Available online at www.scholarsresearchlibrary.com

Scholars Research Library

Archives of Physics Research, 2014, 5 (5):23-30
(http://scholarsresearchlibrary.com/archive.html)

ISSN : 0976-0970
CODEN (USA): APRRC7

Numerical and analytical calculations of efficiency and calibration factor for CR-39 detectors in the chamber diffusion by using Monte-Carlo method and the mean critical angle

Ali Farhan Nadir, Abdul R. H. Subber* and Noori H. N. Al-Hashmi

Department of Physics, College of Education for Pure Science, University of Basrah, Basrah, Iraq

ABSTRACT

Numerical calculation of efficiency and calibration factors for CR39 detectors, in the detection of alpha particle emitted by radon gas and its progenies. A software was employed for the latest version of the Monte Carlo code together with SRIM2013. The results showed that, the initial energy influence is one of the important parameters for the alpha detectors. Three methods have been used to calculate the calibration factor for CR-39 detector, and it is found that, they are consistent. The optimum radius of the cylindrical can is 3.5 cm for 1.5 cm x 1.5 cm CR39 detector. It was found that the calibration factors calculated by mean critical angle and Monte-Carlo methods are very closed, for both radon and thoron.

Keywords: CR-39, calibration factor, detection efficiency, Monte Carlo, SRIM2013

INTRODUCTION

Solid State Nuclear Track Detectors SSNTDs are extensively used in environmental science and technology. Polyallyl diglycol carbonate, $C_{12}H_{18}O_7$, (CR-39) and made with cellulose nitrate materials (LR-115 II) track detectors are the most sensitive and popular detector for recording charge particles and neutrons [1-4]. Therefore, several studies have been carried out to determine the main factors which affect the sensitivity, etching conditions and the properties of the CR-39 and LR-115 polymer as a track detector [5-6]. The passing of alpha particles through CR-39 sample causes ionization of almost all molecules which close to its path in a cylindrical zone. A zone enriched with free radicals and other chemical species is created along the path of the alpha particle. This damage zone is called a latent track [7]. The latent track created by charge particles, can be seen through the chemical etching in the material surface by using an acid or base solution with a certain normality [8]. The optimum etching conditions for CR-39 detectors were 6.25N, NaOH solution at 70^0C for 7h [9]. In the chemical etching, the route along the particle trajectory, track etch rate (V_T), is faster than the rate of etching on the undamaged surface, bulk etch rate (V_b). A pit is formed in the position of each track with etching progress [10].

By using the SSNTDs there are many ways to measure and calculate the radon gas concentration, some of these methods are: 1- Calculation of detection probability. 2- Calculation of the average etching critical angle. 3- Calculation of the calibration factor using spherical coordinates. 4- Calculation of calibration factor using the equilibrium factor of radon and its progenies. 5- Measurement of the sensitivity factor. 6- Calculation of efficiency of the detectors in response to alpha particle [11-14].

Any device used for relative measurement should be calibrated to convert its reading the value of measurements. The response of any radon dosimeter is the calibration factor, in track density per unit integrated concentration (Track.cm^{-2}/Bq.m^{-3}.d) or (cm) in dimensions of length, where 1cm=0.0864 Track.cm^{-2}/Bq.m^{-3}.d. The calibration factor can be used to determine the radon concentration, mass and a area exhalation rate, effective radium content, radon diffusion coefficient and its diffusion length. The calibration factor depends, not only on the geometry of the used configuration (filter and bare) but also, on many other parameters such as type of detector, detector efficiency and the dimension of dosimeter [15-16]. The Mont Carlo approach is more convenient than the analytical one when real physical dimensions of the detector are considered. The Mont Carlo simulation also enables determination of radial track density distribution on detector at the bottom of a diffusion chamber [16].

The aim of the present work is to estimate the CR-39 detector efficiency, mean critical angle and the sensitivity factor by focusing on modeling and the response of the detector to alpha particle using Monte Carlo Method with the latest version of SRIM013 programs. All results were compared with each other to focus on the optimum conditions required to increase the efficiency of the CR-39 detector.

