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Mapping the spatial extend of groundwater declination using Geostatistical techniques: A case study around the area of Rajatalab, Varanasi, U.P., India

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ABSTRACT

The spatial analysis of declination nature of groundwater level for ninety eight (98) wells monitored between March 2004 and February 2006 was carried out using Geostatistics. The data for the groundwater levels were analysed in order to accounts for the spatial variability of the phenomenon. The result of the experimental variogram that characterise the spatial variability showed a low nugget-sill-ratio which implies a strong spatial correlation. A theoretical variogram was used to fit the experimental variogram and to obtain the range of influence. Ordinary Kriging and cross-validation were used to map the spatial variability of groundwater declination and to asses the accuracy of the theoretical model respectively. The result of the Kriged map showed a high declination in groundwater level around the northeast to the southern part of the study area.

Key words: Geostatistics, groundwater level, kringing, variogram, mapping.

INTRODUCTION

Information about geological phenomenon are usually sketchy or very limited. In real world it may be very difficult if not impossible to get exhaustive information at every desire point because of practical constrain. This situation may even be particularly acute when dealing with regional scale or restricted area. Geostatistics provides a set of probabilistic techniques, which are useful to detect and find the mode of patterns of spatial dependence of attribute values at locations not sampled [1]

The emphasized on the use of geostatistics for the better management and conservation of water resources and sustainable development of any area was stated by [2]. They reported that geostatistical methods are good tools for water resources management and can effectively be used to derive the long term trends of the groundwater. A good estimate of water table was observed by [3] as a crucial requirement for exploring water resources for environmental protection and for construction. Work on the application of geostatistics to study groundwater level shows that, monitoring of groundwater is the principal source of information on the effects of hydrologic stresses on groundwater [4]. The study further ascertain that water level data collected over periods or days to months are useful for the such purposes, however data collected over years to decades are required to address the long term effects of aquifer for development and to compile a hydrologic record that defines water level fluctuation. The groundwater level were monitored by [5] during twelve months and used the kriging interpolation method to estimate the groundwater level at unmeasured points and wells for each of the months, and hence established that water level being directly measured, is an important parameter for the study of aquifer system and their dynamic behaviour.

In this study we applied the principle of geostatistics to map out the spatial variability of the declining nature of the groundwater level in the study area and to determine the direction of groundwater level drop.

2.0 Geostatistics Principle

The goal of geostatistical analysis is to predict values where no data have been collected for a spatially dependent data. If the data are spatially independent, there is no possibility to predict values between them [6]. Hydrologic data such as rainfall, water level, effective recharge, aquifer characteristics etc, are all function of space (and time) and often display a high spatial variability called heterogeneity. This variability is not in general random. It is a general rule that these properties display a so called "scale effect" i.e if we take measurement at two different points the difference in the measured value decreases as the two points come closer. The semivariogram plays a central role in the analysis of geostatistical data using the kriging method. The semivariogram, γ(h), was defined by [7] as a graph (and/or formula) describing the expected difference in value between pairs of samples with a given relative orientation, it is expressed as;

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[Z(x_i) - Z(x_i + h)\right]^2 \qquad \text{(i)}$$

Where $Z(x_i)$ and $Z(x_i + h)$ are values of variable Z at x_i and x_i+h respectively, x_i and x_i +h are positions in two dimensions, and N(h) is the total number of experimental pairs that are separated by a distance h.

Prior to the geostatistical estimation, we require a model that enables us to compute a semivariogram value for any possible sampling interval. The most commonly used models are spherical, exponential, Gaussian, and pure nugget effect [8]. The selected model influences the prediction of the unknown values, particularly when the shape of the curve nearby the origin differs significantly. The steeper the curve nearby the origin, the more influence the closest neighbors will have on the prediction [9]. The adequacy of the fitted models is checked on the basis of cross-validation tests. According to [6], cross-validation sequentially omits a point, predicts its value using the rest of the data, and then compares the measured and predicted values. The calculated statistics serve as diagnostics that indicate whether the model is reasonable for map production.

