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European Journal of Applied Engineering and Scientific Research, 2015, 4 (2):1-9
(http://scholarsresearchlibrary.com/archive.html)

ISSN: 2278 – 0041
CODEN(USA): EJAEAJ)

Numerical solution of convective and radiation heat transfer from a vertical rectangular fin

Akeel Mohammed Ali Morad

Basra Technical College, Refrigeration and Air-conditioning Eng. Dept. ,Basra City, Basra - Ashar, Iraq

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ABSTRACT

The present work presents a numerical solution of combined convective and radiation heat transfer from a vertical rectangular air-cooled aluminum fin. There are three governing equations (Continuity, Momentum and Energy equation) are solved numerically by using (TDMA) for fluid and the fin. Heat transfer by both mixed convection and radiation is considered. Mixed convection effect should be appreciable for low speed air flow over the fin. Radiation heat transfer mode is important for large temperature difference between the fin and the surrounding as well as for high emissivity fin material and low speed air-flow. Conjugate heat transfer should be taken into account as it gives rise to non-uniform heat transfer coefficient the consideration of which is more realistic in contrast with the assumption of uniform heat transfer coefficient in conventional fin theory. Results shown at constant of Re with large value for CCP means increased convective cooling and the fin requires shorter time to attain steady stat temperature.

Key words: Heat Transfer, air-cold fin, vertical

Nomenclature

Greek Symbols

b	Thickness	c	Specific heat
CCP	Convection-conduction parameter ($\sqrt{Re}\, k_f\, L / k_s\, b$)	g	Acceleration due to gravity
Gr	Grashof number ($g\, \beta\, (T_w - T_\infty)\, L^3 / v^2$)	h_c	Convection heat transfer coefficient
h_r	Radiation heat transfer coefficient	h	Total heat transfer coefficient ($=h_c + h_r$)
k	Thermal conductivity	k_s	Thermal conductivity of fin material
K_f	Thermal conductivity of the fluid	L	Length of the fin
Pr	Prandtl number ($= v / \alpha$)	q''_w	Local heat flux from one surface of the fin ($= h(T_w - T_\infty)$)
Re	Reynolds number ($= u_\infty\, L / v$)	T	Temperature
t	Time	u	Velocity of the fluid in x-direction
U	Non-dimensional velocity in x-direction ($=u/ u_\infty$)	υ	Velocity of the fluid in y-direction
u_∞	Free stream velocity	V	Non-dimensional velocity in y-direction ($=\upsilon / u_\infty$)
x	Coordinate along the length of the fin	X	Non-dimensional x-coordinate (=x/L)
y	Coordinate perpendicular to the fin surface	Y	Non-dimensional y-coordinate (=y/L)
ΔX	Grid size in X-direction	ΔY	Grid size in Y-direction
α	Thermal diffusivity ($= k/\rho\, c$)	β	Coefficient of volumetric thermal expansion [$- 1/ \rho \quad (\delta \rho / \delta T)_p$
ε	Emissivity of the fin material	υ	Kinematics viscosity
ρ	Density	σ	Stefan-Boltzman constant, $5.67 * 10^{-8}$ $w/m^2\, k^4$
θ	Non-dimensional temperature ($= T - T_\infty / T_b - T_\infty$)	τ	Dimensionless time ($\alpha\, t / L^2$)
Δ τ	Non-dimensional time-step		

Subscripts

b	Base of the fin	c	Convection
f	Fluid	p	Constant pressure
r	Radiation	s	Fin material
w	Wall or fin surface	∞	Ambient