THEORETICAL WORK

1. MEAN CRITICAL ANGLE

One of the basic definitions of the efficiency is the probability of detection of charge particle by the detector or it is the ratio between the solid angle Ω of the incident particle to the total solid angle 4π.

Let we assumed that a detector with the area A_D fixed on the surface of a sphere and there is a radioactive source positioned in the center of the sphere, using spherical coordinate [17];

$$dP = \frac{dA}{4\pi r^2} = \frac{r^2 \mathrm{Sin}\theta \, d\theta \, d\varphi}{4\pi r^2} = \frac{\sin\theta \, d\theta \, d\varphi}{4\pi} \tag{1}$$

$$\therefore P = \frac{1}{2}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta \qquad \theta_1 \le \theta \le \theta_2 \;\;, \quad 0 \le \varphi \le 2\pi \tag{2}$$

According to the definition of the critical angle; $0 \le \theta \le \theta_c$, we have;

$$P = \tfrac{1}{2}\int_0^{\theta_c} \sin\theta \, d\theta = \tfrac{1}{2}(1-\cos\theta_c) \tag{3}$$

Or, in other words

$$\varepsilon = \frac{1}{2}(1-\cos\theta_c) \tag{4}$$

where θ_c is the critical angle. However, some researchers used other relation of efficiency like [18];

$$\varepsilon = \tfrac{1}{2}(1-\sin\theta_c^*) \tag{5}$$

$$\text{where } \theta_c^* = \tfrac{\pi}{2} - \theta c \tag{6}$$

The detector efficiency (ε) depends on the critical angle (θ_c) for track registration [19].

1.1. CALCULATION OF THE MEAN CRITICAL ANGLE

To calculate the mean critical angle; one needs to find an analytical expression for the relation of critical angle with alpha particle energy or range. This energy can expressed in term of the range of the particle inside the detector, CR39 in the present case. It is found the polynomial of the range versus energy, the most suitable polynomial which describe this case is[20]:

$$R_D(E) = b_o + b_1 E_{Res} + b_2 E_{Res}^2 + b_3 E_{Res}^3 + b_4 E_{Res}^4 + b_5 E_{Res}^5 \quad \text{For CR} - 39 \tag{7}$$

where R (in mm) is the range of alpha particles , E_{Res}(MeV) is an alpha particle residual energy in the range (0.1 to 10 MeV), b_i (i=0...5) are fitting parameters and their values are listed in Table (1).

Table1. The best fit parameters for b_n

b_o	b_1	b_2	b_3	b_4	b_5
$9.51342*10^{-4}$	0.00396	$2.69929*10^{-4}$	$1.37928*10^{-4}$	$-1.42234*10^{5}$	$5.04952*10^{-7}$

The response function $V(R_D)$ (track etching ratio) for a certain value of R_D , can be adopted from [21];

$$V_{CR} = 11.6 \, R_D^{-0.464} \quad \text{for CR} - 39 \tag{8}$$

It follows the relation between the response function and the critical angle written as;

$$\cos\theta_c(x) = \frac{1}{V_{CR}} \quad (9)$$

$$\theta_c = \cos^{-1}\left(\frac{1}{V(R_D)}\right) \quad (10)$$

According to the above information, one can create a drawing between θc and $E_{RES.}$ and introduced fitting to curve as shown in Figure (1), in order to create an analytical relation between these parameters, the fitting polynomial is written as;

$$\theta_c(E_{Res}) = a_o + a_1E_{Res} + a_2E_{Res}^2 + a_3E_{Res}^3 + a_4E_{Res}^4 + a_5E_{Res}^5 + a_6E_{Res}^6 + a_7E_{Res}^7 + a_8E_{Res}^8 + a_9E_{Res}^9 \quad \text{for CR} - 39 \quad (11)$$