Kriging is an interpolation method to estimate values at unmeasured locations. It uses information from the theoretical model fitted to the experimental semivariogram to find an optimal set of weights [10]. The kriging is based on the intrinsic assumption. It considers both the distance and the degree of variation between known data points when estimating values in unknown areas. It attempts to minimize the error variance and set the mean of the prediction errors to zero so that there is no over- or under-estimates. One advantage of kriging is that it calculates the mean square interpolation error. This interpolation error has zero values on the observation points and increases as the observation uncertainty increases or the observation data decreases [11].

In kriging interpolation techniques, the interpolated value of Z at any point p is given as the weighted sum of the measured value and it is expressed as [4];

$$Z^*(x_p) = \sum_{i=1}^{n} \lambda_i Z(x_i) \qquad i = 1,2,3,\ldots\ldots\ldots,N \qquad \text{(ii)}$$

Where, $Z^*(x_p)$ is the kriged value at location x_p, $Z(x_i)$ is the known value at location x_i, λ_i is the weight for the observation Z at location x_i.

Solving the set of equation below gives an unbiased estimation in ordinary kriging.

$$\begin{cases} \sum_{i=1}^{n} \lambda_i \gamma(x_i, x_j) - \mu = \gamma(x_i, x_j) \\ \sum_{i=1}^{n} \lambda_i = 1 \end{cases} \qquad \text{(iii)}$$

Where μ is the langrange multiplier and $\gamma(x_i,x_j)$ is the value of the semivariogram between two points x_i and x_j.

The minimum error of estimation is also measure for the accuracy of estimates, which is known as estimation variance, and is given as;

$$\sigma_k^2(x_o) = \sum_{i=1}^{N} \lambda_i \gamma(x_i, x_o) + \mu \qquad \text{(iv)}$$

3.0 Study Area

The study area is situated in the South – Western part of Varanasi district in Uttar Pradesh, India. Its lies between $25^o14'$ – $25^o20'$ northing and $82^o47'$ – $82^o54'$ easting with a total area of approximately 144 Sq Km. The ground elevation above mean sea level varies between 75.51m and 82.58m, with gradient towards the eastern part of the study area. The river Ganga and its main tributaries flow near the study area in the South Eastern part and the Northern part respectively.

The groundwater resources of the area have been exploited mostly through shallows dug-wells, hand-pumps, dug-cum-bore wells and bore wells for both domestics and irrigation purposes. There has been a continuous declination in the water table in the study area [12].

The main dependence of agriculture on irrigation is mainly due to uneven distribution of rainfall. A study of the water balance condition by [13] for Varanasi district shows that there is a water surplus only in the months of August and September. Further, soil moisture recharge occurs only in July and August, indicating the great necessity for irrigation in this predominantly agricultural area.

Figure 1 shows the locations of the observations wells in the study area. The Universal Transverse Mercator (UTM) is the coordinate system used in locating the observations wells. The datum of this system is World Geodetic System of 1984 (WGS 1984) upon which Global Positioning System (GPS) measurement were made. The study area is located in zone 44 based on this system.

