \\ 0<x<a}} \left| (-t)^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right| \leq C_{lq} A^k k^{k\alpha}\} \quad (3.9)$$

The smooth function $\phi(t,x)$ defined on I_2 is in $FM_{f,b,c,\alpha}^{\nu}$ if $\phi^{\nu}(t,x) = \phi(-t,x)$ is in $FM_{f,b,c,\alpha}$

3.10. The Space $F^{\nu}M_{f,b,c}^{\beta}$

We define this space as,

$$F^{\nu}M_{f,b,c}^{\beta} = \{\phi : \phi \in E_- / \sigma_{b,c,k,q,l}\phi(t,x) = \sup_{\substack{-\infty<t<0 \\ 0<x<a}} \left| (-t)^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right| \leq C_{kq} B^l l^{l\beta}\} \quad (3.10)$$

Here $\phi^{v}(t,x)=\phi(-t,x)$ is in $F^{v}M^{\beta}_{f,b,c}$

3.11. The Space $F^{v}M^{\beta}_{f,b,c,\alpha}$

Combining the conditions of (3.9) and (3.10) we get

$$F^{v}M^{\beta}_{f,b,c,\alpha}=\{\phi:\phi\in E_{-}/\rho_{b,c,k,q,l}\phi(t,x)=\underset{\substack{-\infty<t<0\\0<x<a}}{Sup}\left|(-t)^{k}\lambda_{b,c}(x)x^{q+1}D_t^{l}D_x^{q}\phi(t,x)\right| \leq CA^{k}k^{k\alpha}B^{l}l^{l\beta}\} \tag{3.11}$$

Where the constants A,B,C depend on the testing function ϕ.

3.12. The Space $F^{v}M_{f,b,c,\gamma}$

It is given by

$$F^{v}M_{f,b,c,\gamma}=\{\phi:\phi\in E_{-}/\mu_{b,c,k,q,l}\phi(t,x)=\underset{\substack{0<t<\infty\\-a<x<0}}{Sup}\left|t^{k}\lambda_{b,c}(x)(-x)^{q+1}D_t^{l}D_x^{q}\phi(t,x)\right| \leq C_{lk}A^{q}q^{q\gamma}\} \tag{3.12}$$

4. Distributional Generalized Fourier-Finite Mellin Transforms (FM_fT)

For $(t,x)\in FM^{*\beta}_{f,b,c,\alpha}$, where $FM^{*\beta}_{f,b,c,\alpha}$ is the dual space of $FM^{\beta}_{f,b,c,\alpha}$ It contains all distributions of compact support. The distributional Fourier-Finite Mellin transform is a function of $f(t,x)$ is defined as $FM_f\{f(t,x)\}=F(s,p)=\langle f(t,x),e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\rangle$ (4.1)

Where, for each fixed t $(0<t<\infty)$, x $(0<x<a), s>0$ and $p>0$, the right hand side of (4.1) has a sense as an application of $f(t,x)\in FM^{*\beta}_{f,b,c,\alpha}$ to $e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\in FM^{\beta}_{f,b,c,\alpha}$.

5. RESULTS ON COUNTABLE UNION SPACE

5.1. Theorem: For a real numbers b_1,b_2,c_1 and c_2 such that $b_1\leq b_2$ and $c_2\leq c_1$ then $FM_{f,b_2c_{2,\alpha}}\subset FM_{f,b_1c_{1,\alpha}}$ and the induced topology on $FM_{f,b_2c_{2,\alpha}}$ is weaker than the original topology that is $T_{b_1,c_{1},\alpha}$ / $M_{f,b_2c_{2,\alpha}}\subset T_{b_2,c_2,\alpha}$.

Proof: Consider ,

$$\gamma_{b,c,k,q,l}\phi(t,x)=\underset{I}{Sup}\left|t^{k}\lambda_{b_1,c_1}(x)x^{q+1}D_t^{l}D_x^{q}\phi(t,x)\right|$$
$$\leq\underset{I}{Sup}\left|t^{k}\lambda_{b_1,c_1}(x)x^{q+1}D_t^{l}D_x^{q}\phi(t,x)\right|=\gamma_{b_2,c_{2,k,q,l}}\phi(t,x)$$

Hence $FM_{f,b_2c_{2,\alpha}}\subset FM_{f,b_1c_{1,\alpha}}$ if $b_1\leq b_2$ and $c_2\leq c_1$

Second part of the proof is simple and hence omitted. This completes the proof.

