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Method of Hermite series expansion for solving the relativistic linear quantum simple harmonic oscillator problems

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ABSTRACT

In this paper, the relativistic linear quantum simple harmonic oscillator problem is solved by the method of Fourier Hermite series to derive the exact analytical solutions of the relativistic linear quantum simple harmonic oscillator. The first profound physical result of this work is the discovery of indefinitely fine corrections to the well known sequence of Schrodinger's quantum mechanical eigenenergies which become significant as the oscillator moves faster and faster compared to the speed of light in vacuo, especially the subatomic and elementary particles. the result implies corresponding hitherto unknown results in the areas of theoretical and experimental physics of oscillations and vibrations, such as Solid State Physics, Statistical and Thermal Physics and Particle Physics.

Keywords: Relativistic linear quantum Simple Harmonic Oscillator, relativistic Eigen energies, Hermite Series Expansion, relativistic wave functions,

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INTRODUCTION

Two important facts were established before the discovery of quantum mechanics. The first fact was based on the realization that the allowed values of energy between sub-atomic bodies are discrete- they involve quantum jumps. This tract began with Max Planck's work on black-body radiation, and was greatly supported by Niels Bohr's theory of the hydrogen atom and was carried to fruition by Werner Heisenberg's discovery of the matrix version of quantum mechanics.

The second fact began with the discovery of wave-particle duality of matter by Albert Einstein. Wave equation was discovered by Erwin Schrodinger, and de Broglie Schrödinger waves were interpreted as waves of probability by Max Born. This was the waves mechanics version of quantum mechanics [1].

Schrodinger decided to find a wave equation for matter that would give particle-like propagation when the wave-length becomes comparatively small. Paul A.M. Dirac produced a wave equation for the electron that combined relativity with quantum mechanics in 1928. In Schrodinger's wave equation, the kinetic energy used is non-relativistic and hence does not satisfy the requirements of the special theory of relativity. Dirac demonstrated that an electron has additional quantum numbers, m_s which is not a whole integer and can have only the values +1/2 and -1/2. It corresponds to an additional form of angular momentum ascribed to spinning motion.

The principles of correspondence and complementarily in the old quantum theory, which also preceded quantum mechanics was developed by Neil Bohr in 1923. He applied the principle to the problem of atomic emission and absorption spectra. Subsequently, after a consistent quantum mechanics had been created, the characteristics of atomic spectra were explained on a deeper foundation, with the essential features of the mathematical apparatus being determined by the correspondence principle.

The importance of this principle is not limited to mechanics alone. The principle is largely used in other areas like quantum electrodynamics and elementary particle theory and undoubtedly will be an integral part of any new theoretical scheme. This study is centered in solving the relativistic linear quantum simple harmonic oscillator problem. It aims to achieve the following objectives:

- Derive the exact analytical solutions of the relativistic linear quantum simple harmonic oscillator using the method of fourier hermite series solutions
- Derive the relativistic Eigen energies of the relativistic linear quantum simple harmonic oscillator using the method of fourier hermite series solutions
- Derive the relativistic wave functions of the relativistic linear quantum simple harmonic oscillator using the method of fourier hermite series solutions
- To investigate the physical consequences and applications of the wave functions and the energies such as application in quantum statistical mechanics.
- Application of the 4th order energy to generalization of the quantum partition functions of the thermodynamic system and the corresponding generalization of the thermodynamic potential of thermodynamic system

THEORY AND APPLICATIONS

For linear simple harmonic oscillator, the quantum mechanical energy wave equation is given as:

$$i\hbar\frac{\partial}{\partial t}\psi = \left\{\frac{-\hbar^2}{2m_o}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m_o\omega_o^2x^2\right\}\psi \qquad (1.1)$$

where $\boldsymbol{\psi}$ is the quantum mechanical wave function for the linear simple harmonic oscillator, which is subject to the conditions of uniqueness and regularity everywhere and continuity across all boundaries and normalization.

