Modified OPLL in Microwave Signal Generation Technique

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ABSTRACTS

The modified Optical Phase Lock Loop, as suggested in this paper, considerably improve the performance of the microwave signal generation technique over the earlier technique with the help of commercially available DFB lasers with linewidth of about 4.0 MHz. This modified OPLL seems a realistic approach for generating reference signal for future advanced communication systems with active phased array antennas. This opto-electronic technique provides the advantage of low weight, small size, flexible, immunity to EMI and above low cost because the proposed technique significantly reduces the requirement of narrow linewidth lasers. Moreover, it has been observed that bulk and surface acoustic waves in the power and frequency ranges of a few mW and KHz respectively are capable of modifying both the cavity length and refractive index of the material of semiconductor lasers.

Keywords: Optical Phase Lock Loop, phased array antennas, photo detector, Microwave Generator, Laser modulator, Optical Amplifier, photo diode.

INTRODUCTION

Future generation space based communication systems are required to generate a large number of shaped re-configurable and re-positional antenna beams for earth stations and mobile receivers. To meet this challenge large aperture active phased array antenna systems are needed, because in this approach RF circuit-functions are transferred to the antenna platform. High-density integration of these circuits is made possible with the help of GaAs microwave monolithic integrated circuit technology. The antenna system will have a transmitting module and a receiving module with each of the elements or a small group of radiating elements. These circuits will require a frequency reference to local oscillators in the sub-arrays to have them frequency and phase synchronized. Fig.1 is a conceptual representation of the technique [1-4].

Space based systems, where size, weight, power consumption, and complexity are extremely costly commodities; conventional distribution networks using semi-rigid coaxial transmission line for reference signals will make the system prohibitively heavy, bulky, and susceptible to electromagnetic interference. Also these electrical links suffer from considerable insertion loss. For example, the typical dimension of a coaxial cable is 0.355 cm in diameter whose attenuation varies from 17.0 dB/10 meters at 10 GHz to 170 dB/10 meters at 100 GHz, whereas that of an optical cable is only 0.01 dB/10 meters. Moreover, the optical cables are small, flexible, lightweight, and immune to electromagnetic interference. Usually the optical link is passive in nature but the link can play an active role by providing some form of control functions, like adjusting phase delay to the microwave/millimeter wave systems. Naturally, a need has been felt for finding alternative to the electrical distribution systems based on the use of optical/opto-electronic principles.
To the knowledge of the authors, it appears that there are three ways of generating microwave/millimeter wave signal using light wave techniques, viz., direct, injection locked and indirectly synchronized techniques. This is illustrated in Fig. 2. The direct method is based on beating of two inherent optical sources. In this method very narrow line-width lasers are required. This system is more costly, consumes more power and is larger in dimension than commercially available DFB lasers having the advantage of potential optoelectronic integration.

![Fig. 1 Optical transport of microwave signals directly modulated fibre optic link](image)

In the injection locked method, a microwave frequency modulated output from a highly stabilized laser is used to injection lock a relatively wide line-width laser using side band lock technique [1, 5, and 6]. Theoretically the idea is charming but implementation is not easy. It is worthwhile to note that injection locking is stable through out the whole locking range for low values of injection locking ratio (ILR). For medium values of the ILR, the stable zone of locking becomes very narrow and becomes stable again over the whole locking zone for very high injection ratio. This demands high power of the master laser. Moreover, in both these systems external cavity lasers are used. This requires high mechanical stability – a condition difficult to fulfill in satellites. Whereas the method, we propose, does neither require external cavity lasers nor use narrow line width lasers. This is based on a modified optical phase locking technique which is shown in Fig. 3.