INTRODUCTION

Multi-mode heat transfer continues to be a fertile area of research due to its application in several fields. The most frequent application is one in which an extended surface has been used specifically to enhance the heat transfer rate between a solid and an adjoining fluid. Such an extended surface has been termed a fin. There are several fin applications. Consider the arrangement for cooling engine heads on motorcycles, automobiles, lawnmowers, air-conditioner, refrigerators, and for electric power transformers . Literature reports quite a few studies, numerically, which pertains to interaction between two modes of heat transfer, namely, convection and radiation. To mention a few, [1](Gorski & Plumb) investigated, numerically, the problem of conjugate heat transfer from an isolated heat source in vertical flat plate. Here, the problem was solved using well-known Blasius velocity profile for laminar force convection as the input. However, the transient mixed combined of convection and radiation concerning the geometry of the flat plate has been studied to a lesser degree. For example, [2](Hossain & Takhar) investigated, numerically, the effect of radiation on mixed convection from a vertical rectangular plate. In this context, [3](Rao,et all) have preformed exhaustive numerical studies on conjugate mixed convection with radiation from a vertical fin. [4](Rao, et all) numerical investigation into the fundamental problem of combine conduction-convection-surface radiation from a vertical plate with three identical flush-mounted discrete heat sources. Here resulting partial differential equations are first converted into algebraic form and solved using Guss-Seidel iterative technique. No studies are available in literature that provides a transient combined mixed convection and radiation from a vertical rectangular aluminum fin. The objectives of the present study are (i) to develop a good numerical procedure to handle transient conjugate heat transfer from vertical fin when radiation is taken into account and (ii) to see how important the radiation mode as compared to mixed convection is when the problem is solved for an aluminum fin.

2-Problem Formation

2-1 Physical Description

The fin and the coordinate system are depicted in Fig.1. The fin has a thickness 'b' and length 'L'. The tip of the fin is insulated. The fin is cooled by an air flow having free-stream velocity and temperature, u_∞ and T_∞ respectively. In the beginning, the fin is at the temperature of the surrounding, i.e., at T_∞. Then, suddenly the temperature of the base of the fin is raised to T_b ($T_b > T_\infty$) and thereafter maintained at that temperature.

The analysis is based on the following assumptions:

1. The density of the fluid (air) is constant except in the buoyancy force term where it is assumed to be a function of temperature. Other physical properties are constant.
2. The width of the fin is small and hence one-dimensional heat conduction can be taken.
3. A two-dimensional laminar boundary layer flow exists around the fin.
4. Gravity is the only acting body force.
5. Viscous dissipation and pressure work terms are neglected.
6. Axial conduction in the fluid is neglected as RePr is high.
7. The flow is not affected by the base of the fin.
8. The temperature difference between the fin and the ambient is not very high.

2-2.Governing Non-dimensional Differential Equations

The continuity equation, the x-momentum equation, the energy equation for the fluid and the energy equation for the fin are non-dimensionalised using dimensionless parameters (X, Y, U, V, τ and θ) and are shown as Eq. (1), Eq. (2), Eq. (3) and Eq.(4 a, b) respectively,**[5](Sunden)** as follows:

$$\frac{\delta U}{\delta X}+\frac{\delta V}{\delta Y}=0 \quad \textbf{(1)}$$

$$\frac{1}{\mathrm{Re\,Pr}}\frac{\delta U}{\delta\tau}+U\frac{\delta U}{\delta X}+V\frac{\delta U}{\delta Y}=\frac{1}{\mathrm{Re}}\frac{\delta^2 U}{\delta Y^2}+\frac{Gr}{\mathrm{Re}^2}\theta \qquad \textbf{(2)}$$

$$\frac{1}{\mathrm{Re\,Pr}}\frac{\delta\theta}{\delta\tau}+U\frac{\delta\theta}{\delta X}+V\frac{\delta\theta}{\delta Y}=\frac{1}{\mathrm{Re}}\frac{\delta^2\theta}{\delta Y^2} \qquad \textbf{(3)}$$

$$\frac{\delta\theta_W}{\delta\tau}-\frac{\delta^2\theta_W}{\delta X^2}=2\left[\frac{CCP}{\sqrt{\mathrm{Re}}}\right]\left[\frac{\delta\theta}{\delta Y}\right]_W \qquad \textbf{(4 a)}$$

$$\frac{\delta\theta_W}{\delta\tau}-\frac{\delta^2\theta}{\delta X^2}=-2\left[\frac{CCP}{\sqrt{\mathrm{Re}}}\right]\left[\frac{L}{k_f}\right](h_C+h_r)\theta_W \qquad \textbf{(4 b)}$$

