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Numerical Solution of Two-Dimensional Electrostatic Field Problems

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Abstract

In this paper we compute voltage distribution and charge flux for two-dimensional surface with a curved edge using finite element method. The ease with which the finite element method is implemented on a digital computer system and its flexibility which allows for choosing any desired degree of approximation without having to reformulate the problem is the reason for its consideration in this work. The numerical solutions were obtained using maple software package

Keywords: Finite element method, Partial differential equation, Two-dimensional surface

INTRODUCTION

The desire in this paper is to compute the numerical approximation of two-dimensional partial differential equations (Poisson and Laplace equations) step-by-step. One obvious observation with a two-dimensional domain Ω is, in general a curve. Fourier's law and the heat balance [4] are employed to characterize the temperature distribution and analogue relationship is also available to model field problems as in the area of electrical engineering. Electrical engineers use similar approach when modeling electrostatic fields. See for example [7]. An analogue of Fourier's law can be represented in one dimensional form as

$$D = \sum \frac{dv}{dx} \qquad (1)$$

See for instance [2], where D is called electric field flux density vector, v is the electrostatic potential.

In a similar manner, a Poisson equation for electrostatic fields can be represented in two-dimension as

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\rho_v}{\in} \qquad (2)$$

see [9] where ε is the permittivity of the materials, ρ_v is the volumetric charge density [5]. If the region contains no free charge (that is), then Eq. (2) reduces to Laplace equation written as

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \qquad (3)$$

In what follows we will employ the finite element method to solve Eq. (3). This is because the finite element method takes care of the boundary conditions and allows for the exactness of the results [1]. In addition the finite element method implementation has a great potential for assisting Scientists as well as engineers in making research in their fields and other related scientific and engineering fields that require application of finite element scheme. In the next section we used moderate matrix size which helped us achieve a high degree of accuracy of results.

Method of Solution

We consider a two-dimensional system which consists of triangles and rectangles that are connected to each other at the nodal points on the boundary of the elements. Here we consider a mesh generation which involves the numbering of the elements and the nodes. Nodes numbering is always done column wise from bottom to the top, starting with the leftmost column and proceeding to the next, when each column is done and repeating the procedure when the numbering along a particular column is done. The elements were numbered in accordance with the angle subtended between the two co ordinate's axes; see [6]. The result determines whether the elements will take a number incremented or decremented by one or ignored completely in the case of the surface of the boundary. In fact the element numbering has no effect on the half bandwidth, however, it does have effect on the computer run-time required to assemble the global coefficient matrix [8]. This is the reason for choosing maple symbolic software package which is very reliable, flexible and has a high degree of accuracy of results, always consistent to some extent with standard classical solutions.

Below we considered a two-dimensional system with a voltage of 1000 units along the circular boundary and a voltage of zero (0) along the base as shown in **Fig. 1.**

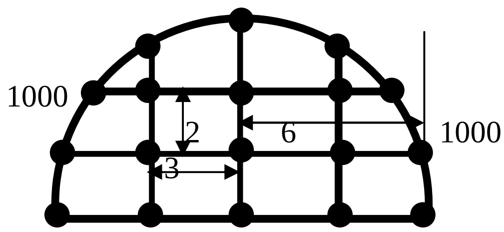

Fig. 1 (a) a circular boundary with a base

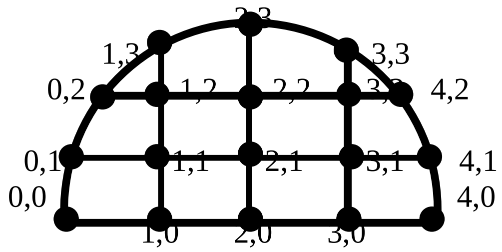

Fig. 1(b) the nodal numbering scheme

If x = 3 and y = 2 then we determine the voltage distribution of each node using Eq. (4)

$$\frac{2}{\Delta x^2}\left[\frac{v_{1,1}-v_{0,1}}{\alpha_1(\alpha_1+\alpha_2)}+\frac{v_{1,1}-v_{2,1}}{\alpha_2(\alpha_1+\alpha_2)}\right]+\frac{2}{\Delta y^2}\left[\frac{v_{1,1}-v_{1,0}}{\beta_1(\beta_1+\beta_2)}+\frac{v_{1,1}-v_{1,2}}{\beta_2(\beta_1+\beta_2)}\right] \tag{4}$$

according to the geometry addition depicted in(Fig. 1b), substituting these values into Eq. (4) yield.