5.2. Theorem: If $\alpha_1<\alpha_2$ and $\beta_1<\beta_2$ then $FM^{\beta_1}_{f,b,c,\alpha_1}\subset FM^{\beta_2}_{f,b,c,\alpha_2}$ and the topology of $FM^{\beta_1}_{f,b,c,\alpha_1}$ is equivalent to the topology induced on $FM^{\beta_1}_{f,b,c,\alpha_1}$ by $FM^{\beta_2}_{f,b,c,\alpha_2}$.

Proof: Let $\phi\in FM^{\beta_1}_{f,b,c,\alpha_1}$ therefore

$$\rho_{b,c,k,q,l}\phi(t,x)=\underset{I}{Sup}\left|t^{k}\lambda_{b,c}(x)x^{q+1}D_t^{l}D_x^{q}\phi(t,x)\right|\leq CA^{k}k^{k\alpha_1}B^{l}l^{l\beta_1}\leq CA^{k}k^{k\alpha_2}B^{l}l^{l\beta_2},$$

Where $k,q,l=0,1,2,3$ ... ...

Hence $\phi\in FM^{\beta_2}_{f,b,c,\alpha_2}$.

Consequently $FM^{\beta_1}_{f,b,c,\alpha_1} \subset FM^{\beta_2}_{f,b,c,\alpha_2}$ the topology of $FM^{\beta_1}_{f,b,c,\alpha_1}$ is equivalent to the topology $T^{\beta_2}_{b,c,\alpha_2} \,/\, FM^{\beta_2}_{f,b,c,\alpha_2}$ It is clear from the definition of the topologies of these spaces.

5.3. Theorem: $FM_{f,b,c} = \bigcup_{\alpha_i,\beta_i=1}^{\infty} FM^{\beta_i}_{f,b,c,\alpha_i}$ and if the space $FM_{f,b,c}$ is equipped with the strict $FM_{f,b,c}$ inductive limit topology defined by the injection maps from $FM^{\beta_i}_{f,b,c,\alpha_i}$ to $FM_{f,b,c}$ then the sequence $\{\phi_n\}$ in $FM_{f,b,c}$ converges to zero iff $\{\phi_n\}$ iscontained in some $FM^{\beta_m}_{f,b,c,\alpha_m}$ and converges to zero.

Proof: Once we show that $FM_{f,b,c} = \bigcup_{\alpha_i,\beta_i=1}^{\infty} FM^{\beta_i}_{f,b,c,\alpha_i}$

Clearly $\bigcup_{\alpha_i,\beta_i=1}^{\infty} FM^{\beta_i}_{f,b,c,\alpha_i} \subset FM_{f,b,c}$

For proving other inclusion , let $\phi \in FM_{f,b,c}$ then

$\rho_{b,c,k,q,l}(\phi) = \sup_{I} \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x) \right|$ is bounded by some number. We can choose the integers α_m and β_m such that $\rho_{b,c,k,q,l}(\phi) \leq C A^k k^{k\alpha_m} B^l l^{l\beta_m}$.

Therefore $\in FM^{\beta_m}_{f,b,c,\alpha_m}$, for some integers α_m and β_m .

Hence $FM_{f,b,c} \subset \bigcup_{\alpha_i,\beta_i=1}^{\infty} FM^{\beta_i}_{f,b,c,\alpha_i}$

Thus $FM_{f,b,c} = \bigcup_{\alpha_i,\beta_i=1}^{\infty} FM^{\beta_i}_{f,b,c,\alpha_i}$

5.4. Definition

Let $\{b_n\}$ and $\{c_n\}$ be monotonic sequence, converging to $w+$ and $z-$ respectively.

Now we define countable union space. $FM_f(w,z,\alpha) = \bigcup_{n=1}^{\infty} FM_{f,b_n,c_n,\alpha}$

Concerning this space we prove next theorem.

5.5. Theorem: The space $FM_f(w,z,\alpha)$ is independent of the choice of the sequence $\{b_n\}$ and $\{c_n\}$. If $FM_f(w,z,\alpha)$ is equipped with the strict inductive limit topology defined by $FM_{f,b_n,c_n,\alpha}$ then the sequence $\{\phi_n\}$ in $FM_f(w,z,\alpha)$ converges to zero iff $\{\phi_n\}$ belongs to some $FM_{f,b_n,c_n,\alpha}$ and converges to zero in that space. Moreover $FM_f(w,z,\alpha)$ is complete.

Proof: Proof is easy and hence omitted.

CONCLUSION

In this paper Fourier-Finite Mellin Transform is generalized in the distributional sense. Twelve testing function spaces using Gelfand Shilov technique are developed. Also some results on countable unionspace are proved.

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