Equation (4 a) is used for only mixed convection case while Eq. (4 b) is used for combined mixed convection and radiation. ' h_c' is the convection heat transfer coefficient and ' h_r' is the radiation heat transfer coefficient derived from the linearised radiation heat flux equation. ' h_r' and ' h_c' **[6](Hottel)** are written as:

$$h_r=\varepsilon\sigma(T_W^2+T_\infty^2)(T_W-T_\infty) \qquad (5\ a)$$

$$h_C=-\frac{k_f\,(\delta\theta/\delta Y)_W}{L\,\theta_W} \qquad (5\ b)$$

'CCP' is called convection-conduction parameter ($\sqrt{\mathrm{Re}}\ k_f\ L/\ k_s\ b$) . In Eqs (1) – (4 a , 4b), an important dimensionless parameters appear: Gr/Re^2 (ratio of buoyancy force to inertia force). Since in the present study air is a participating medium and the fin material, base temperature and ambient temperature are used as parameters, a radiation-convection-conduction parameter has not been introduced.

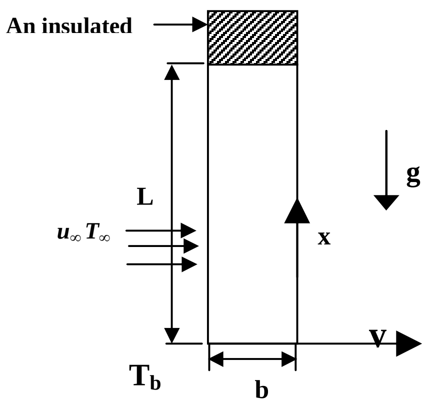

Fig. 1 Problem under consideration

Initial and Boundary Condition **in the dimensionless form [7](Patankar) are as follows:**

<u>At τ =0</u>

Y=0	: U=0 , V=0	(6a)
Y→∞	: U=1	(6b)
X=0	: U=1 , V=0	(6c)
θ_w =0	for all X	(6d)
θ = 0	for all X & Y	(6e)

at $\tau > 0$,

Y=0	: U = 0 , V= 0	(7a)
$Y \to \infty$	: U=1, θ = 0	(7b)
X =0	: U= 1 , V = 0	(7c)
X =0	$\delta \theta_w / \delta X = 0$	(7d)
X =0	: $\theta_w = 1$	(7e)

3- Method of Solution

3-1 Numerical Scheme

The governing differential equations are discredited using finite-difference scheme. A pure implicit method is used to march ahead in time. Implicit scheme is preferred to explicit scheme as the latter produces undesirable limitations on grid sizes and time step. The time step, $\Delta\tau$, the grid size Δ X and Δ Y have been taken as 0.02, 0.1 and 0.002 respectively. Maximum values of X and Y are 1.0 and 0.03 respectively. Uniform grid spacing have been used in X and Y directions. The grid size and time-step are chosen after having performed sufficient numerical experimentations. An 11*16 grid system has been used.

3-2 Algorithm

The mixed convection loop (see Fig. 2) computes convection heat transfer coefficient, ' h_c ' as a function of position and time which means a set of values of ' h_c ' as a function of temperature has been obtained. This is required to solve the fin heat conduction Eq. (4b) which includes radiation effect also. To solve Eq. (4b), the future fin temperature distribution is assumed first. This is required to calculate the radiation heat transfer coefficient, h_r . To make an educated guess, the future fin temperature distribution is assumed to be the one at the end of the time-step ($\tau + \Delta\tau$) for the case of only pure mixed convection. Eq. (4a) and Eq. (4b) are solved by Tri-diagonal Matrix Algorithm (TDMA). The algorithm, in the form of a flow chart is shown in Fig. (2).

