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Isgur-Wise Function for Heavy-Light Mesons within a QCD Quark Model Developed with Linear plus Coulombic Potential.

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ABSTRACT

We have analysed a potential model for quark-antiquark system developed by obtaining analytic solution of Schrodinger equation with linear plus coulombic potential. With the wave function obtained we have studied Isgur-Wise function for heavy light mesons and compared our result with recent theoretical and experimental expectations.

Key words : Heavy-Light Mesons, Isgur-Wise Function, Potential Model.
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INTRODUCTION

Considerable progress has been observed in recent past in the description of hadronic systems and their weak decays. Weak decay of hadrons is an important source of information on the standard model of electro-weak interactions, the nature of weak currents and internal structure of hadrons. For systems with only lighter composite quarks (u , d or s) , chiral perturbation theory is found to be effective in explaining the hadronic system. For heavy-light systems (like meson with one heavy quark), progress has also been there in phenomenological front [1].

Understanding physics of hadrons containing one heavy quark is useful in the determination of physical quantities like elements of CKM matrix [2]. HQET [3] shows that for semi-leptonic decays in the infinite quark mass limit, the spins of heavy and light degrees of freedom decouples and all the form factors can be expressed in terms of a single universal function - called the Isgur-Wise Function (IWF) [4].

In the low energy regime, QCD is non-perturbative. In this infrared energy region, approaches like QCD sum rules[5], lattice QCD [6] are effective to solve QCD. Another useful tool in QCD in this low energy regime is the construction of phenomenological models by employing basic properties of QCD, which in turn is used to predict the properties of hadrons like its mass, its form factors , its decay constant etc. In this connection, potential models involving potential between heavy and light quarks has been very successful in the study of hadron spectroscopy. To construct a potential model, the choice of reasonable potential is very important. In this context, *linear plus coulombic* potential (Cornell potential)[7] has been very popular and useful in such phenomenological study. Due to lack of generally accepted exact solution for the quark equation of motion with *linear plus coulombic* potential, the wave function for heavy-light mesons have been calculated earlier with such potential by applying quantum mechanical perturbation theory [8-12]. But then, there always remains some margin of uncertainty. On the contrary, exact wave function calculation by solving Schrodinger equation with *linear plus coulombic* potential is more preferable as far as accuracy is concerned.

In this paper, we work with an exact analytic solution of Schrodinger equation for mesons with linear + coulombic potential reported earlier [13]. With the analytic wave function, we calculate Isgur-Wise function for heavy-light mesons and its derivatives like slope (charge radii) and curvature (convexity parameter). We then compare our result with other theoretical and experimental expectations [12, 24-31].

After this introduction in section 1, section 2 describes the brief formalism. Section 3 contains calculations and results and final discussion is mentioned in section 4.

2 Formalism:
2.1 Potential Model:
As stated in section 1, for the construction of potential model, choice of potential is of utmost importance. The generally accepted potential for describing a quark-antiquark system is having *linear* + *coulombic* form , better known as Cornell potential [7]:

$$V(r) = -C_F\frac{\alpha_s}{r} + br + c \tag{1}$$

Here α_s is the QCD coupling consatnt and C_F is the colour factor in QCD algebra [14] given by :

$$C_F = \frac{N_C^2 - 1}{2N_c} \tag{2}$$

N_C is the colour quantum number; for $N_C = 3$, we have $C_F = \frac{4}{3}$.

There are other potentials also like Richardson potential [15] , but the best suited potential for $Q\bar{Q}$ system is believed to be the Cornell potential. Getting exact analytic solution of Schrodinger equation for such a QQ system with this *Linear* + *coulombic* potential is a hard nut to crack; and quantum mechanical perturbation techniques (Dalgarno's method of perturbation [16], variationally inspired perturbation theory[17]) are commonly employed to extract meson wave function by solving Schrodinger equation with such a potential.