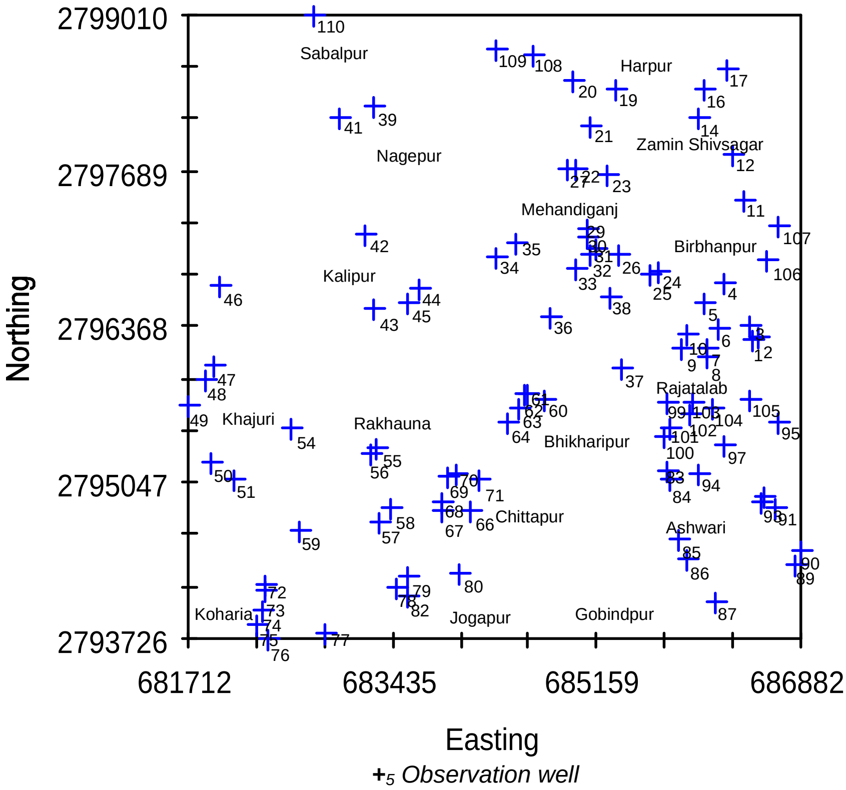


Figure 1 Location of the observation wells and Villages

MATERIALS AND METHODS

4.1 DATA

The data used in this work were derived from a Coca – cola project of groundwater levels around Rajatalab, Varanasi carried out in year 2004 - 2007. The data consist of the groundwater elevation measurement above mean sea level in meters for 110 wells monitored continually from March 2004 to February 2007. For the purpose of this work, the difference between the maximum and minimum groundwater elevation for ninety eight (98) continuous yielding wells was computed for two years period (March 2004 – February 2006). The difference of the result obtains at each well from the previous year's result represent the net loss or gain in groundwater elevation over the preceding year. A positive result means an increase in depth to water from the ground surface (or decrease in groundwater elevation), and a negative result means a decrease in depth to water from the ground surface (increase in groundwater elevation). For convenience, measurements from March 2004 to February 2005 and March 2005 to February 2006 are referred to as year 2004 and 2005 measurement respectively.

4.2 METHODOLOGY

A descriptive statistics was first carried out on the data to asses the behaviour of the data. According to [6], descriptive statistics helps one to have a preliminary judgement of the data set and to decide suitable approach for further analysis.

The next task of investigation is to identify the semivariogram of the investigated variable in space or time. This task is carried out by determining the estimated semivariogram of the data collected, by grouping the available pair-values into a number of lags or distance classes in accordance with their in-between distances using equation (i). The experimental semivariogram obtained is known at discrete points, in order to have a continuous function, it was modeled with theoretical models of GS^+ software version 9 [14]. Values at unmeasured location are further estimated using the kriging interpolation, equation (ii).

In order to check the accuracy of the kriging method, a cross-validation of our estimated groundwater elevation data is done at the known points. The criteria used for accurate prediction in the cross-validation of this work are the root mean square error and standard error of estimation which should be close to one and be as small as possible respectively.

RESULTS AND DISCUSION

The result for the descriptive statistics carried out is given in table 1.

Table 1: Descriptive statistics of the data set

	Year 2004 Minimum	*Year2004 Maximum*	*Year 2005 Minimum*	*Year 2005 Maximum*	*GW Declination*
Number of values	98	98	86	86	86
Number of missing values	0	0	12	12	12
Minimum	59.21	63.81	57.16	62.66	-4.26
Maximum	74.19	78.80	73.55	78.70	4.87
Mean	68.09	71.80	66.77	70.59	0.16
Median	68.46	72.26	67.04	71.07	-0.04
Variance	11.97	8.15	12.83	12.10	3.05
Standard deviation	3.46	2.86	3.58	3.48	1.75
Skew	-0.50	-0.04	-0.44	-0.24	-0.004
Kurtosis	-0.46	0.75	-0.51	-0.05	0.22

The missing values are wells that have dried up throughout the year or during some months of the observation period. From the descriptive statistics, it can be seen that the variable is normally distributed, because geostatical interpolation works well with data that are normally distributed [6]. This can be observed from the similarities between the central statistical parameter, and also the low skewness and kurtosis values.

The experimental semivariogram was computed for year 2004 and 2005 maximum and minimum groundwater elevation values along with the groundwater declination. The semivariogram was computed in different direction to detect any anisotropy of spatial variability, but there was no much significant spatial variability detected. Thus isotropic variation was considered and modelled using Exponential model, the models are shown in figure 2 and parameters of the modelled semivariogram given in table 2.