The numerical results have been obtained using the data listed in table (1). The fin emissivity value at the base temperature T_b is used. All other properties are evaluated at the average of the base and the ambient temperature, i.e. at ($T_b + T_\infty$) /2. **[8](Anderson).**

Table 1. Input Data

Fin (Aluminum)	value
Length of the fin (L) Base temperature(T_b) Thermal conductivity(k_s) Emissivity (ε)	= 0.1 m = 773 k = 204 w/m-k = 0.049
Fluid (Air)	**value**
Prandtl number (Pr) Temperature(T_∞) Reynolds number (Re) Thermal conductivity(k_f) Kinematics viscosity (υ)	= 0.708 = 300 k = 5000 = 0.023 w/m.k =15.89E-6 m^2/s

RESULTS AND DISCUSSION

Figure (3) shows the rise of temperature vs time at X = 0.5 of the fin and for Gr/Re^2 = 2.0. The curves are shown for various convection-conduction parameters (CCP = 1, 5 , 10 and 15). It is seen that higher CCP values results in lower fin temperature and the fin requires shorter time to attain steady state temperature. This is because higher CCP means increased convective cooling or increased insulation. In the present case, since Re, k_f , k_s ,L are constants, therefore for larger values of CCP, only the thickness 'b' can be lower implying greater surface area per unit thickness of the fin which results in higher convective cooling. Figure (4) shows the transient temperature of the fin at X = 0.5 for different buoyancy parameters (Gr/Re^2 = 2.0 and 10) and for CCP = 5.0. For higher Gr/Re^2 , the temperature of the fin is less and the amount of time the fin takes to reach steady state is also lower. This is because for higher Gr/Re^2 , the contribution of the natural convection part towards the cooling of the fin increases. Figure (5) steady state fin temperature profile for pure forced convection i.e. Gr/Re^2 = 0 at different CCPs are presented. This is because when Gr/Re^2 is positive, the buoyancy force assisted by the main forced flow increases the mixed convection heat transfer resulting in lower fin temperature.