1. **System Configuration**

   Basically the proposed system works on the principle of indirect optical phase locking using a modified optical phase locked loop (OPLL). Incidentally an OPLL is the optical analog of an electrical/electronic phase-lock loop in the sense that here the locking and locked sources are lasers (optical oscillators) instead of electrical oscillators in a conventional phase-lock loop (PLL). It operates on the same principle as that of a PLL. The structure of the proposed optical PLL (OPLL) is shown in Fig.4. The loop components are: Lasers, Photo-detector, Phase-detector, Phase-modulator, etc. Although analytically equivalent to a homodyne OPLL, the heterodyne OPLL has two mean advantages, (1) The phase detector works at a frequency which can be far lower than that of the lasers and as such well developed microwave phase detectors can be used. (2) A narrow band pass filter can be used at the input stage of the phase detector. This rejects the dc drifts at the phase detector and helps to improve the signal-to-noise ratio. But if the bandwidth of the band pass filter is not judiciously chosen it contributes to the loop propagation delay that deteriorates the loop performance in many ways.

   The outputs of the transmitter laser and the VCO laser, shining on the photo-detector, generate a beat signal at a frequency corresponding to the frequency offset between the two sources. This beat signal and the microwave reference signal after being detected by the phase detector produce a phase error signal at the output of the phase detector. The loop filter processes this signal and modulates the instantaneous frequency of the VCO laser or slave laser.
Fig. 3 Modified optical phase locking technique.

Fig. 5.2 Methods of generating microwave signal.
To understand the operation of the system in simple physical terms [7] let us assume to begin with that the instantaneous frequency difference as well as the phase difference between the two inputs to the phase detector is zero. The output of the phase detector which is proportional to sine of the instantaneous phase difference is zero. If the frequency or frequencies one or both laser sources tries or try to drift, this change of frequency between the two will be felt as a phase difference with respect to the reference signal and it will produce a voltage in correspondence to a measure of the phase difference. This voltage will correct the frequency of the VCO laser in such a way so as to reduce the error in frequency of the beat with respect to the reference frequency to nothing. There is a limit to this frequency error up to which the frequency of the slave laser can be controlled. This limit is called the hold-in range.

As noted the heterodyne OPLL works on the same principle of an OPLL, but its performance in detail depends on the properties of the various loop component particularly laser and photo-detector. This is explained in the following:

**Laser:** Although stimulated emission and feedback are the basis of laser oscillation, yet spontaneous emission is an inevitable aspect of laser emission. Each spontaneous event causes a sudden jump of random and signs in the electromagnetic field generated by the laser. This generates laser phase noise and the output field \( E_o(t) \) of a laser is analytically represented as

\[
E_o(t) = \sqrt{2P_o} \cos(\omega_o t + \gamma(t))
\]

where \( P_o \) is the detected average power, \( \omega_o \) is the angular frequency of laser oscillation and \( \gamma(t) \) is the random phase. The corresponding noise frequency spectra are characterized by white, flicker and random walk noises and the two-sided spectral densities are defined as

\[
S_{pn}(f) = S_{fn}(f) + S_{an}(f) + S_{rn}(f)
\]

where

\[
S_{fn}(f) = \frac{K_f}{|f|}
\]
where \( K_f \) and \( K_r \) are constants giving the strength of flicker frequency noise and random walk frequency noise respectively. \( \delta \nu \) is the Lorentzian spectral width. Typically the line width of the commercially available laser lies in the range of 5 to 50 MHz whereas that of the microwave source is on the order of 1.0 Hz. For a narrow and medium bandwidth system \( S_{\text{out}}(f) \) is used to design an OPLL.

**Photo-detector:** A laser beam of average power \( P_o \) shining on a PIN photo-detector gives rise to photo-current, the average value of which is given by

\[
I_{\text{ph}} = R P_o
\]

where \( R \) is the responsivity of the detector. The output of the photo-detector is contaminated with noise, the principal components of which are shot noise, dark current noise and surface leakage current noise. Shot noise is generated due to statistical nature of generation and collection of photo-detectors. Dark current arises from the holes or electrons that are thermally generated at the pn junction of the photo-diode. Surface leakage current noise is dependent on surface defects, surface area and bias voltage.