The relativistic linear momentum $\vec{P}$ and "kinetic energy" $\acute{T}$ of a relativistic linear simple harmonic oscillator of non rest mass m_o are given by Rosser (1964).

$$\vec{P} = \left[1-\frac{u^2}{c^2}\right]^{\frac{-1}{2}} m_o\vec{u} \qquad (1.2)$$

and

$$\acute{T} = \left[1-\frac{u^2}{c^2}\right]^{\frac{-1}{2}} m_o c^2 \qquad (1.3)$$

From (1.2) and (1.3), the kinetic energy operator $\hat{T}$ of the relativistic linear simple harmonic oscillator is given by

$$\hat{T} = m_o c^2 - \frac{\hbar^2}{2m_o}\frac{\partial^2}{\partial x^2} - \frac{\hbar^4}{8m_o^3c^2}\frac{\partial^4}{\partial x^4} - \cdots \qquad (1.4)$$

Replacing the kinetic energy in (1.1) with the first and the second terms of the right hand side of (1.4) it becomes

$$i\hbar\frac{\partial}{\partial t}\psi_2 = \left\{m_o c^2 - \frac{\hbar^2}{2m_o}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m_o\omega_o^2x^2\right\}\psi_2 \qquad (1.5)$$

where ψ_2 is the second order quantum mechanical wave function for the relativistic quantum linear simple harmonic oscillator. This is the second order quantum mechanical wave equation for the relativistic linear quantum simple harmonic oscillator.

Similarly, replacing the kinetic energy operator in (1.1) with the first, second and the third terms of the right hand side of (1.4) it becomes

$$i\hbar\frac{\partial}{\partial t}\psi_4 = \left[m_o c^2 - \frac{\hbar^2}{2m_o}\frac{\partial^2}{\partial x^2} - \frac{\hbar^4}{8m_o^3c^2}\frac{\partial^4}{\partial x^4} + \frac{1}{2}m_o\omega_o^2x^2\right]\psi_4 \qquad (1.6)$$

where ψ_4 is the fourth order quantum mechanical wave function for the relativistic linear quantum simple harmonic oscillator.

MATERIALS AND METHODS

In this study, Hermite series expansion of the fourth order was applied to quantum quantum mechanical wave equation for the relativistic linear quantum simple harmonic oscillator. Consider the relativistic linear simple harmonic oscillator quantum mechanical wave equation of the fourth order given by

$$0 = F_4^{1111}(\xi) - 8\lambda_4\xi F_4^{111}(\xi) + \left[(-12\lambda_4 + \alpha_4) + 24{\lambda_4}^2\xi^2\right]F_4^{11}(\xi) + \left[\left(48{\lambda_4}^2 - 4\alpha_4\lambda_4\right)\xi - 32{\lambda_4}^3\xi^3\right]F_4^{1}(\xi) + \left\{\left(12{\lambda_4}^2 - 2\alpha_4\lambda_4 + \frac{E_4 - m_0c^2}{\epsilon_4\, {a_4}^4}\right) + 16{\lambda_4}^4\xi^4\right\}F_4(\xi) \qquad (1.7)$$

where

$$\lambda_4 = \pm\frac{1}{2} \qquad (1.8)$$

$$\alpha_4 = \frac{\epsilon_2}{\epsilon_4\, {a_4}^2} \qquad (1.9)$$

$$a_4 = \left(\frac{m_0 w_0}{\hbar}\right)^{1/2} \qquad (1.10)$$

$$\epsilon_4 = \frac{\hbar^4}{8{m_0}^3c^2} \qquad (1.11)$$

and

$$\epsilon_2 = \frac{\hbar^2}{2m_0} \qquad (1.12)$$

Towards the solution of equation (1.7) it should be noted that the state of the relativistic linear quantum simple harmonic oscillator is contained within the space $L_2(-\infty,\infty)$ of all square integrable functions over the interval $(-\infty,\infty)$. Also, the Hermite polynomials constitute a sequence of complete orthogonal functions in the space. It therefore follows that there exist constants A_n such that

$$F_4(\xi) = \sum_{n=0}^{\infty} A_n H_n(\xi) \qquad (1.13)$$

Hence simplifying using the well known recurrence properties of the Hermite polynomials we obtain