Table 2. The best fitting parameters a_n

a_5	-0.15045
a_6	0.02089
a_7	-0.00178
a_8	$8.46819*10^{-5}$
a_9	$-1.72089*10^{-6}$

a_0	85.09436
a_1	-8.17899
a_2	3.77285
a_3	-2.0352
a_4	0.69029

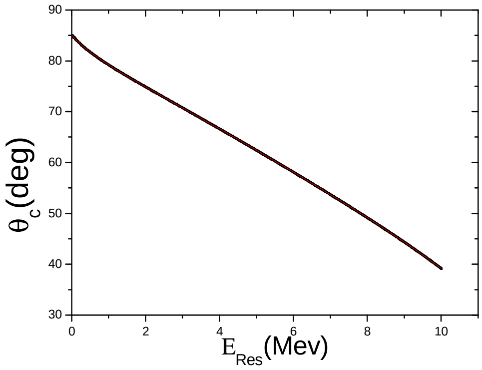

Figure 1. Variation of the CR-39 critical angle(θ_c) versus the residual energy of alpha-particles.

The values of best fitting parameters are listed in Table (2).

The mean value of the critical angle for CR-39 detector can be adobted from [22];

$$<\theta_c>= \frac{1}{E_i}\int_0^{E_i} \theta_c(E_{Res})\, dE_{Res} \quad \text{for CR} - 39 \quad (12)$$

where E_i the initial energy of alpha particle.

This equation can transform to polynomial form and written as;

$$<\theta_c>= a_o + \frac{1}{2}a_1E_i + \frac{1}{3}a_2E_i^2 + \frac{1}{4}a_3E_i^3 + \frac{1}{5}a_4E_i^4 + \frac{1}{6}a_5E_i^5 + \frac{1}{7}a_6E_i^6 + \frac{1}{8}a_7i + \frac{1}{9}a_8E_i^8 + \frac{1}{10}a_9E_i^9 \quad (13)$$

The best fitting parameters are listed in table (2).

Table 3. The radon gas parameters

Nuclide	$E_{\alpha_i}(Mev)$	$<\theta_c^o>$	ε_i
Radon group			
Rn^{222}	5.49	71.97	34.52
Po^{218}	6.00	70.97	33.63
Po^{214}	7.68	67.33	30.73
Thoron group			
Rn^{222}	6.28	70.30	33.14
Po^{216}	6.78	69.24	32.28
Bi^{212}	6.08	70.72	33.49
Po^{212}	8.78	64.98	28.85

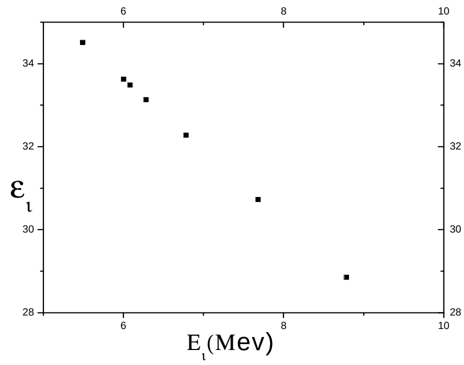

Figure 2. The variation of the CR-39 detecto efficiency with the alpha Partic initial energy using the mean critical angle calculation

Table (3) contains radon gas calculated parameters; radon group energies and thoron group energies and the mean values of θ_c together with the calculated values of the efficiency. From Figure (2), we can see that the efficiency of the detector decreases as the energy of the charge particle increases.

2. ANOTHER DEFINITION OF EFFICIENCY

The detector efficiency defined as the ratio between number particles reach the surface of the detector and create latent tracks on the detector body (N_d), to the total number of charged particles emitted by the source (N_o) [23].

$$\varepsilon = \frac{N_d}{N_o} \tag{14}$$

In the case of more than one detector in the dosimeter, the equation becomes;

$$\varepsilon = \frac{N_d}{\frac{N_o}{i}} \tag{15}$$

Where i is the number of detectors inside irradiation chamber.

An accurate and precise calibration of detection efficiency is very important for quantitative measurements.