In this paper we work with *Linear* + *coulombic* potential to study the Isgur-Wise function of heavy light mesons by considering some previously developed analytic solution of Schrodinger equation [13] by considering simplest form of *linear* + *coulombic* potential as:

$$V(r) = -\frac{a}{r} + br \tag{3}$$

Comparing (3) with Cornell potential of equation (1), we find $a = \frac{4\alpha_s}{3}$ and scale factor c = 0. Given this potential, we construct the analytic form of ground state (l = 0) wave function of meson following reference [13]:

$$\Psi(r) = Nexp[-\alpha r^{3/2} - \beta r] \tag{4}$$

with :

$$\alpha = \frac{2}{3}\sqrt{2\mu a} \tag{5}$$

$$\beta = \mu b \tag{6}$$

Here, N is the normalisation constant and μ is the reduced mass of the meson consisting of a quark and antiquark and is given by :

$$\mu = \frac{m_q m_Q}{m_q + m_Q} \tag{7}$$

It is to be mentioned that, in the infinite heavy quark mass limit ($m_Q \to \infty$) ,

$$\mu = \lim_{m_Q \to \infty} \frac{m_q m_Q}{m_q + m_Q} \approx m_q \quad (8)$$

Considering relativistic effect [18] on the wave function, the total relativistic wave function is given by:

$$\Psi_{rel}(r) = N exp[-\alpha r^{3/2} - \beta r](\frac{r}{a_b})^{-\epsilon} \quad (9)$$

Here,

$$a_b = \frac{3}{4\mu\alpha_s} \;\; and \;\; \epsilon = 1 - \sqrt{1 - (\frac{4\alpha_s}{3})^2} \quad (10)$$

2.2 *Isgur-Wise Function*:

In case of semi-leptonic decay of hadrons (mesons) , in the infinite mass limit, a new symmetry called spin-flavored symmetry, will emerge and the Heavy Quark Effective Theory (HQET) will be suitable. In this theory, the strong interactions of the heavy quarks are independent of its spin and mass[19] and all the form factors are completely determined, at all momentum transfers, in terms of only one elastic form factor function, the universal Isgur-Wise function $\xi(v, v')$. $\xi(v, v')$ depends only upon the four velocities v_ν and v_ν' of heavy particle before and after decay. This $\xi(v, v')$ is normalized at zero recoil [20]. If y = $v_\nu . v_\nu'$, then, for zero recoil $\xi(1) = 1$. In explicit form IW function can be expressed as :

$$\xi(y) = 1 - \rho^2(y-1) + C(y-1)^2 + ... \quad (11)$$

ρ^2 is the slope parameter and is given by -

$$\rho^2 = -\frac{\delta\xi(y)}{\delta y}|_{y=1} \quad (12)$$

ρ is known as the charge radius.

C is the convexity parameter given by -

$$C = \frac{\delta^2\xi(y)}{\delta y^2}|_{y=1} \quad (13)$$

The calculation of this IWF is non-perturbative in principle and is performed for different phenomenological wave functions for mesons [21]. This function depends upon the meson wave function and some kinematic factor, as given below :

$$\xi(y) = \int_0^\infty 4\pi r^2 |\Psi(r)|^2 \cos(pr) dr \quad (14)$$

where $\cos(pr) = 1 - \frac{p^2r^2}{2} + \frac{p^4r^4}{24} + \cdots\cdots$ with $p^2 = 2\mu^2(y-1)$. Taking cos(pr) up to $O(r^4)$ we get,

$$\xi(y) = \int_0^\infty 4\pi r^2 |\Psi(r)|^2 dr - [4\pi\mu^2 \int_0^\infty r^4 |\Psi(r)|^2 dr](y-1) + [\frac{2}{3}\pi\mu^4 \int_0^\infty r^6 |\Psi(r)|^2 dr](y-1)^2 \quad (15)$$

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Equations (11) and (15) give us :

$$\rho^2 = [4\pi\mu^2 \int_0^\infty r^4 |\Psi(r)|^2 dr] \quad (16)$$

$$C = [\frac{2}{3}\pi\mu^4 \int_0^\infty r^6 |\Psi(r)|^2 dr] \quad and \quad (17)$$

$$\int_0^\infty 4\pi r^2 |\Psi(r)|^2 dr = 1 \quad (18)$$

From equation (18) we can obtain the normalization constants N for the wave-function.