$$0=\sum_{n=0}^{\infty}2^4(n+1)(n+2)(n+3)(n+4)A_{n+4}\,H_n(\xi)-\sum_{n=1}^{\infty}2^4n(n+1)(n+2)A_{n+2}\,H_n(\xi)$$
$$-\sum_{n=0}^{\infty}2^5(n+1)(n+2)(n+3)(n+4)A_{n+4}\,H_n(\xi)$$
$$+(-6+\alpha_4)\sum_{n=0}^{\infty}2^2(n+1)(n+2)A_{n+2}\,H_n(\xi)+\sum_{n=2}^{\infty}6n(n-1)\,A_nH_n(\xi)$$
$$+\sum_{n=0}^{\infty}12(n+1)^2(n+2)\,A_{n+2}H_n(\xi)+\sum_{n=1}^{\infty}12n(n+1)(n+2)A_{n+2}\,H_n(\xi)$$
$$+\sum_{n=0}^{\infty}24(n+1)(n+2)(n+3)(n+4)A_{n+4}\,H_n(\xi)+(12-2\alpha_4)\sum_{n=1}^{\infty}nA_n\,H_n(\xi)$$
$$+(12-2\alpha_4)\sum_{n=0}^{\infty}2(n+1)(n+2)A_{n+2}\,H_n(\xi)-\sum_{n=3}^{\infty}(n-2)\,A_{n-2}H_n(\xi)$$
$$-\sum_{n=1}^{\infty}2n\,(n+1)A_nH_n(\xi)-\sum_{n=1}^{\infty}2n^2\,A_nH_n(\xi)-\sum_{n=0}^{\infty}4(n+1)(n+2)^2\,A_{n+2}H_n(\xi)$$
$$-\sum_{n=2}^{\infty}2n(n-1)\,A_nH_n(\xi)-\sum_{n=0}^{\infty}4(n+1)^2(n+2)\,A_{n+2}H_n(\xi)$$
$$-\sum_{n=1}^{\infty}4n(n+1)(n+2)\,A_{n+2}H_n(\xi)-\sum_{n=0}^{\infty}8(n+1)(n+2)(n+3)(n+4)\,A_{n+4}H_n(\xi)$$
$$+\left(3-\alpha_4+\frac{E_4-m_0c^2}{\epsilon_4\,a_4{}^4}\right)\sum_{n=0}^{\infty}A_nH_n(\xi)+\sum_{n=4}^{\infty}\frac{1}{16}A_{n-4}H_n(\xi)+\sum_{n=2}^{\infty}\frac{1}{8}(n+1)A_{n-2}H_n(\xi)$$
$$+\sum_{n=2}^{\infty}\frac{1}{8}nA_{n-2}H_n(\xi)+\sum_{n=0}^{\infty}\frac{1}{4}(n+1)(n+2)A_nH_n(\xi)+\sum_{n=2}^{\infty}\frac{1}{8}(n-1)A_{n-2}H_n(\xi)$$
$$+\sum_{n=0}^{\infty}\frac{1}{4}(n+1)^2\,A_nH_n(\xi)+\sum_{n=1}^{\infty}\frac{1}{4}n(n+1)A_nH_n(\xi)+\sum_{n=0}^{\infty}\frac{1}{2}(n+1)(n+2)(n+3)A_{n+2}H_n(\xi)$$
$$+\sum_{n=3}^{\infty}\frac{1}{8}(n-2)A_{n-2}H_n(\xi)+\sum_{n=1}^{\infty}\frac{1}{4}n(n+1)A_nH_n(\xi)$$
$$+\sum_{n=1}^{\infty}\frac{1}{4}n^2A_nH_n(\xi)$$
$$+\sum_{n=0}^{\infty}\frac{1}{2}(n+1)(n+2)^2A_{n+2}H_n(\xi)+\sum_{n=2}^{\infty}\frac{1}{4}n(n-1)A_nH_n(\xi)+\sum_{n=0}^{\infty}\frac{1}{2}(n+1)^2(n$$
$$+2)A_{n+2}H_n(\xi)+\sum_{n=1}^{\infty}\frac{1}{2}n(n+1)(n+2)A_{n+2}H_n(\xi)$$
$$+\sum_{n=0}^{\infty}(n+1)\,(n+2)(n+3)(n+4)A_{n+4}H_n(\xi)\qquad(1.14)$$