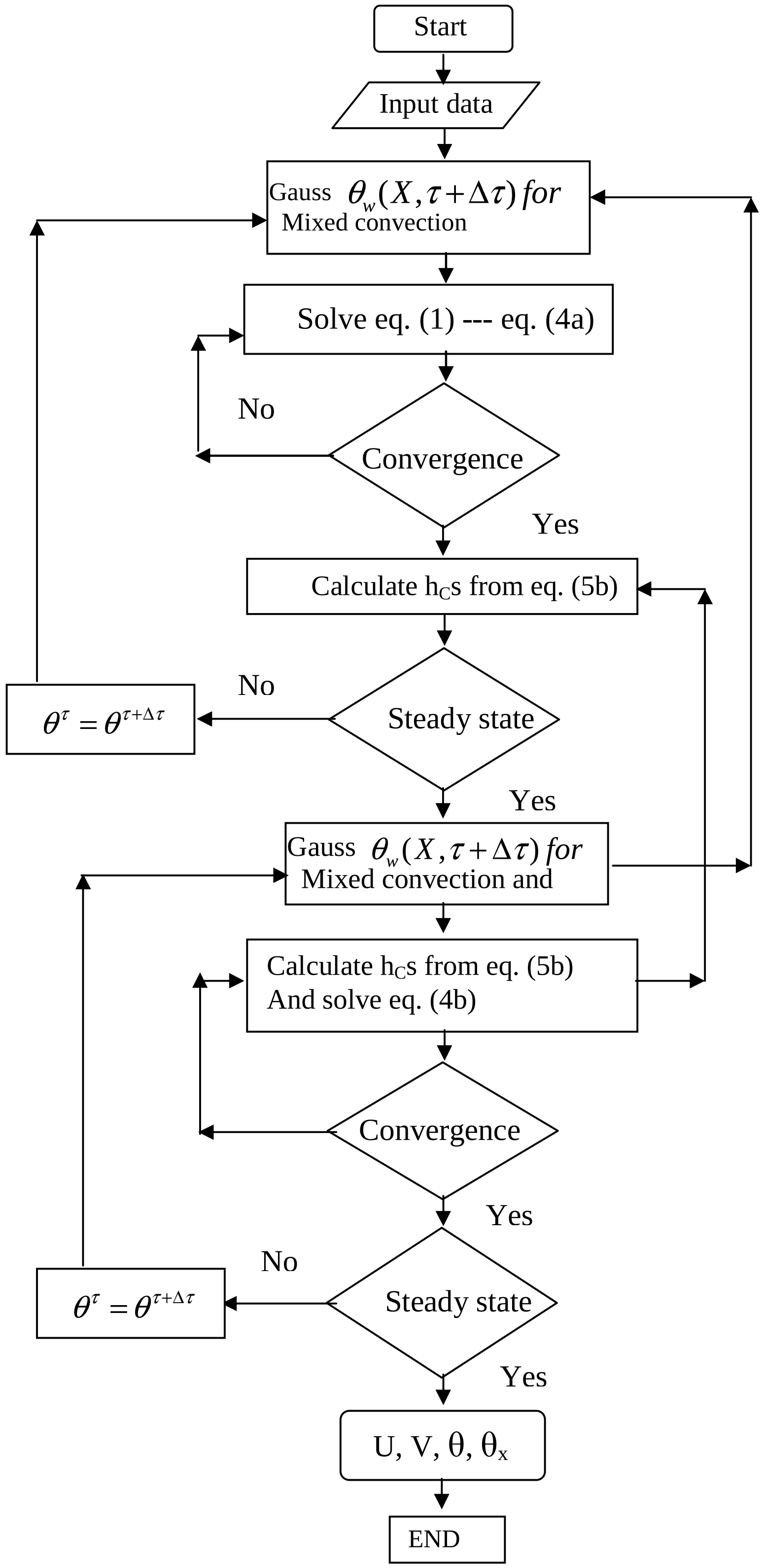

Fig. 2 Flow chart of the solution procedure

Figure (6) indicates vertical velocity profiles of the fluid in the steady state for Gr/Re^2 = 0, 2 and 10. The velocities are higher for greater Gr/Re^2 as increased Gr/Re^2 means buoyancy effect is more. For Gr/Re^2 = 10, the velocity profile reaches a peak (U = 1.15) and then decreases gradually to U = 1.0, the reason being the flow is aided much more by the buoyancy force in comparison with the case of Gr/Re^2 = 2.0.