### 2. System Equation

With reference to Fig. 2, let us denote the outputs of the two DFB lasers of the system as

\[
E_x(t) = \text{the output of the } T_x \text{ laser} = \sqrt{2 P_R} \cos(\omega_1 t + \alpha(t))
\]

\[
E_{v}(t) = \text{the output of the VCO laser after phase modulation} = \sqrt{2 P_R} \cos(\omega_2 t + \theta(t) + \psi(t) + \beta(t))
\]

where

- \( P_R \) = power output of the \( T_x \) laser
- \( P_o \) = phase modulator output power
- \( \alpha(t) \) = phase modulation due to phase noise of the \( T_x \) laser
- \( \beta(t) \) = phase modulation due to phase noise of the DFB laser
- \( \theta(t) \) = phase modulation of the DFB laser by the phase detector output \( V_{d}(t) \), as a result of the instantaneous frequency control.
- \( \psi(t) \) = phase modulation of the DFB laser obtained as a result of application of the phase detector output \( V_{d}(t) \) to the phase modulator.

Denoting the output of the microwave reference source as \( 2\cos \omega_1 t \), it is not difficult to show that the phase detector output can be expressed as

\[
V_{o}(t) = \left[ \sqrt{P_R P_o} R r \sin \phi(t) + n(t) \right]
\]

where

\[
\phi(t) = (\omega_1 - \omega_2 - \omega_x) t + \alpha(t) - \beta(t) - \theta(t) - \psi(t)
\]

\( R \) is the responsivity of the photo-detector, \( r \) is the load resistance of the photo-detector and \( n(t) \) is a noise process mainly due to shot noise. The spectral density of \( n \) is given by
\[ S_n(f) = 4eR(P_o + P_R)r^2 \]  
\[ e \text{ being the electronic charge. Further,} \]
\[ \frac{d\theta}{dt} = K_vV_o(t) \]  
and
\[ \psi(t) = K_pV_o(t), \]
where \( K_v \) and \( K_p \) are, respectively, the sensitivities of the laser VCO and the phase modulator. 

Using (6) through (9), the governing phase equation of the system is written in mixed notations
\[ \frac{d\phi}{dt} = \Omega - F(s)e^{-s\tau}(K_v + sK_p)2\sqrt{P_oP_R}Rr\sin\phi + n(t) + \frac{da}{dt} - \frac{d\beta}{dt} \]  
where \( F(s) = \) the loop filter transfer function,
\[ \Omega = \omega_1 - \omega_2 - \omega_x. \]

The term \( e^{-s\tau} \) indicates the effect of the loop delay \( (\tau) \). Putting  
\[ 2K_vRr\sqrt{P_oP_R} = K; \quad \frac{K_p}{K_v} = \tau_p \quad \text{and} \quad N(t) = \frac{n(t)}{Rr\sqrt{P_oP_R}} \]  
the equation (9a) can be written as
\[ \frac{d\phi}{dt} - KF(s)e^{-s\tau}(1 + s\tau_p)[\sin\phi + N(t)] + \frac{da}{dt} - \frac{d\beta}{dt} \]  
Referring to (9b) and (7), it is seen that the spectral density of \( N(s) \) is given by
\[ S_N(f) = \frac{e(P_o + P_R)}{RPP_oP_R} \]  

First Order & Delay
The governing equation is a higher order stochastic delay-differential equation. The exact solution is not known. However, it has been observed that the delay modifies the system behavior many ways. To audit how it does, let us consider a conventional first order OPLL \( (F(s)=1) \). Thus (cf. (10a))
\[ \frac{d\phi}{dt} = \Omega - K\sin\phi(t - \tau) + \frac{d}{dt}(\alpha - \beta) \]

Here it can be proved that for stability of the noise-free loop i.e. when \( \alpha = \beta = 0 \) is given by the following conditions [7]
\[ \Omega \leq K \]
\[ 0 \leq K\tau\cos\phi_s \leq \frac{\pi}{2} \]  
where
\[ \phi_s = \arcsin\left(\frac{\Omega}{K}\right) \]

In many cases, like ours, the effect of \( N(t) \) is negligible compared to that of phase noise. Therefore, the variance of the phase error for a delay-less OPLL

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That is
\[ \sigma_\phi^2 = 2\pi \frac{\delta \nu}{K} . \] (12)

Thus it seen from (11) & (12) the phase error cannot be reduced by increasing the value of the loop gain (K), the highest value of which is set by the stability condition, (cf.(11)) namely, \( \frac{\pi}{2\tau} \) with \( \Omega = 0 \). In the following we will examine how the inclusion of the phase modulator helps to overcome this difficulty.

