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On the nature of vacuum fluctuation and squeezed state of light

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ABSTRACT

All quantum states of light exhibit field fluctuation even in the vacuum state. There are many physical phenomena which result from vacuum fluctuation. The most prominent example is the spontaneous radiation. In quantum optics and laser physics people have succeeded in producing theoretically even a squeezed state of light that fluctuates at a lower noise level than the vacuum fluctuation. The application of squeezed light will provide new opportunities for high precision measurements. In the present work we consider the quantum superposition of state $|\psi_{01}\rangle = \alpha|0\rangle + \beta|1\rangle$ *and work out the variances of the quadrature operators* $\hat{X}_1$ *and* $\hat{X}_2$*. We have shown that for some values of parameters* α *and* β*, the quadrature variances become less than the vacuum fluctuation. This indicates the principle of manipulating vacuum.*

Key Words: *Vacuum fluctuation, quantum optics, laser physics, squeezed state.*

INTRODUCTION

The principle of superposition in the language of Schrodinger is one of the greatest mysteries of quantum mechanics. It is not fully understood yet, but it gives rise to some visible effects. This principle is at the heart of quantum mechanics. In classical physics we do not speak of superposition of possible states of a system rather we assume that the physical attributes of a system objectively exist even if unknown. As Einstein might say, the moon really is there when nobody looks. But in quantum mechanics it appears necessary to abandon the notion of an objective local reality[1,2]. Instead, a quantum system is described by a state vector which may be expanded into a coherent superposition of the eigenstates of some observable,

$$|\Psi\rangle = \sum_i c_i |\psi_i\rangle \qquad (1)$$

Where the coefficients c_i are probability amplitudes. The probability that a measurement of that observable finds the system in state ψ_i is $|c_i|^2$. But the state vector of Eq.(1) is not merely a reflection of our ignorance of the true state of the system before a measurement but rather of its objective indefiniteness. The system has no objectively definite state prior to a measurement. The acts of measurement “collapse” the state vector to one of the eigenstates.

The basic feature of the superposition principle is that probability amplitude can interfere: a feature that has no analog in classical physics.

The above lines conform to the Copenhagen interpretation (some would say dogma) of quantum mechanics[3]. It is certainly the case that such superposition states are not observable in the everyday world of classical physics. We do not observe macroscopic objects in coherent superposition states and therefore it may be comforting to conclude that the superposition principle operates only on the microscopic scale, at a level inaccessible to everyday experience. It has been realized recently that the interference between states of light composing a quantum superposition state gives rise to various nonclassical effects[4-11]. In particular, it has- been shown that squeezing i.e.., a reduction of quadrature fluctuations below the level associated with the vacuum[12], higher order squeezing[13], as well as sub-Poissonian photon statistics[14] and oscillations of the photon number distribution[11], emerge from a superposition of coherent states. In the present work we consider the quantum superposition of state $|\psi_{01}\rangle = \alpha|0\rangle + \beta|1\rangle$ and work out the variances of the quadrature operators $\hat{X}_1$ and $\hat{X}_2$. We have shown that for some values of parameters α and β, the quadrature variances become less than the vacuum fluctuation. This indicates the principle of manipulating vacuum. The calculation of the variances in a quantum state leads to the determination of the total noise of that state. The knowledge of the noise level of a state is essential to estimate the value of such a state in practice. According to Schumaker[15] , the variances in a single-mode state are defined as the mean-square uncertainties in the real and imaginary parts of the annihilation operator of the mode. Hence the total noise of the state is given by the sum of the variances in that state.