The equation that represents the fitted isotropic theoretical (Exponential) model was derived from [15] as;

$$\gamma(h) = C_o + C[1 - \exp(-h / A_o)] \qquad \text{(v)}$$

where $\gamma(h)$ = semivariance for interval distance class h,

h = lag interval,
C_o = nugget variance,
$C_o + C$ = sill,

A_o = range parameter. (Practical range in the exponential variogram model is usually assumed to be the point at which the model attains about 95% of the sill (C_o+C), which can be estimated as $3A_o$)

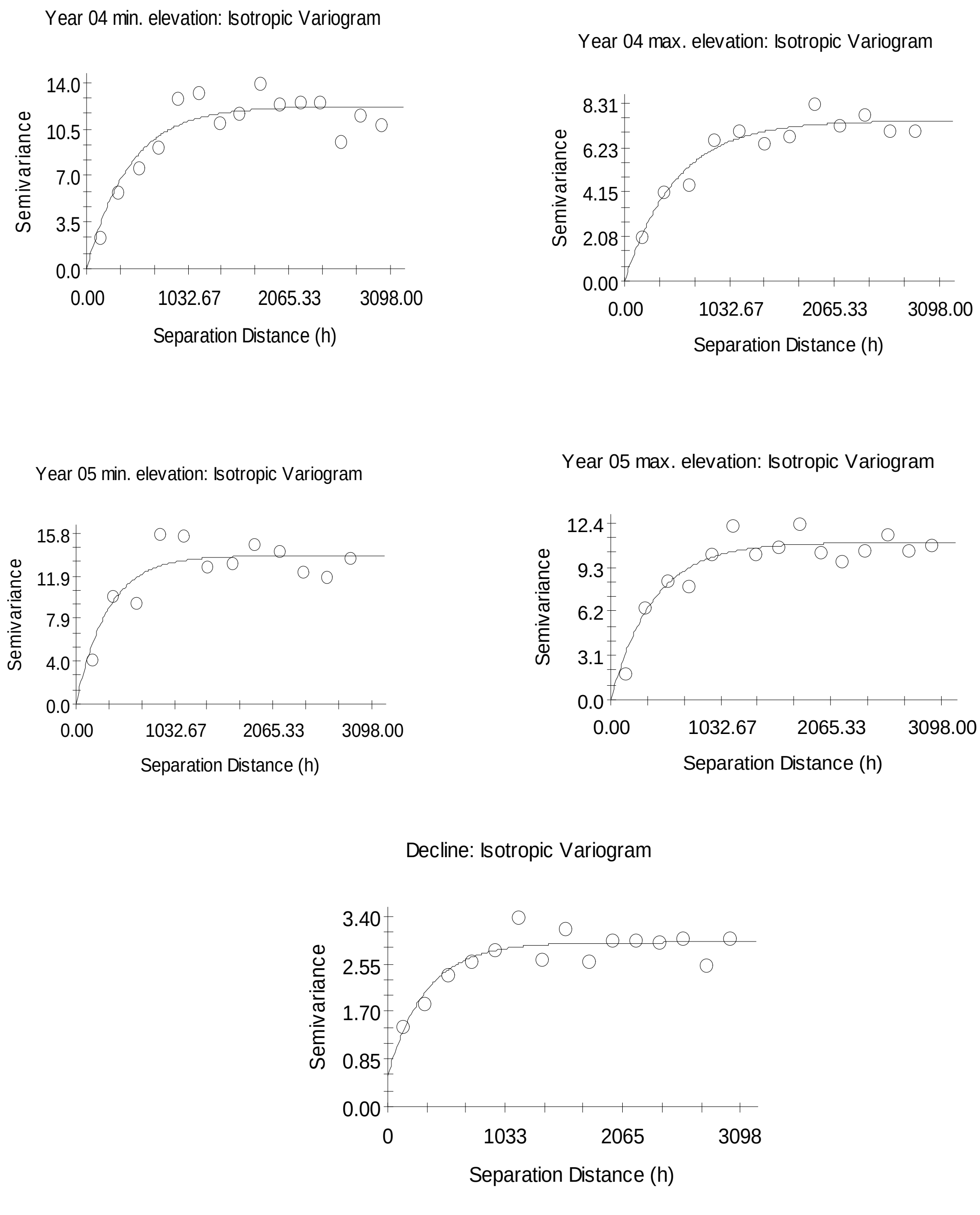


Figure 2 Semivariogram of the maximum, minimum and groundwater declination

Table 2: Modelled parameters

Parameter	*Year 2004 Minimum*	*Year2004 Maximum*	*Year 2005 Minimum*	*Year 2005 Maximum*	*GW Declination*
Nugget effect (C_o)	0.01m	0.21m	0.01m	0.01m	0.56m
Sill	12.23m	7.48m	13.74m	11.07m	5.87m
Nugget to sill ratio	0.001	0.03	0.001	0.001	0.1
Range (A_o)	432m	506m	322m	399m	338m

From the result in table 2, the very low nugget effect shows the absence of variability in groundwater elevation at short distances which mean there is an insignificant small-scale variability measurement error, thus the fitted semivarigram represents the spatial structure of the groundwater elevations very well. The low nugget-to-sill ratios also suggest a very strong spatial dependence in accordance with [16].

Ordinary kriging was applied for estimating the maximum and minimum value of groundwater elevation across the study area for the two years. Figure 3(a,b,c,d) shows the kriged estimation map and the standard error associated with the estimation. The kriged estimation map of the maximum value is an indication of how the study area is been recharged while the minimum value shows the extend at which water is been extracted from the study area every year. The map shows a short fall in groundwater recharge in the study area and more extraction of the resources within the period of study.

Figure 4(a,b) is a kriged map of the decline nature of groundwater level in the study area with its standard error of estimation and the cross-validation graph. It can be observed that, there is a declination in groundwater around the north western and the southern part of the study area. Ten wells (Wells No. 31, 47, 48, 56, 57, 59, 62, 85, 86, 87) out of the twelve wells that dried up during the study period are observed to be located around these areas of decreasing groundwater level, while two of the wells are found within area of increasing groundwater elevation. The two wells (well no. 4 and 24 around Birbhanpur village) are probable not drilled deep enough.

The map of the standard error shows a minimum values around the observation wells and these values gradually increase toward area of less or no observation wells. Since the standard error of prediction is assumed to be normally distributed, it can therefore be stated that a 95% confidence interval for the true groundwater level declination at any point within the study area is ±3.10 of the estimated value.