3. MONT-CARLO METHOD

Whenever you need to make an estimate, radon exposure rate, alpha particle probability, or decision where there is significant uncertainty in your results, you'd be well advised to consider Monte Carlo simulation. Monte Carlo methods are a broad class of computational algorithms that relies on repeated random sampling to obtain numerical results; typically one runs simulations many times over in order to obtain the distribution of an unknown probabilistic entity. Random number generators are used in computer simulations, which are small programs which generate random numbers as outputs by using some suitable algorithms. The creation of randomly distributed number can be done by Fortran language (CALL RANDUM_NUMBER (RN)), where RN is random number $0 \leq RN < 1$.

Since alpha decays are a statistical application connected to the probability of decay for a certain nucleus, which mean it is a random distribution, or in another one can apply the Monte-Carlo simulation of such operation [24].

Let us assume that alpha particle hit the detector by polar angle θ and from random directions presented by the solid angle φ, and according to this assumption, the application of Monte-Carlo simulation depends on; a- The effective volume in front of the detector, which is fully distributed with alpha particles. b- The coordinates of the point. c- Number of detectors present.

In the case of circular shape detector and the effective volume of the container is a cylinder, (which is appreciable in this case), with length R_i and radius q, which is equal to the radius of the detector, Figure 3.

Any point P inside the effective volume has a coordinates P(r,t,θ,φ), where r , t represent the distances of the point P from cylinder axis and the detector respectively, and θ,φ are the coordinates of the incident angle of alpha particles. The point P(r,t,θ,φ) is falling inside the effective volume if it follows the conditions;

$$\left.\begin{array}{ll} r = q\sqrt[2]{RN_1} & 0 \leq RN_1 < 1 \\ t = R_i RN_2 & 0 \leq RN_2 < 1 \\ \mathrm{COS}\,\theta = RN_3 & 0 \leq RN_3 < 1 \\ \varphi = 2\pi RN_4 & 0 \leq RN_4 < 1 \end{array}\right\} \qquad (16)$$

Where NR_1, NR2,NR3,NR4 are random numbers.

Let, we will have N points inside the effective volume, having coordinates $P(r_j, t_j, \theta_j, \varphi_j)$. For large values of N, for more precise results these points will consider as emission points of alpha particle inside the effective volume.

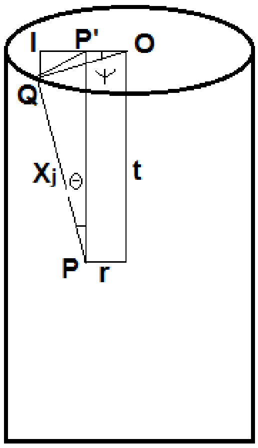

Figure 3 The cylendrical effective volume

After creations of these points inside the volume**,** we can apply certain rules that these points should obey in order to be sure that any emit alpha particle will reach the detector and record a latent track on it (remember that, some of particles reach the detector, but do not record any track due to the etching condition). From Figure 3, we can write;

$$\left.\begin{array}{l} x_j = \frac{t_j}{\mathrm{COS}\,\theta_j} \\ \theta_j = \mathrm{COS}^{-1}(RN_3) \end{array}\right\} \qquad (17)$$

where x_j is the distance that alpha particle travel from the point $P(r_j, t_j, \theta_j, \varphi_j)$ to the detector surface. So the conditions are;

$$\left.\begin{array}{ll} \textit{1-} & q > \overleftrightarrow{QO} \\ \textit{2-} & x_i < R_i \\ \textit{3-} & \theta_j < \theta_c \\ \textit{4-} & R_D^j \cos\theta_j > h \\ \textit{5-} & E_{Res} < E_i \end{array}\right\} \qquad (18)$$

where

$$\overleftrightarrow{QO} = \sqrt[2]{r^2 + t^2 + t^2\tan^2\theta + 2rt}\tan\theta\cos\varphi \quad (19)$$

$$E_{Res} = E_i - E_{x_j} \quad (20)$$

$\overleftrightarrow{QO}$ is the distance between the detector center O and the point Q, R_i is the maximum range of alpha particles in air (mm), θ_j is the incident angle of alpha particle on the detector surface, h is the thickness of the removed layer from the surface of the detector (µm) during the etching process, $E_{RES.}$ (MeV) is the residual energy of the alpha particle after distance X in the air, and E_i (MeV) is the initial energy of the particle emitted from the source.