2. Superposition of State $|\psi_{01}\rangle = \alpha|0\rangle + \beta|1\rangle$

Now let us consider the superposition of state $|\psi_{01}\rangle = \alpha|0\rangle + \beta|1\rangle$ where α and β are complex and satisfy the condition $|\alpha|^2 = |\beta|^2 = 1$. We proceed to calculate the variance of the quadrature operators $\hat{X}_1$ and $\hat{X}_2$. Quadrature operators are defined as

$$\hat{X}_1 = \frac{1}{2}\left(\hat{a} + \hat{a}^\dagger\right)$$

$$\hat{X}_2 = \frac{1}{2i}\left(\hat{a} - \hat{a}^\dagger\right)$$

$$\hat{X}_1^2 = \frac{1}{4}\left(\hat{a}^{\dagger 2} + \hat{a}^2 + 2\hat{a}^\dagger\hat{a} + 1\right)$$

$$\hat{X}_2^2 = -\frac{1}{4}\left(\hat{a}^{\dagger 2} + \hat{a}^2 - 2\hat{a}^\dagger\hat{a} - 1\right)$$

$$|\Psi_{01}\rangle = \alpha|0\rangle + \beta|1\rangle$$

Where $|\alpha|^2 = |\beta|^2 = 1$

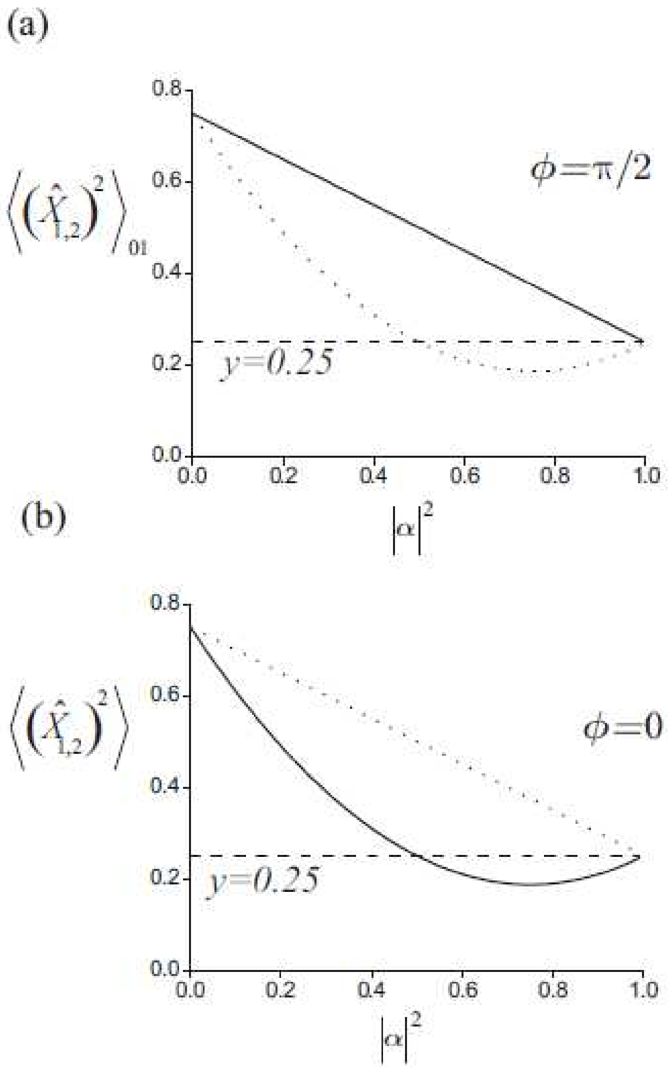
$$\left\langle \hat{X}_1 \right\rangle_{01} = \frac{1}{2}\left(\alpha^*\beta + \alpha\beta^*\right)$$
$$\left\langle \hat{X}_2 \right\rangle_{01} = \frac{1}{2i}\left(\alpha^*\beta - \alpha\beta^*\right)$$

$$\langle \hat{a}^{\dagger 2} \rangle_{01} = 0$$
$$\langle \hat{a}^{2} \rangle_{01} = 0$$
$$\langle \hat{a}^{\dagger}\hat{a} \rangle_{01} = |\beta|^2$$