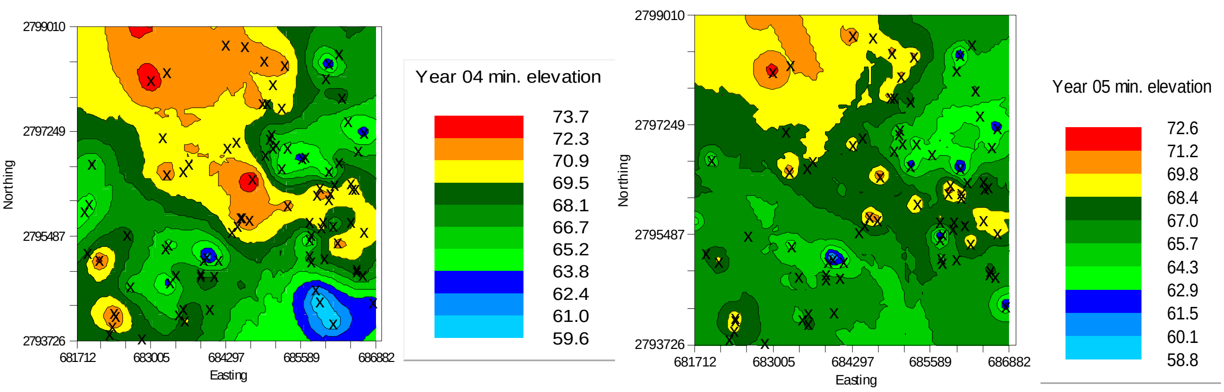
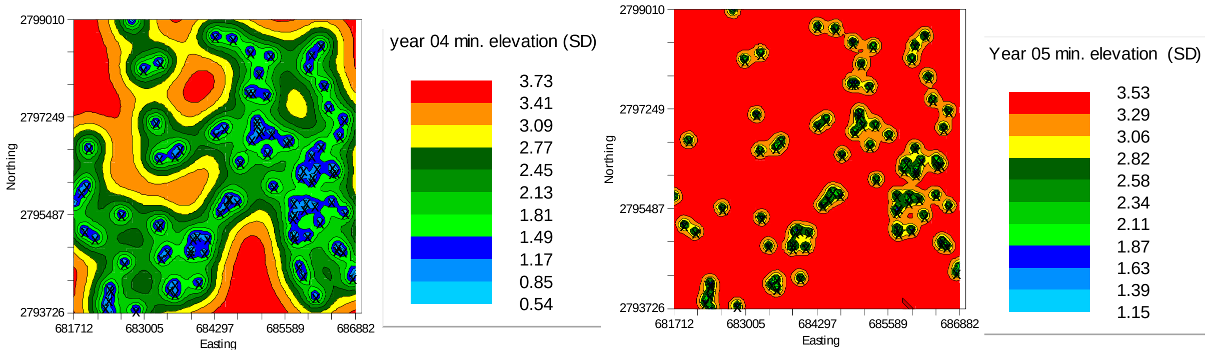
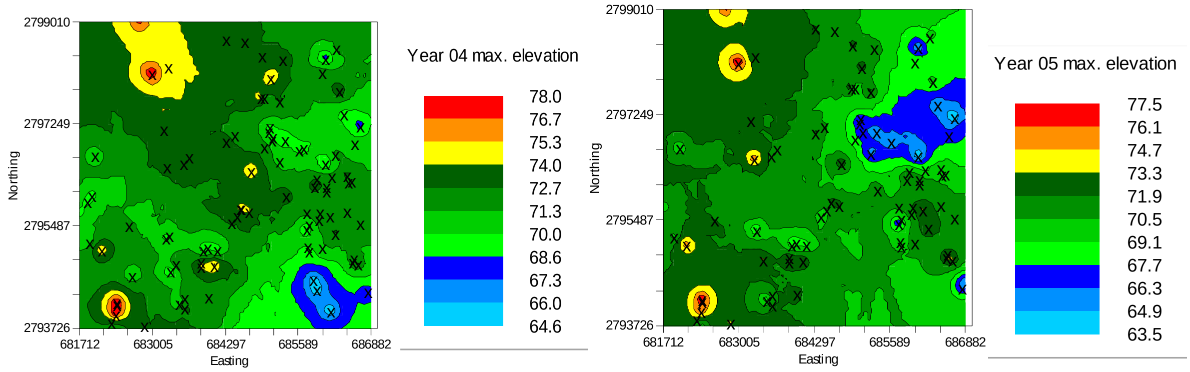


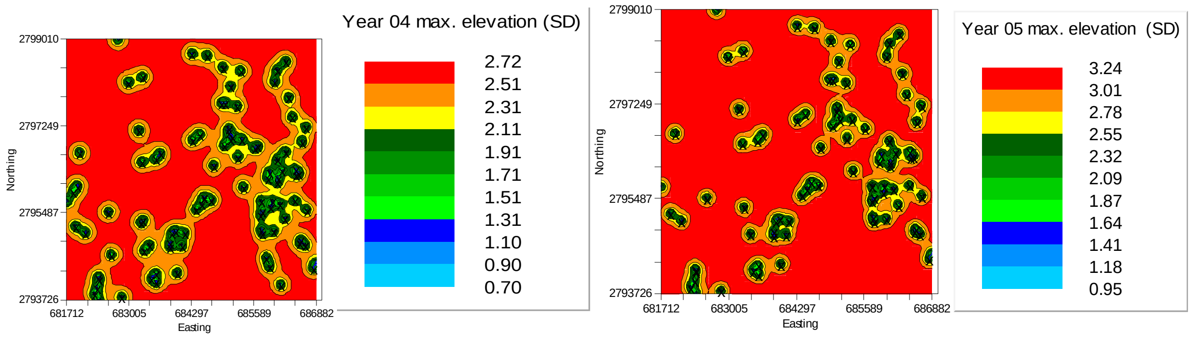
Figure 3(a) Minimum groundwater elevation estimation for year 2004 and year 2005



b) Standard error of estimation for year 2004 and 2005 minimum groundwater elevation



c) Maximum groundwater elevation estimation for year 2004 and year 2005



d) Standard error of estimation for year 2004 and 2005 maximum groundwater elevation