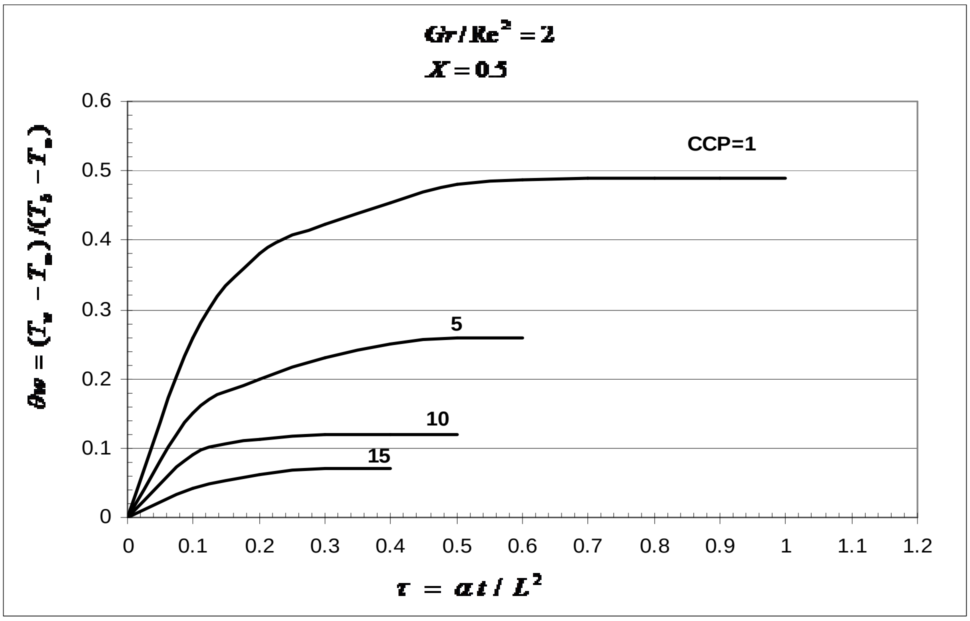

Fig. 3, Temperature distribution vs time plots at X=0.5 of the fin for various CCP values

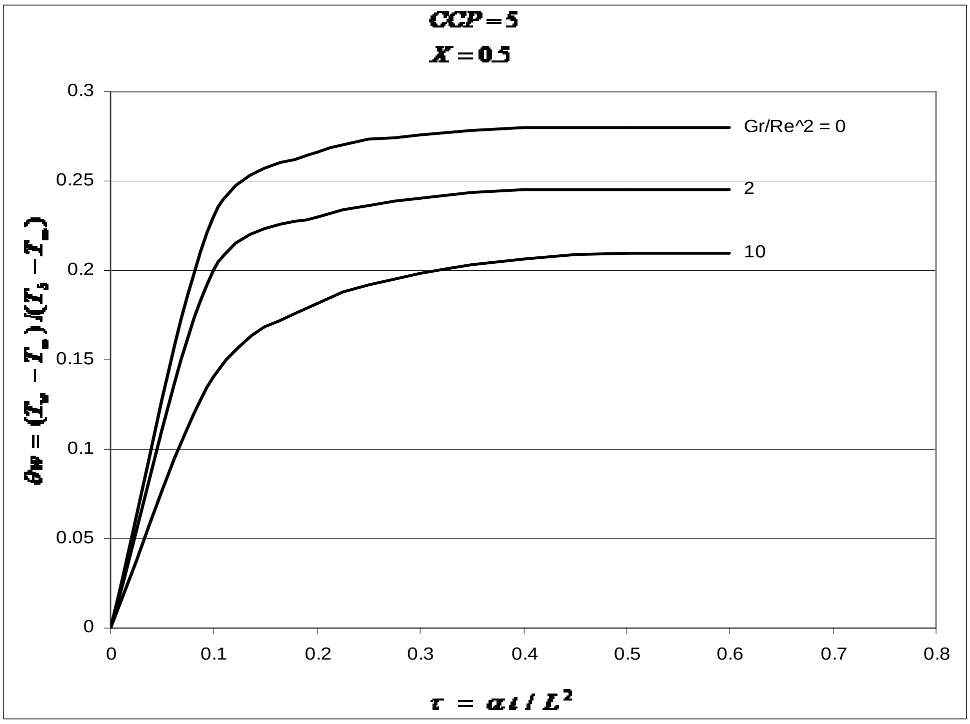

Fig. 4, Temperature distribution vs time plots at X=0.5 of the fin different Gr/Re^2 values