From the coefficients of $H_0(\xi)$ we obtain the recurrence equation

$$0=\left(\frac{15}{4}-\alpha_4+\frac{E_4-m_0c^2}{\epsilon_4\,a_4{}^4}\right)A_0+6A_2+24A_4\qquad(1.15)$$

From the coefficients of $H_1(\xi)$ we obtain the recurrence equation

$$0=\left(\frac{51}{4}-3\alpha_4+\frac{E_4-m_0c^2}{\epsilon_4\,a_4{}^4}\right)A_1+6A_3+120A_5\qquad(1.16)$$

Generally,

$$0=\sum_{n=2}^{\infty}\left[\left(\frac{E_4-m_0c^2}{\in_4\,a_4{}^4}-(2n+1)\alpha_4+\frac{3}{2}n^2+\frac{15}{2}n+\frac{15}{4}\right)A_n+(n^4+10n^3+35n^2+50n+24)A_{n+4}-\left(6n^3+23n^2+\frac{39}{2}n+55\right)A_{n+2}+\left(-\frac{5}{8}n+\frac{7}{4}\right)A_{n-2}+\frac{1}{16}A_{n-4}\right] \quad (1.17)$$

GROUND LEVEL OF THE FOURTH ORDER

For the ground level of the relativistic linear quantum simple harmonic oscillator of the fourth order let us choose the coefficient of A_0 in the recurrence equation (1.15) to vanish

$$\frac{15}{4}-\alpha_4+\frac{E_4-m_0c^2}{\in_4\,a_4{}^4}=0 \quad (1.18)$$

$$E_{4,0}=m_0c^2+\alpha_4\in_4\,a_4{}^4-\frac{15}{4}\in_4\,a_4{}^4 \quad (1.19)$$

$E_{4,0}$ is the fourth order quantum mechanical energy of the ground level of the relativistic linear quantum simple harmonic oscillator

or explicitly using (1.9), (1.10), and (1.11)

$$E_{4,0}=m_0c^2+\frac{1}{2}\hbar w_0-\frac{15}{32}\frac{\hbar^2 w_0{}^2}{m_0c^2} \quad (1.20)$$

FIRST LEVEL OF THE FOURTH ORDER

For the first level of the relativistic linear quantum simple harmonic oscillator of the fourth order let us choose the coefficient of A_1 in the recurrence equation (1.16) to vanish

$$\frac{51}{4}-3\alpha_4+\frac{E_4-m_0c^2}{\in_4\,a_4{}^4}=0 \quad (1.21)$$

$$E_{4,1}=m_0c^2+3\alpha_4\in_4\,a_4{}^4-\frac{51}{4}\in_4\,a_4{}^4 \quad (1.22)$$

$E_{4,1}$ is the fourth order quantum mechanical energy of the first level of the relativistic linear quantum simple harmonic oscillator

or explicitly using (1.9), (1.10), and (1.11)

$$E_{4,1}=m_0c^2+\frac{3}{2}\hbar w_0-\frac{51}{32}\frac{\hbar^2 w_0{}^2}{m_0c^2} \quad (1.23)$$

GENERAL LEVEL OF THE FOURTH ORDER

It is now obvious that for the general level of the relativistic linear quantum simple harmonic oscillator we choose the coefficients of A_n in the recurrence relation (1.17) to vanish

$$0=\frac{E_4-m_0c^2}{\in_4\,a_4{}^4}-(2n+1)\alpha_4+\frac{3}{2}n^2+\frac{15}{2}n+\frac{15}{4} \quad (1.24)$$

It follows from (1.24) that the fourth order quantum mechanical eigen energy of the n^{th} level of the relativistic linear quantum simple harmonic oscillator, denoted by $E_{4,n}$ is given by

$$E_{4,n}=m_0c^2+\left(n+\frac{1}{2}\right)\hbar w_0-\frac{\hbar^2 w_0{}^2}{8m_0c^2}\left(\frac{3}{2}n^2+\frac{15}{2}n+\frac{15}{4}\right);\ n=0,1,2,.,.,. \quad (1.25)$$