$0.12132v_{1,1} - 121.32 + 0.11438\, v_{1,1} - 0.11438v_{2,1} + 0.25v_{1,1} + 0 + 0.25v_{1,1} - 0.25v_{1,2}$

Similar approaches were applied to the remaining interior nodal points. In this way six simultaneous equations were obtained which we expressed in a matrix form as

$$\begin{bmatrix} 0.735700 & -0.11438 & 0 & -0.25000 & 0 & 0 \\ -0.11111 & 0.722222 & -0.11111 & 0 & -0.25000 & 0 \\ 0 & -0.11438 & 0.735700 & 0 & 0 & -0.25000 \\ -0.31288 & 0 & 0 & 1.28888 & -0.149070 & 0 \\ 0 & -0.25000 & 0 & -0.11111 & 0.722222 & -0.11111 \\ 0 & 0 & -0.31288 & 0 & -0.149070 & 1.28888 \end{bmatrix}\begin{bmatrix} v_{1.1} \\ v_{2,1} \\ v_{3,1} \\ v_{1,2} \\ v_{2,2} \\ v_{3,2} \end{bmatrix}=\begin{bmatrix} 121.32 \\ 0 \\ 121.32 \\ 826.92 \\ 2.50 \\ 826.92 \end{bmatrix}$$

Using maple software yields the following results as

$v_{1,1} = 521.19$, $v_{2,1} = 421.85$ $v_{3,1} = 521.19$, $v_{1,2} = 855.47$, $v_{2,2} = 755.40$, $v_{3,2} = 855.47$

In order to compute the flux density (D), the relationships are given below.

$$D_x = -\in \frac{v_{i+1,j}-v_{i-1,j}}{(\alpha_1+\alpha_2)\Delta x} \tag{5a}$$

$$D_y = -\in \frac{v_{i,j+1}-v_{i,j-1}}{(\beta_1+\beta_2)\Delta y} \tag{5b}$$

For node (1,1), the flux density is computed as

$$D_x = -2\frac{421.85 - 1000}{(0.94281+1)3} = 198.4 \qquad (6)$$

In a similar manner, this in turn can be use to calculate the electric density vector using Eq. (7) below.

$$D = \sqrt{D_x^{\ 2} + D_y^{\ 2}} \qquad (7)$$

The direction of the flux can be determine using Eq. (8) below [3]

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) \qquad (8)$$

The electric flux density vector D = 471.5 and the flux direction ϴ = -65.1^{o} from Eq. (7) and (8) above.

Results

The solution of the other nodes is given in Table

Table 1: Results for the other nodes

Node	D $_x$	D $_y$	D	θ
2,1	0.000	-377.7	3777.7	-90.0
3,1	-198.4	-427.7	471.5	245.1
1,2	109..4	-299.6	281.9	-69.1
2,2	0.000	-289.1	289.1	90.1
3,2	-109.4	-299.6	318.6	249.9

Solution of the Laplace equation with correction factors for the irregular boundaries is shown in (Fig. 2) below

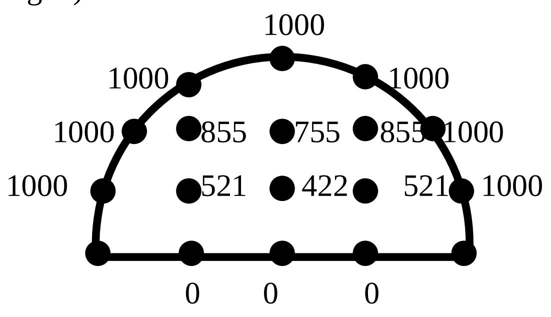

Fig. 2 (a) Potentials

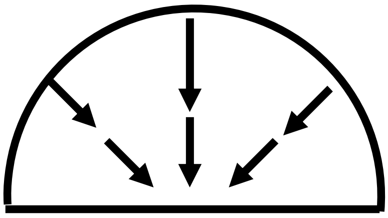

Fig. 2(b) Flux

CONCLUSION

The computation of voltage distribution and charge flux using Laplace equation for a two-dimensional surface with a curved edge were achieved in relation to boundary problems. All the distributed voltage and charge flux falls within the boundary limit, no any single value exceeds the required boundary. The same method can also be employ to determine the voltage distribution and change flux for three dimensions or more with a curved edge, if desired. This type of problem is usually associated or drawn from engineering problem. Their numerical approximations by the use of numerical method for solving partial differential equations especially finite element methods usually demonstrate a high degree of reliability; efficiency and accuracy as it accommodate unequal spacing and equal spacing in the discretisation procedure.

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