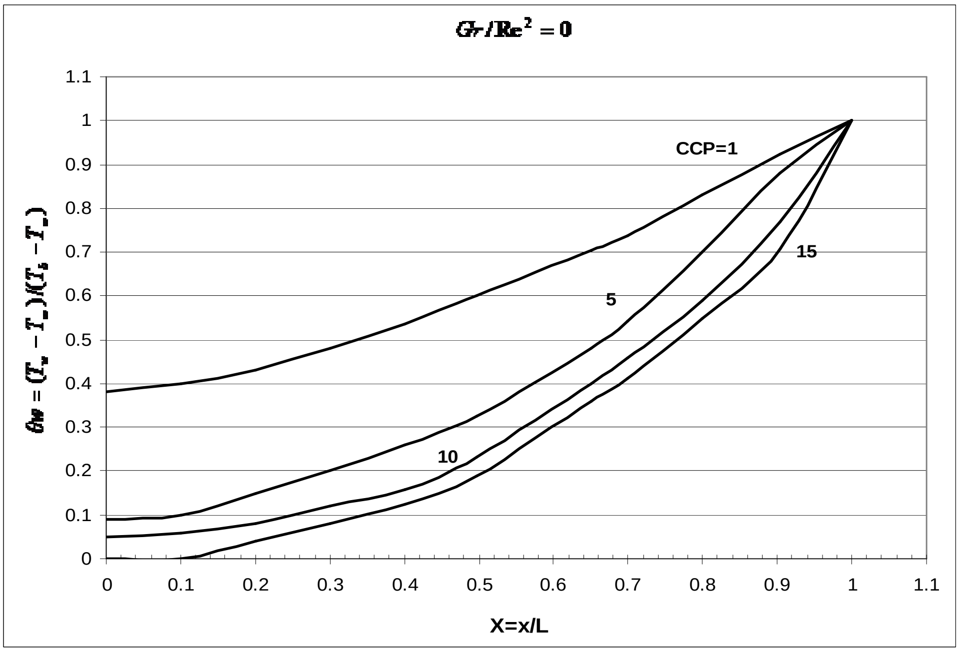

Fig. 5, Steady state of the fin temperature distribution at different CCP values.

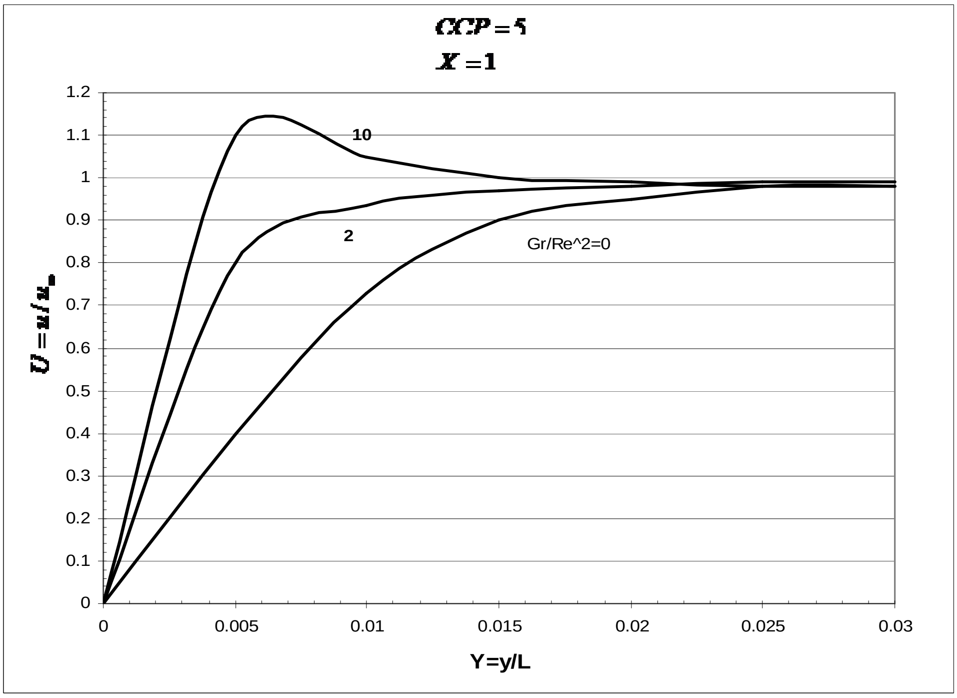

Fig. 6, vertical velocity profiles of the fluid for steady state at X=1 along Y of fin with various Gr/Re^2