The above conditions need to: a- describe the range of alpha particle in the air by the polynomial [20];

$$R(E) = c_o + c_1E + c_2E^2 + c_3E^3 + c_4E^4 + c_5E^5 \quad \text{for air} \quad (21)$$

Table 4. The best fit parameters a_n

c_o	c_1	c_2	c_3	c_4	c_5
0.7163	2.92139	0.15825	0.10016	-0.01017	$3.37885*10^{-4}$

b- Describe the energy consumed by alpha particle in travelling a distance x_j in the air, by the polynomial;

$$E(x) = a_o^* + a_1^*x + a_2^*x^2 + a_3^*x^3 + a_4^*x^4 + a_5^*x^5 \quad (22)$$

Table 5. The best fit parameters a_n^*

a_o^*	a_1^*	a_2^*	a_3^*	a_4^*	a_5^*
-0.20964	0.34209	-0.00932	$1.92765*10^{-4}$	$-2.05317*10^{-6}$	$8.5793*10^{-9}$

C- Calculate the residual energy from;

$$E_{Res} = E_i - E_{x_j}$$

d- Calculate R_D; the range of alpha particles in the detector
e- Calculate the mean etching critical angle $<\theta_c>$

RESULTS AND DISCUSSION

Tables (6,7,8,9) show the detector efficiency for radon and thoron and their progenies calculated by Monte-Carlo method, and Figure (4) shows the behavior of the efficiency with energy of alpha particles. We can see that the behavior of the efficiency is the same for both methods of calculations (Monte-Carlo and Mean Critical Angle). The efficiency increases linearly with the detector diameter D. Combine the methods for calculations of efficiency, one can see that; the efficiencies are almost equal if we used detector radius 3cm as shown in Figure 5, which is the range of alpha particle emitted from radon in air [20].

Table 6. The detector efficiency

$D = 1.5\ cm \quad h = 0\ \mu m$		
Nuclide	$E_{\alpha_i}(Mev)$	ε_i
Radon group		
Rn^{222}	5.49	15.88
Po^{218}	6.00	14.09
Po^{214}	7.68	9.82
Thoron group		
Rn^{222}	6.28	13.21
Po^{216}	6.78	11.82
Bi^{212}	6.08	13.83
Po^{212}	8.78	7.98

Table 7. The detector efficiency

$D = 1.7\ cm \quad h = 0\ \mu m$		
Nuclide	$E_{\alpha_i}(Mev)$	ε_i
Radon group		
Rn^{222}	5.49	26.64
Po^{218}	6.00	24.42
Po^{214}	7.68	18.03
Thoron group		
Rn^{222}	6.28	23.22
Po^{216}	6.78	21.18
Bi^{212}	6.08	24.07
Po^{212}	8.78	14.95

Table 8. The detector efficiency

$D = 3\ cm \quad h = 0\ \mu m$		
Nuclide	$E_{\alpha_i}(Mev)$	ε_i
Radon group		
Rn^{222}	5.49	26.64
Po^{218}	6.00	24.42
Po^{214}	7.68	18.03
Thoron group		
Rn^{222}	6.28	23.22
Po^{216}	6.78	21.18
Bi^{212}	6.08	24.07
Po^{212}	8.78	14.95

Table 9. The detector efficiency

$D = 6\ cm \quad h = 0\ \mu m$		
Nuclide	$E_{\alpha_i}(Mev)$	ε_i
Radon group		
Rn^{222}	5.49	35.15
Po^{218}	6.00	33.59
Po^{214}	7.68	28.21
Thoron group		
Rn^{222}	6.28	32.73
Po^{216}	6.78	31.28
Bi^{212}	6.08	33.35
Po^{212}	8.78	24.75

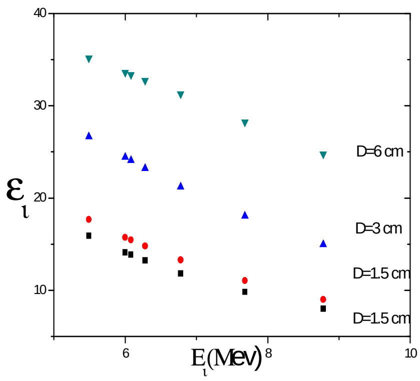

Figure 4. variation of the CR-39 detector efficiency function of the alpha particles initial energy using Monte Carlo.