3. Stability of the Proposed OPLL

For the loop to act as a reference signal source, it must satisfy the condition of deterministic stability as well as stochastic stability. For deterministic stability, the loop parameters are to be fixed in such a way that the loop does not break into oscillations in a noise-free environment. Whereas stochastic stability means that the loop does not often steps out of synchronism due to random fluctuation.

a) Deterministic Stability

It is known that the introduction of a delay element in a phase locked loop modifies the behavior of the loop in many ways. Of them the most serious is the phenomenon of false or spurious locking, in which the loop slips into false locking instead of locking to the instantaneous phase of the reference signal. Nonlinear analysis of this mode of operation of a heterodyne PLL (i.e. when the phase error is large) is both time and space consuming. Moreover in this context, when the phase error is small, it is worthwhile to look into the stability of the linearized loop. We thus refer to the closed loop transfer function (H(s)) and rewrite it as

\[ H(s) = \frac{G(s)}{1 + G(s)} \] (13)

where G(s), the open loop gain of the PLL, is given by

\[ G(s) = KF(s)(1 + s\tau_p)\exp(-s\tau)/s . \] (14)

It is known that the system becomes unstable if the locus of G(j\omega) passes through or encloses the point (-1+j0) in the complex plane, the x and y-coordinates of the point respectively refer to the real and imaginary parts of G(j\omega). That is, for stability of the loop operation the following conditions need to be satisfied [8-10]

\[ \angle G(j\omega) = -\pi . \] (15)

and

\[ \left| G(j\omega) \right| = 1 . \] (16)

Therefore, referring to (14), (15), (16) and taking

\[ F(s) = (1 + s\tau_2)/s\tau_1 , \]

one finds that

\[ \tan^{-1}(\omega\tau_2) + \tan^{-1}(\omega\tau_p) = \omega\tau \] (17)

\[ K^2 \frac{[1 + \omega^2\tau_2^2][1 + \omega^2\tau_p^2]}{\omega^4\tau_1^2} < 1 \] (18)
Taking $\xi = 0.707$ and putting $\omega_n^2 = \frac{K}{\tau_1}$, $2\xi = \omega_n\tau_2$, it is easily shown that

$$\frac{(1 + 2z^2)(1 + z^2(\omega_n\tau_p)^2)}{z^4} < 1$$  \hspace{1cm} (19)

$$\tan^{-1}\left(\sqrt{2z}\right) + \tan^{-1}\left[\frac{z\omega_n\tau_p}{1 - 2(\omega_n\tau_p)^2}\right] = z(\omega_n\tau).$$  \hspace{1cm} (20)

where $z = (\omega/\omega_n)$.

For a standard second order loop with $\tau_p = 0$, the critical value of $\omega_n\tau$ for loop stability can be easily found by equating the left hand side of (18) to unity to find the value of $z$, which when substituted in (19) leads to the required value of $\omega_n\tau$. Thus,

$$z^4 = 2z^2 + 1$$

i.e. $z = \sqrt{1 + \sqrt{2}} = 1.54$

Using this value of $z$ in (20), one arrives at

$$\omega_n\tau = 0.736$$  \hspace{1cm} (21)

Now for the case when $\tau_p \neq 0$, we write (17) for the critical condition as

$$\frac{(1 + 2z^2)(1 + z^2(\omega_n\tau_p)^2)}{z^4} = z^4$$

i.e.

$$y^4 + y^2\left(2 + (\omega_n\tau_p)^2\right) - \left(1 - 2(\omega_n\tau_p)^2\right) = 0.$$

where $y = 1/z$.

From (21) and (22) one can easily calculate the value of $z$ for different values of $\omega_n\tau_p$, which on substitution in (20), leads to the required value of $\omega_n\tau$. The variation of the maximum value of $\omega_n\tau$ against $\omega_n\tau_p$ is shown in Fig.5.3.