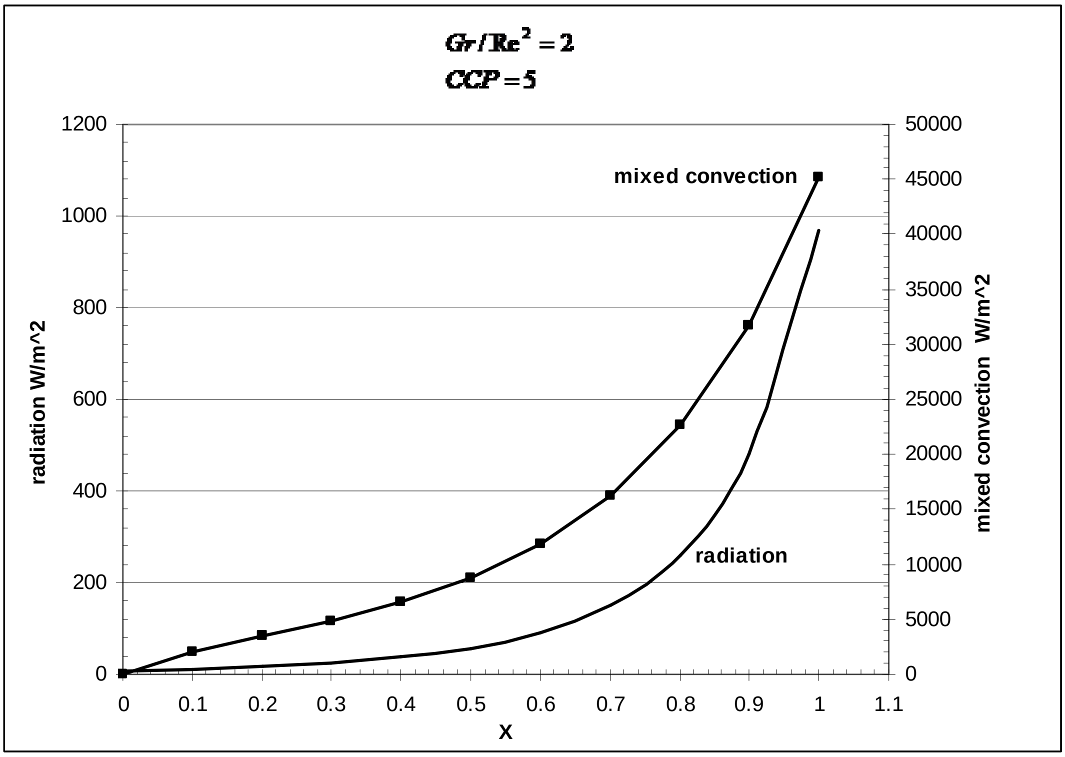

Fig 7, steady state local heat fluxes long the length of the fin due to mixed convection and radiation

Fig. (7) shows the local heat fluxes long the length of the fin due to mixed convection and radiation that the contribution due to radiation is much smaller compared to that due to mixed convection. This is because, the temperature difference in this case between the fin and the fluid is not extremely high, the emissivity of the fin material is low and also Reynolds number is high (Re = 5000). The radiation effect can be significant for very high temperature difference, low speed flow of the fluid and high emissivity fin material. However, the concept of radiation heat transfer coefficient, h_r would be inappropriate to use for large temperature difference between the fin and the ambient.

CONCLUSION

A numerical investigation of the influence of buoyancy force parameter (Gr/Re^2) and the convection-conduction parameter, CCP ($\sqrt{Re}\, k_f L / k_s b$) on the transient combined mixed convection and radiation heat transfer from air-cooled rectangular aluminum fin has been reported. It is observed that both transient and steady state heat transfer characteristics, local heat fluxes and the fin temperature are greatly affected by both CCP and Gr/Re^2 . However, for the input data of the problem under study, heat transfer by radiation is found to be very small compared to that by mixed convection.

The present numerical result has been validated against an earlier limiting case numerical solution.

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