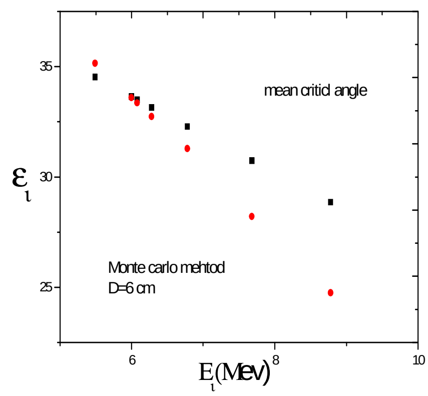

Figure 5. variation of the CR-39 detector efficiency as function of the alpha particles initial energy using Monte Carlo the mean critical angle.

4.1 THE RELATION BETWEEN THE CALIBRATION FACTOR AND THE EFFICIENCY

The mathematical and physical relation connects the efficiency and the calibration factor is written as .

$$K = \varepsilon R_\alpha \qquad (23)$$

where R_α is the range of alpha particle in the air, K is the calibration factor measured by unit length.

The calculated values of the calibration factor using the two methods are present in Table (10). It is obvious from the table that, the values are nearly the same when the detector diameter equal to 6cm. The values of touring group are always higher than that for reading group and this is in consistence with the nature of thoron. Means; the radioactivity concentration of thoron group always less than that of radon group, due to the efficiency of thoron group is less than radon. Initial energies of thron group always larger than radon group.

Table 10. The calibration factor

Nuclide	$K_{\theta_c}\left(\frac{Track/cm^2}{Bq\,m^{-3}}\right)$	$K_{Monte}\left(\frac{Track/cm^2}{Bq\,m^{-3}}\right)$ $D = 6cm$	$K_{Monte}\left(\frac{Track/cm^2}{Bq\,m^{-3}}\right)$ $D = 3cm$	$K_{Monte}\left(\frac{Track/cm^2}{Bq\,m^{-3}}\right)$ $D = 1.7cm$	$K_{Monte}\left(\frac{Track/cm^2}{Bq\,m^{-3}}\right)$ $D = 1.5cm$
Radon group	0.33	0.32	0.22	0.14	0.13
Thoron group	0.49	0.46	0.31	0.19	0.17

The values of K measured by many researchers using different experimental methods and criteria were in the rages 0.1 to 0.4 $Tr.cm^{-2}/(Bq.m^{-3}.d)$ for CR-39 detector [23].

However, the calculation of the calibration factor for CR39 detector using the analytical relation [24] ;

$$K = \frac{1}{4}\ r\left(2\sin\theta_c - \frac{r}{R_\alpha}\right) \qquad \text{For} \quad r < R_\alpha \qquad (24)$$

$$K = \frac{1}{4}\ R_\alpha \quad \sin^2\theta_c \qquad \text{For} \quad r \geq R_\alpha \qquad (25)$$

Where r is the radius of the can (chamber diffusion)

Table 11. The calibration factor

Nuclide	$K\left(\frac{Track/cm^2}{Bq\,m^{-3}\,d}\right)$, r=3.5 cm
Radon group	0.22
Thoron group	0.32

Many experimental works to measure the calibration factor were performed and different values have been given, some

CONCLUSION

In the present work we calculate the efficiency and the calibration factor and it is found that;
1- The efficiency of the detector depends on the mean critical angle, which is depends on the type of the detector and alpha particle energy
2- The efficiency depends on the response function $V(R_D)$
3- The optimum diameter of emanation can be 7 cm to 3 cm x 3 cm, CR39 detector.

