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Extended Abstract

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On the dynamical foundations of the Lidov-Kozai Theory

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The Lidov–Kozai theory developed by each of the authors independently in 1961–1962 is based on qualitative methods of studying the evolution of orbits for the satellite version of the restricted three-body problem (Hill's problem). At present, this theory is in demand in various fields of science: in the field of planetary research within the Solar system, the field of exoplanetary systems, and the field of high-energy physics in interstellar and intergalactic space. This has prompted me to popularize the ideas that underlie the Lidov–Kozai theory based on the experience of using this theory as an efficient tool for solving various problems related to the study of the secular evolution of the orbits of artificial planetary satellites under the influence of external gravitational perturbations with allowance made for the perturbations due to the polar planetary oblateness.

The theory of Lidov (Lidov 1961, 1963) and Kozai (Kozai 1962) is based on a study of the evolution of the orbit of a body with a small mass m2 around a body with a larger mass m1 under the influence of external gravitational perturbations from a third body with a mass m3. This problem is called the restricted three-body problem (Hill's problem). The Lidov– Kozai theory was developed by each of the authors independently. Lidov's theory is based on a study of the secular evolution of artificial Earth satellites (AESs) under the influence of gravitational perturbations from external bodies (the Moon and the Sun), while Kozai's theory is based on a study of the secular evolution of asteroid orbits under the influence of gravitational perturbations (from Jupiter and Saturn). Lidov's results were published in 1961 and presented in 1962 at an international symposium in Paris, while Kozai's results were published in 1962. These authors derived expressions for the perturbing function of Hill's problem and expansions of this function in a series in powers of the ratio $|r2 - r1| / |r3 - r1|$, where r1, r2, and r3 are the radius-vectors of the corresponding bodies in inertial space. The qualitative methods of studying perturbations in the Lidov– Kozai theory are based on this perturbing function. The secular component of the perturbing function was obtained by Lidov and Kozai based on the first three terms of the expansion of the perturbing function for Hill's problem twice averaged over the periods of the motion of the second and third bodies relative to the first body (assuming the periods to be incommensurable). Three first integrals and a completely integrable system of Lagrange differential equations correspond to the secular component of the perturbing function. The presence of a resonance in the periods of the motion of each of the bodies (m2 and m3) relative to the central body m1 leads to various peculiarities of the orbital evolution of the body m2. The problem of resonant orbital evolution is considered in the book by Shevchenko (2017), which is devoted to applying the Lidov–Kozai theory in studies of the Solar system dynamics and exoplanetary research. To justify the application of these approximate methods to solving practical problems, I analyzed the scope of the theory under consideration from a study of the secular evolution of high-apogee orbits, where external gravitational perturbations play a major role. In the course of these studies (Prokhorenko 2007) I managed to prove the theorems on the manifolds of initial conditions whereby the secular evolution of the eccentricity leads or (does not lead) to a collision of the satellite with the surface of the central body.

Then, the problem of estimating the regions of predominant influence of the gravitational perturbations from external bodies and predominant influence of the gravitational field due to the polar oblateness of the central body naturally arose (Prokhorenko 2010, 2011). The integrable cases of the mixed problem obtained in 1974 by Lidov and Yarskaya (1974) were used for these studies. As a result, I managed to obtain the boundaries of the regions of predominant influence of the factors under consideration on the scale of possible semimajor axes of the orbits of planetary satellites.

The dynamics of the two-body problem, which clearly demonstrates the action of Newton's and Kepler's laws underlying the universe, should be mastered to describe the dynamics of the restricted threebody problem. Describing the orbital evolution under the influence of a third body is reduced to describing the evolution of Keplerian orbital elements, which remain constant in the two-body problem. The evolving Keplerian orbital elements are called the osculating ones.

It should be noted that at the initial stage of near-Earth space exploration the influence of external gravitational perturbations was taken into account only for high-apogee orbits. The goal of my previous work (Prokhorenko, 2006) was my aspiration to draw attention to the question that the influence of external gravitational perturbations should be taken into account even when launching satellites into orbits with semimajor axes belonging to the region of predominant influence of the gravitational perturbation from planetary oblateness. In this case, the fact that in the mentioned region the evolution of angular orbital elements occurs under the predominant influence of planetary oblateness, whose pattern is determined by the trends described by Okhotsimskii et al. (1957), should be taken into account. Some examples were demonstrated in Prokhorenko (2006) to illustrate this idea.

Bottom Note: *This work is partly presented at International Conference on Planetary Science and Particle Physics*, August 27-28, 2018, Boston, USA