GROUND LEVEL WAVE FUNCTION

The fourth order quantum mechanical energy of the ground level of the relativistic linear quantum simple harmonic oscillator as obtained is

$$E_{4,0} = m_0c^2 + \frac{1}{2}\hbar w_0 - \frac{15}{32}\frac{\hbar^2 {w_0}^2}{m_0c^2} \tag{1.26}$$

and from the coefficients of H_O and H_1 in (1.14), the first recurrence relations becomes

$$A_{o,4} = -\frac{1}{4}A_{0,2} - \frac{1}{32}A_{0,0} \tag{1.27}$$

$$A_{o,5} = -\frac{1}{20}A_{o,3} - \left(\frac{13}{160} - \frac{1}{60}\alpha_4\right)A_{o,1} \tag{1.28}$$

Substituting (1.27) and (1.28) into (1.13), as in the case of the non-relativistic problem, the ground level wave function $F_{4,0,}$ of the relativistic linear quantum simple harmonic oscillator is obtained as the A_0 series obtained from the recurrence equations and the ground level eigen energy

$$F_{4,0,0} = A_{0,0}H_0 + A_{0,1}H_1 + A_{0,2}H_2 + A_{0,3}H_3 - \left(\frac{1}{4}A_{0,2} + \frac{1}{32}A_{0,0}\right)H_4 - \left[\frac{1}{20}A_{0,3} + \left(\frac{13}{160} - \frac{1}{60}\alpha_4\right)A_{0,1}\right]H_5 \tag{1.29}$$

Simplifying equation (1.29), we obtain

$$F_{4,0,0} = A_{0,0}\left(H_0 - \frac{1}{32}H_4 \ldots.\right) + A_{0,1}\left[H_1 - \left(\frac{13}{160} - \frac{1}{60}\alpha_4\right)H_5 \ldots.\right] + A_{0,2}\left[H_2 - \frac{1}{4}H_4 \ldots.\right] + A_{0,3}\left[H_3 - \frac{1}{20}H_5 \ldots.\right] \tag{1.30}$$

Thus we obtain four linearly independent solutions given by

$$\left(F_{4,0,0}\right)_0 = A_{0,0}\left(H_0 - \frac{1}{32}H_4 \ldots \ldots..\right) \tag{1.31}$$

$$\left(F_{4,0,0}\right)_1 = A_{0,1}\left[H_1 - \left(\frac{13}{160} - \frac{1}{60}\alpha_4\right)H_5 \ldots.\right] \tag{1.32}$$

$$\left(F_{4,0,0}\right)_2 = A_{0,2}\left[H_2 - \frac{1}{4}H_4 \ldots.\right] \tag{1.33}$$

$$\left(F_{4,0,0}\right)_3 = A_{0,3}\left[H_3 - \frac{1}{20}H_5 \ldots.\right] \tag{1.34}$$

The appropriate choice of the ground level wave function is the first independent solution given by

$$\left(F_{4,0,0}\right)_0 = A_{0,0}\left(H_0 - \frac{1}{32}H_4 \ldots \ldots..\right) \tag{1.35}$$

FIRST LEVEL WAVE FUNCTION

The fourth order quantum mechanical energy of the first level of the relativistic linear quantum simple harmonic oscillator is

$$E_{4,1} = m_0c^2 + \frac{3}{2}\hbar w_0 - \frac{51}{32}\frac{\hbar^2 {w_0}^2}{m_0c^2} \tag{1.36}$$

Using (1.36) and from the coefficients of H_O and H_1 in (1.14), the recurrence relations becomes

$$A_{1,4} = -\left(\frac{1}{32} + \frac{\hbar w_0}{24} - \frac{\hbar^2 {w_0}^2}{16}\right)A_{1,0} - \frac{1}{4}A_{1,2} \tag{1.37}$$

$$A_{1,5} = -\frac{1}{20}A_{1,3} - \left(\frac{39}{480} - \frac{\alpha_4}{60} + \frac{\hbar w_0}{120} - \frac{\hbar^2 {w_0}^2}{80m_0c^2}\right)A_{1,1} \qquad (1.38)$$