Thus from (21) and Fig.5.3 it is concluded that about 14% increase in the value of $\omega_n\tau$ (i.e. 0.98 compared to 0.73) is possible with $\omega_n\tau_p = 0.4$. That is, for the same value of the loop natural frequency the loop can accommodate a larger value of the loop delay without throwing the loop into unstable operation.

b) Stochastic Stability

Note that although the PLL has been assumed to be working in the linear mode by virtue of the small rms error ($1^\circ$-$2^\circ$), the PLL slips cycles (i.e. the phase error of the PLL at times exceeds $\pi / 2$). This happens because the phase fluctuation is a random process, and thus there is always a certain probability that the phase error may exceed $\pi / 2$ radians. The exact analytical expression for evaluating the average time $T_{av}$ between such events cannot be found in this case of a second order PLL with delay. However, the exact expression is available for a delay less first order PLL [7-10]. And for a delay less second order PLL, certain experimental results indicate that the expression...
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for $T_w$ of a delay less first order loop can be applied, provided the corresponding expression for the loop bandwidth is used. The authors also feel that this concept can be applied to a loop with time delay. Thus we write

$$T_{av} = \frac{\pi \exp\left(\frac{2}{\sigma_\phi^2}\right)}{4B_n}$$

(23)

where

$$B_n = \int_0^\infty |H(j\nu)|^2 \, df$$

(24)

and

$$(\sigma_\phi)^2 = \frac{\Delta V}{\pi} \int_0^\infty \frac{|1 - H(j\nu)|^2}{f^2} \, df$$

(25)

Note that in writing (25) we have ignored the effect of shot noise. Putting $\frac{2\pi f}{\omega_n} = y$ in the expression for $H(s)$ we rewrite (24) and (25) as

$$B_n = f_n \int_0^\infty |H(jy)|^2 \, dy.$$  

(26)

and

$$\sigma_\phi^2 = \frac{2\Delta V}{\omega_n} \int_0^\infty \frac{|1 - H(jy)|^2}{y^2} \, dy.$$  

(27)

where

$$1 - H(jy) = -\frac{y^2}{y^2 + \left(1 + j\nu y\right)^2 \left(1 + j\omega_n\nu y\right) \exp(-jy\omega_n\nu)}.$$  

(28)

Replacing $\omega_n$ by the frequency $\omega_z$ at which the open loop gain of a second order OPLL assumes 0dB gain, i.e.,

$$\omega_n = \frac{\omega_z}{\sqrt{1 + \sqrt{2}}},$$  

(29)

and taking the value of $T_{av}$ as 10 yrs, a value sufficient to give reliable operation in a practical OPLL [11], we get,

$$\Delta V = \frac{4.0438 f_z}{\int_0^\infty \frac{1 - H(jy)\nu}{y^2} \left[19.37 + \ln\left(1 + j\nu y\right)^2 \nu dy\right]}.$$  

(30)

This is the maximum value of the summed line width for reliable operation of the OPLL. Note also that the Eqn. (30) gives the maximum value of the summed line width to achieve the desired receiver performance.

The dependence of the line width on the loop delay and how it can be controlled through a phase modulator are shown in Fig.4 and Fig.5. From these figures it is easily appreciated, the proposed system can be easily built with the help of commercial lasers with line widths ranging between 5 MHz and 50 MHz.

**CONCLUSION**

The modified OPLL, as suggested, considerably improve the performance of the microwave signal generation technique over the earlier technique [2] with the help of commercially available DFB lasers with linewidth of about 4.0 MHz instead of 80 KHz of [2]. This modified OPLL seems a realistic approach for generating reference signal
for future advanced communication systems with active phased array antennas. This opto-electronic technique provides the advantage of low weight, small size, flexible, immunity to EMI and above low cost because the proposed technique significantly reduces the requirement of narrow linewidth lasers. But it needs precise control of ambient temperature because typically the frequency change coefficient is around 3.5 MHz/C°. Moreover, it has been observed that bulk and surface acoustic waves in the power and frequency ranges of a few mW and KHz respectively are capable of modifying both the cavity length and refractive index of the material of semiconductor lasers. These changes have negligible effects on the intensity of laser output but causes frequency modulation of the optical carrier frequency [12]. Thus mechanical shock and vibrations may affect the preference.

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