Acknowledgment
The authors would like to thanks physics department and the college of education for pure sciences for their financial support and encouragement.

REFERENCES

[1] N. Dwaikat, G. Safarini, M. El-Hasan and T. Iida, *Nuclear instrument and Methods A*, **2007**, 574, 289-291.
[2] Ji.Young Lee and J. Heung jo, *J. of the Korean Physical Society*, **2007**, 51(1), 426-430.
[3] A. Kulkarni, Vyas, CK H Kim, PC Kalsi, T Kim and V. Manchanda, *Sensors and Actuators B*, **2012**, 161, 697-701.
[4] M. R. Rezaie, M. Sohrabi and A. Negarestani , *Radiat. Meas.* **2013**, 50, 103-108.
[5] A. S. Alghamdi and K. A. Aleisa, *Radiation Measurements*,**2014**, 62, 35-40.
[6] A. F. Saad, R. M. Abdallah and N. A. Hussein, *Applied Radiation isotopes*,**2013**,72, 163-168
[7] R. Schweikani,SA. Durrani and T. Tsuruta, *Nucl. Tracks Radiat.Meas*, **1993**,22, 153-156.
[8] D. Nikezic and KN. Yu, *Materials Science and Engineering.* **2004**, R46, 51.
[9] N. F. Salih and M. S. Jaafar M. S, *International journal of science and engineering Research*,**2012**, 3(9), 1-8.
[10] S. A. Durrani and R.K. Bull, *Nuclear Track Detection: Principles, Methods and Applications*, Pergamon press, Oxford,**1987**, 52-54.
[11] D. Nikezic,K. n. Yu and J. M. Stajic, *Review of Scientific Instruments*, **2014**, 85, 022102,
[12] T. Siiskonen and R. Pollanen, *arXiv:nucl-ex/*, **2005**,0505005V1.
[13] M. R. Deevband, P. Abdolmaleki,M. R. Kardan,H. R. Khosravi, M. Taheri, F. azeri and N. Ahmadi, *Iran J. Radia. Res.*,**2011**, 9(2), 95-102.
[14] D. Nikolopoulos, S. Kottou, E. Petraki, E. Vogiannis and H. Yannakopoulos, *J. Phys. Chem. Biophys*., **2014**,4(2), 200-211.
[15] M. Mansy , M. A. Shafa, H. M. Eissa, S. U. El-Kamees, M. Abo-Elmaged, *Radiation Measurements*, **2006**, 41, 222-228.
[16] D.Nikezic and K.N. Yu , and J. M. Stajic, *Review of Scientific Instruments*,**2014**, 85,022102.
[17] M. A. Misdaq, S. Berrazzouk, A. Elharti, F. Ait Nouh, W.Bourzik, *Journal of Radioanlytical and Nuclear Chemistry*,**2000**, 246(2), 395-401.
[18] K.N. Yu, D.Nikezic , F.M.F. Ng, J.K.C.Leung, *Radiation measurements*, 2005,40, 560-568.
[19] Murtadha S. Al-Nafiey, Mohamad S. Jaafar, Sabar Bin Bauk, N.F. Salih, *International Journal of Scientific & Engineering Research*,**2012**, 3(10).
[20] Ali Farhan Nader; Abdul R.H. Subber; Noori.H.N. Al-Hashimi, *IOSR Journal of Engineering (IOSRJEN)*,**2014**, 4(9), 51-55 .
[21] M. A. Misdaq and C. Satif, *Jointly published by Elsevier Science S. A., Lausanne and AkademiaiKiado, Budapest*,**1996**, 207(1), 107-116.
[22] M. A. Misdaq, A. Bakhchi, A. Ktata, A. Merzouki, N. Youbi, *Applied Radiation and Isotopes*,**1999**, 51, 209-215.
[23] M. A. Misdaq , F. Aitnouh, H. Khajmi, H. Ezzahery, S. Berrazzouk, *Applied Radiation and Isotopes*,**2001**, 55,205-213.
[24] D.Nikezic and K.N. Yu, *Computer Physics Research Trends*, **2007**, ISBN:978-160021-595-7.