Substituting (1.37) and (1.38) into (1.13), as in the case of the non-relativistic problem the first level wave function $F_{4,1,}$ of the relativistic linear quantum simple harmonic oscillator is obtained as the A_1 series obtained from the recurrence equations and the first level eigen energy given

$$F_{4,1,0} = A_{1,0}H_0 + A_{1,1}H_1 + A_{1,2}H_2 + A_{1,3}H_3 - \left[\left(\frac{1}{32} + \frac{\hbar w_0}{24} - \frac{\hbar^2 {w_0}^2}{16}\right)A_{1,0} + \frac{1}{4}A_{1,2}\right]H_4 - \left[\frac{1}{20}A_{1,3} + \left(\frac{39}{480} - \frac{\alpha_4}{60} + \frac{\hbar w_0}{120} - \frac{\hbar^2 {w_0}^2}{80m_0c^2}\right)A_{1,1}\right]H_5 \qquad (1.39)$$

Simplifying, equation (1.39) becomes

$$F_{4,1,0} = A_{1,0}\left[H_0 - \left(\frac{1}{32} + \frac{\hbar w_0}{24} - \frac{\hbar^2 {w_0}^2}{16}\right)H_4 \dots.\right] + A_{1,1}\left[H_1 - \left(\frac{39}{480} - \frac{\alpha_4}{60} + \frac{\hbar w_0}{120} - \frac{\hbar^2 {w_0}^2}{80m_0c^2}\right)H_5 \dots.\right] + A_{1,2}\left[H_2 - \frac{1}{4}H_4 \dots.\right] + A_{1,3}\left[H_3 - \frac{1}{20}H_5 \dots.\right] \qquad (1.40)$$

We also obtain four linearly independent solutions expressed as:

$$\left(F_{4,1,0}\right)_0 = A_{1,0}\left[H_0 - \left(\frac{1}{32} + \frac{\hbar w_0}{24} - \frac{\hbar^2 {w_0}^2}{16}\right)H_4 \dots.\right] \qquad (1.41)$$

$$\left(F_{4,1,0}\right)_1 = A_{1,1}\left[H_1 - \left(\frac{39}{480} - \frac{\alpha_4}{60} + \frac{\hbar w_0}{120} - \frac{\hbar^2 {w_0}^2}{80m_0c^2}\right)H_5 \dots.\right] \qquad (1.42)$$

$$\left(F_{4,1,0}\right)_2 = A_{1,2}\left[H_2 - \frac{1}{4}H_4 \dots.\right] \qquad (1.43)$$

$$\left(F_{4,1,0}\right)_3 = A_{1,3}\left[H_3 - \frac{1}{20}H_5 \dots.\right] \qquad (1.44)$$

The appropriate choice of the first level wave function is the second independent solution given by

$$\left(F_{4,1,0}\right)_1 = A_{1,1}\left[H_1 - \left(\frac{39}{480} - \frac{\alpha_4}{60} + \frac{\hbar w_0}{120} - \frac{\hbar^2 {w_0}^2}{80m_0c^2}\right)H_5 \dots.\right] \qquad (1.45)$$

RESULTS AND DISCUSSION

The fourth order relativistic quantum mechanical wave equation for the relativistic linear quantum simple harmonic oscillator is given as

$$i\hbar\frac{\partial}{\partial t}\psi_4(x,t) = \left[m_oc^2 - \frac{\hbar^2}{2m_o}\frac{\partial^2}{\partial x^2} - \frac{\hbar^4}{8m_o^3c^2}\frac{\partial^4}{\partial x^4} + \frac{1}{2}m_o\omega_o^2x^2\right]\psi_4(x,t) \quad (1.46)$$

Equation (1.46) is then reduced by separation of variables to give

$$0 = {F_4}^{1111}(\xi) - 8\lambda_4\xi {F_4}^{111}(\xi) + \left[(-12\lambda_4 + \alpha_4) + 24{\lambda_4}^2\xi^2\right]{F_4}^{11}(\xi) + \left[\left(48{\lambda_4}^2 - 4\alpha_4\lambda_4\right)\xi - 32{\lambda_4}^3\xi^3\right]{F_4}^{1}(\xi) + \left\{\left(12{\lambda_4}^2 - 2\alpha_4\lambda_4 + \frac{E_4 - m_0c^2}{\epsilon_4\ {\alpha_4}^4}\right) + 16{\lambda_4}^4\xi^4\right\}F_4(\xi) \qquad (1.47)$$