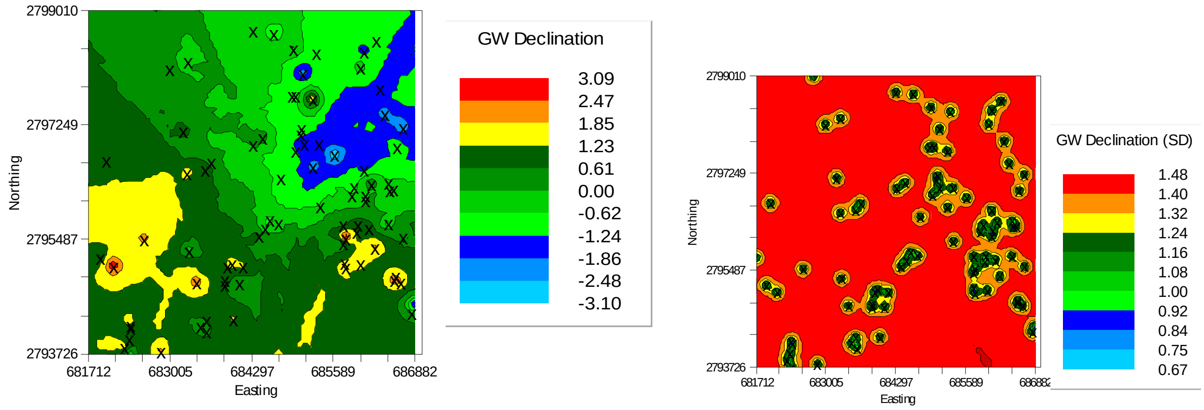
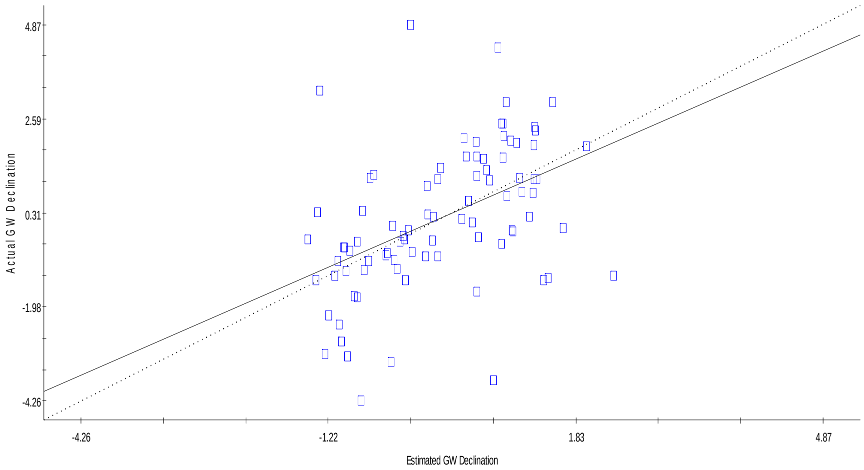


Figure 4(a) Groundwater declination and standard error of estimation



Regression coefficient = 0.861 (SE = 0.179 , r2 =0.216,
y intercept = 0.02, SE

CONCLUSION

The spatial analysis of groundwater declination was carried out in the study area using the groundwater level data monitored during two years. This analysis showed a short fall in recharge from the maximum elevation data obtained at each producing well and more extraction of groundwater probably for domestics and agricultural purposes during the period of studies. The declination in groundwater was estimated in the study area and found to have a declination around the north western and the southern part of the study area; almost 90% of the dried well encounters during the study period are situated in this area. An acceptable estimation criterion of the declination pattern was obtained using ordinary kriging and cross-validation which give a confidence level of prediction. For better understanding of the behaviour of the groundwater in the study area, a volume estimation of the resources at a given area and the temporal variation is recommended for further studies.

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