$$\left\langle \hat{X}_1^2 \right\rangle_{01} = \frac{1}{4}\left(2|\beta|^2 + 1\right)$$
$$\left\langle \hat{X}_2^2 \right\rangle_{01} = \frac{1}{4}\left(2|\beta|^2 + 1\right)$$

$$\left\langle \left(\Delta\hat{X}_1\right)^2 \right\rangle_{01} = \frac{1}{4}\left[2|\beta|^2 + 1 - (\alpha^*\beta)^2 - (\alpha\beta^*)^2 - 2|\alpha|^2|\beta|^2\right]$$
$$= \frac{1}{4}\left[3 - 4|\alpha|^2 + 2|\alpha|^4 - 2|\alpha|^2(1-|\alpha|^2)\cos(2\phi)\right]$$
$$\left\langle \left(\Delta\hat{X}_2\right)^2 \right\rangle_{01} = \frac{1}{4}\left[2|\beta|^2 + 1 + (\alpha^*\beta)^2 + (\alpha\beta^*)^2 - 2|\alpha|^2|\beta|^2\right]$$
$$= \frac{1}{4}\left[3 - 4|\alpha|^2 + 2|\alpha|^4 + 2|\alpha|^2(1-|\alpha|^2)\cos(2\phi)\right]$$

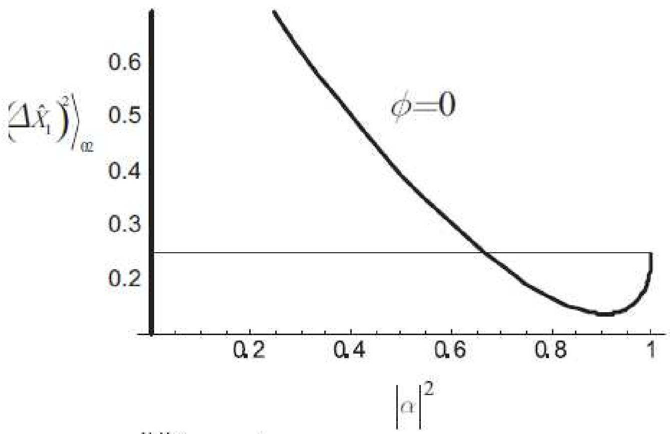
In figures a and b below we plot $\left\langle (\Delta\hat{X}_1)^2 \right\rangle_{01}$ (solid line) for φ=$\frac{\pi}{2}$ and $\left\langle (\Delta\hat{X}_{2)}{}^2 \right\rangle_{01}$ (doted line) for φ=0 respectively. Clearly the quadratures go below the quadrature variances of the vacuum .

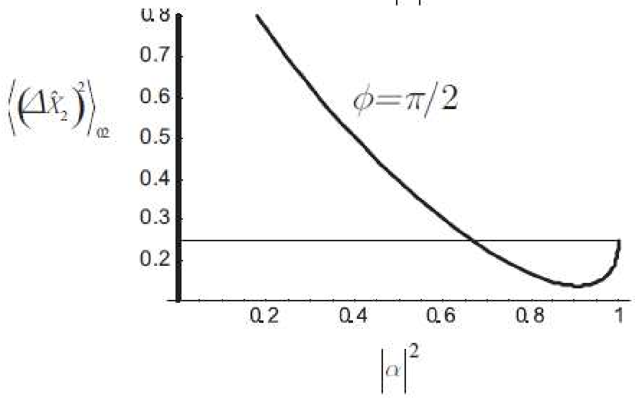

(a)
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
φ=π/2
y=0.25
|α|²
(b)
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
φ=0
y=0.25
|α|²

$$\left\langle \hat{X}_1 \right\rangle_{02} = 0 = \left\langle \hat{X}_2 \right\rangle_{02}$$