RESULTS

With the wave function given by equation (18), we have explored the $\xi(y)$ and its derivatives for B and D sector mesons. We have taken $b = 0.183 GeV^2$ from charmonium spectroscopy [22] and $\alpha_s = 0.22$ [23]. The results for slope and curvature of different mesons are shown in table 1. The results of ρ^2 and C in different models and collaborations are referred in table 2.

Table 1: Result of N, ρ^2 and C with $\Psi_{rel}(r)$.

mesons	N	ρ^2	C
D	0.2074	0.5510	0.0738
B	0.2250	0.6261	0.1105
D_S	0.2450	0.7322	0.1390
B_s	0.2724	0.8901	0.2821
B_c	0.3465	1.2983	0.6322

Table 2: Results of slope and curvature of $\xi(y)$ in different models and collaboration

Model / collaboration	Value of slope	Value of curvature
Ref [12]	0.7936	0.0008
Le Youanc et al [24]	≥ 0.75	≥ 0.47
Skryme Model [25]	1.3	0.85
Neubert [26]	0.82 0.09	–
UK QCD Collab. [27]	0.83	–
CLEO [28]	1.67	–
BELLE [29]	1.35	–
HFAG [30]	1.17 ± 0.05	–
Huang [31]	1.35 ± 0.12	–

The variation of IWF with y for different mesons are shown in figure-1. Graphs confirm the fact that boundary condition for zero recoil ($\xi(1) = 1$) is maintained all through.

We have calculated the slope and curvature of B and D sector mesons using the analytic wave function of mesons of ref [13]. We find that our results are improved than that of ref[12]. However, when compared with recent results of Heavy Flavour Averaging Group (HFAG, ref[30]), our results for D, B, D_s, B_s mesons fall short of the expectation. However, the value of slope parameter for B_C meson tallies close to the result of HFAG. This is due to the fact that mass of B_c meson is much higher compared to other B and D sector mesons. Our analysis of IWF reveals that the zero recoil condition is maintained.

While analysing the limitation of the present formalism, we take a note of the fact that while extracting analytical form of the wave function in ref [13] with *linear+coulombic* potential, some additional counter terms are incorporated in the potential function while fixing the values of α and β. This in turn may have sacrificed the purity of the nature of our *linear + coulombic* potential to some extent. However, due to absence of exact analytic wave function for mesons with Cornell potential, our choice has been limited to ref [13].Further improvement of the result with exact wave function for pure Cornell potential is under consideration.

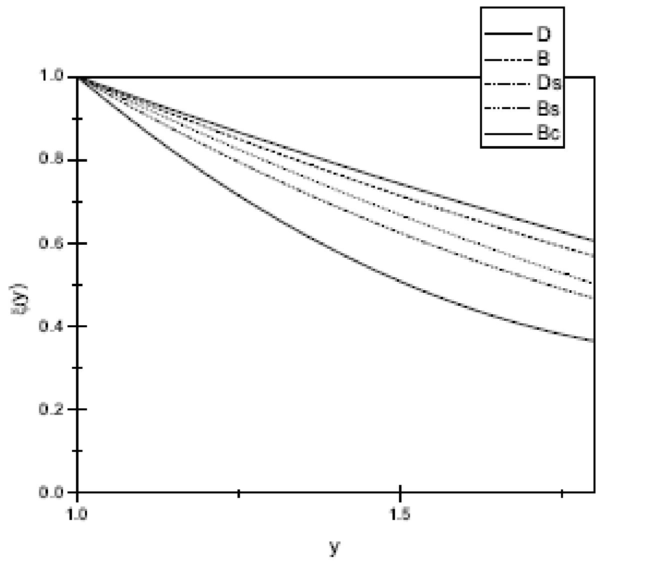

Figure 1: Variation of IWF with y

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