We then seek the solution of (1.47) as a Hermite series of the form

$$F_4(\xi) = \sum_{n=0}^{\infty} A_nH_n(\xi) \qquad (1.48)$$

Hence the relativistic Eigen energies are obtained as

$$E_{4,0} = m_0c^2 + \frac{1}{2}\hbar w_0 - \frac{15}{32}\frac{\hbar^2 {w_0}^2}{m_0c^2} \qquad (1.49)$$

$$E_{4,1} = m_0c^2 + \frac{3}{2}\hbar w_0 - \frac{51}{32}\frac{\hbar^2 {w_0}^2}{m_0c^2} \qquad (1.50)$$

$$E_{4,n} = m_0c^2 + \left(n+\frac{1}{2}\right)\hbar w_0 - \frac{\hbar^2 {w_0}^2}{8m_0c^2}\left(\frac{3}{2}n^2 + \frac{15}{2}n + \frac{15}{4}\right); n = 0,1,2,.,.,. \quad (1.51)$$

from this method we obtain the wavefuctions

$$\left(F_{4,0,0}\right)_0 = A_{0,0}\left(H_0 - \frac{1}{32}H_4 \dots\dots..\right) \quad (1.52)$$

for the ground level and

$$\left(F_{4,1,0}\right)_0 = A_{1,1}\left[H_1 - \left(\frac{39}{480} - \frac{\alpha_4}{60} + \frac{\hbar w_0}{120} - \frac{\hbar^2 {w_0}^2}{80m_0c^2}\right)H_5 \dots.\right] \quad (1.53)$$

for the first level

Carefully observing each level of the fourth order quantum mechanical eigen energies of the relativistic linear quantum simple harmonic oscillator, it may be noted that $E_{4,2}$ is a unique (hitherto unknown in any previous theory of quantum mechanics) and physically most elegant and interesting and natural (based upon the experimental physical facts available) generalization or extension of the second order quantum mechanical eigen energy of the relativistic linear quantum simple harmonic oscillator $E_{2,n}$. Herein lie profound physical and experimental interest.

The first notable physical result of our work in is the discovery of the indefinitely fine revisions or corrections of the well known sequence of Schrodinger's quantum mechanical eigen energies or the linear quantum simple harmonic oscillator which are more significant as the rest mass of the oscillator becomes smaller (or equivalently, as the oscillator moves faster and faster compared with the speed of light in vacuo)-especially the subatomic and elementary particles.

It is most interesting and instructive to note that the smaller the rest mass of the oscillator (or equivalently, the faster it moves compared with the speed of light in vacuo) the more significant are the fourth order correction terms to the second order relativistic quantum mechanical eigen energies.

Secondly, it may also be noted that the harmonic oscillator is the greatest showpiece of quantum mechanics. For it is the most common system in nature, both in the macro and micro worlds. Consequently, the revolution in this study implies corresponding hitherto unknown revolutions in all areas of theoretical and experimental physics of oscillations and vibrations such as follows

- Solid state physics
- Statistical physics
- Thermal physics
- Elementary particle physics

Recommendation

This work can now be extended to the derivation of the eigen energies and eigen functions of the linear simple harmonic oscillator correct to all orders of $\hbar^{2n}$; n=1,2,3,….

For instance it is obvious that our sixth order relativistic quantum mechanical energy wave function of the lsho, denoted by $U_6(x)$ will satisfy an equation of the form.

$$0 = \epsilon_6\, {U_6}^{111111}(x) + \epsilon_4\, {U_6}^{1111}(x) + \epsilon_2\, {U_6}^{11}(x) + \left[(E_6 - m_0c^2) - \frac{1}{2}m_o\omega_o^2x^2\right]U_6(x) \quad (1.54)$$

subject to the condition of uniqueness and regularity everywhere and continuity across all boundaries and normalization, where E_6 is the sixth order relativistic quantum mechanical energy of the linear simple harmonic oscillator and ϵ_6 is constant of order 6 in $\hbar$ and hence their physical and mathematical applications.

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