$$\begin{aligned}\left\langle \left(\Delta \hat{X}_1\right)^2 \right\rangle_{02} &= \left\langle \hat{X}_1^2 \right\rangle_{02} \\ &= \frac{1}{4}\left(|\alpha + \sqrt{2}\beta|^2 + 3|\beta|^2\right) \\ &= \frac{1}{4}\left[5 - 4|\alpha|^2 + 2\sqrt{2|\alpha|^2(1-|\alpha|^2)}\cos\phi\right]\end{aligned}$$

$$\begin{aligned}\left\langle \left(\Delta \hat{X}_2\right)^2 \right\rangle_{02} &= \left\langle \hat{X}_2^2 \right\rangle_{02} \\ &= \frac{1}{4}\left(|\alpha - \sqrt{2}\beta|^2 + 3|\beta|^2\right) \\ &= \frac{1}{4}\left[5 - 4|\alpha|^2 - 2\sqrt{2|\alpha|^2(1-|\alpha|^2)}\cos\phi\right]\end{aligned}$$





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In figures c and d we plot $\left\langle (\Delta\hat{X}_1)^2 \right\rangle_{02}$ for φ=0 and $\left\langle (\Delta\hat{X}_{2)}{}^2 \right\rangle_{02}$ for φ=$\frac{\pi}{2}$ respectively. Clearly the quadratures go below the quadrature variance of the vacuum. From what has been worked out above it appears that the uncertainty principle is not violated.

RESULT AND DISCUSSION

We have considered in the present work the linear superposition of two states. We have calculated the variance of $\hat{X}_1$ and $\hat{X}_2$ and plotted the graphs containing information about $\left\langle \left(\Delta\hat{X}\right)^2 \right\rangle$ and $|\alpha|^2$. We have noted that there are minimum values of expectations for the parameters α and β for which quadrature variance become less than for a vacuum state. This is the indication that there are squeezed states under the present circumstances. It is worthwhile to note that the measurement comes directly from the superposition of states.

The coherence of a state can also be qualitatively estimated from this. Classically an electromagnetic field consists of waves with well defined amplitude and phase. Such is not the case where we treat the field quantum mechanically. There are fluctuations associated with both the amplitude and phase even in the vacuum state. An electromagnetic field in a number state $|n\rangle$ has well defined amplitude but completely uncertain phase whereas a field in a coherent state has equal amount of uncertainties in the two variables. Equivalently, we can describe the field in terms of the two conjugate quadrature components. The uncertainties in the two conjugate variables satisfy the Heisenberg uncertainty principle such that the product of the uncertainties in the two variables is equal to or greater than half the magnitude of the expectation value of the commutator of the variables. A field in a coherent state is a minimum uncertainty state with equal uncertainties in the two quadrature components. In principle, it is possible to generate states in which fluctuations are reduced below the symmetric quantum limit in one quadrature component. This is accomplished at the expense of enhanced fluctuations in the canonically conjugate quadrature, such that the Heisenberg uncertainty principle is not violated. Such states of the radiation field are called squeezed state.

In this connection we would like to bring in an analogy of vacuum fluctuation and squeezed state with some non physics context. Vacuum fluctuation and squeezed states are the measurement in quantum domain. As a non physics analogy we consider the most elementary subject of measurement, i.e., measurements with the help of main scale and vernier scale concept. Vernier scale provides under normal circumstances a scope for measurement within one millimeter which is not otherwise possible in usual meter scale. Now the vacuum fluctuation which is a measurement of fluctuation of electromagnetic field does not provide necessary means to measure extremely small noise like gravity waves etc., but squeezed state does. Therefore main scale is the analog of vacuum fluctuation and squeezed state is the analog of vernier scale. We should emphasize here that this is for analogy only. Indeed, no analogy exists for vacuum fluctuation and squeezed state in classical case. It is worthwhile to mention that under normal circumstances vernier scale provides a measurement up to 1/10th of a millimeter but in case of squeezed state the quantitative measurement is not yet provided in theory.

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