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Operation transform formulae for generalized two dimensional fractional cosine transform

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ABSTRACT

Transforms with cosine and sine functions as the transform kernels represent an important area of analysis. It is based on the so-called half-range expansion of a function over a set of cosine or sine basis functions. Because the cosine and the sine kernels lack the nice properties of an exponential kernel, many of the transform properties are less elegant and more involved than the corresponding ones for the Fourier transform kernel. In this paper fractional cosine transform is extended in the distributional generalized sense. Operational transform formulae of generalized two dimensional fractional cosine transform are discussed.

Keyword: fractional cosine transform, fractional sine transform, fractional Fourier transform.

INTRODUCTION

In recent years, there has been an enormous effort put in the definition and analysis of fractional or fractal operator. Fractional calculus is for example a flourishing field of active research. The idea of fractional power of the Fourier operator appears in the mathematical literature as early as 1929[2, 3, and 4]. It has been rediscovered in quantum mechanics [5, 6], optics [7, 8, and 9] and signal processing [10]. The boom in publications started in the early years of the 1990's and it is still going on. A recent state of the art can be found in [11] which contain an extensive bibliography. See also [12]. However, it is not only the Fourier transform that has been fractionalized. The term fractal or fractional is now available in almost everywhere: geometry optics, mechanics, signal processing, numerical analysis, calculus.

The fractional Fourier transforms which is generalization of ordinary Fourier transform. The fraction FT gives a more complete representation of the signal in a phase space and enlarges the number of ordinary FT. In addition to the, FT, the Cosine and Sine transform (CT,ST), which are based on half-range expansion of a function over Cosine and Sine basis functions , respectively are also important tools in signal processing. Despite of some lack of elegance of their properties with respect to the FT, the CT and ST have their own areas of applications.

1.1. Two dimensional generalized fractional Cosine transform

Two dimensional fractional Cosine transform with parameter αf(x, y) denoted by $F_C^{\alpha}(x,y)$ perform a linear operation given by the integral transform.

$$F_C^{\alpha}\{f(x,y)\}(u,v) = \int_0^{\infty}\int_0^{\infty} f(x,y)K_{\alpha}(x,y,u,v)\,dx\,dy \quad (1.1)$$

Where the kernel,

$$K_c^{\alpha}(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy). \quad (1.2)$$

1.2. The test function space E

An infinitely differentiable complex valued function $\emptyset$ on R^n belongs to $E(R^n)$ if for each copactset$I \subset S_{a,b}$, where,

$$S_{a,b} = \{x, y: x, y \in R^n, |x| \leq a, |y| \leq b, a > 0, b > 0\}, I \in R^n$$

$$\gamma_{E_{p,q}}(\emptyset) = \sup_{x,y} \left|D_{x,y}^{p,q} \emptyset(x,y)\right| < \infty \text{Where, p, q =1, 2, 3....}$$

Thus $E(R^n)$ will denote the space of all$\emptyset \in E(R^n)$ with support contained in $S_{a,b}$

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member ofE^*, the dual space of E.

2. Distributional two-dimensional fractional Cosine transform

The two dimensional distributional fractional Cosine transform of $f(x,y) \in E^*(R^n)$ defined by

$$F_c^\alpha\{f(x,y)\} = F^\alpha(u,v) = \langle f(x,y), K_\alpha(x,y,u,v)\rangle \qquad (2.1)$$

$$K_c^\alpha(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux).\cos(cosec\alpha.vy) \quad (2.2)$$

Where , RHS of equation (2.1) has a meaning as the application of $f \in E^*$to$K_\alpha(x,y,u,v) \in E$.

3. Properties of kernel of two-dimensional fractional Cosine transform

3.1 Linearity property:

$$\boldsymbol{F_c^\alpha\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = k_1 F_c^\alpha(f_1(x,y)) + k_2 F_c^\alpha(f_2(x,y))}$$

Proof: $F_c^\alpha\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = \int_0^\infty \int_0^\infty (k_1 f_1(x,y) + k_2 f_2(x,y))\ K_c^\alpha(x,y,u,v)dxdy$

$$F_c^\alpha\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = \int_0^\infty \int_0^\infty (k_1 f_1(x,y))\ K_c^\alpha(x,y,u,v)dxdy + \int_0^\infty \int_0^\infty (k_2 f_2(x,y))\ K_c^\alpha(x,y,u,v)dxdy$$

$$F_c^\alpha\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = k_1 \int_0^\infty \int_0^\infty f_1(x,y)\ K_c^\alpha(x,y,u,v)dxdy + k_2 \int_0^\infty \int_0^\infty f_2(x,y)\ K_c^\alpha(x,y,u,v)dxdy$$

$$F_c^\alpha\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = k_1 F_c^\alpha(f_1(x,y)) + k_2 F_c^\alpha(f_2(x,y))$$

3.2 To prove $\boldsymbol{k_\alpha(x,u,y,v) = k_\alpha(y,v,x,u)}$

Proof: Consider

$$k_\alpha(x,u,y,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\cos(cosec\alpha.vy)$$

$$k_\alpha(x,u,y,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(y^2+v^2+x^2+u)cot\alpha}{2}} \cos(cosec\alpha.vy)\cos(cosec\alpha.ux)$$

$$k_\alpha(x,u,y,v) = k_\alpha(y,v,x,u)$$

3.3 To prove $\boldsymbol{k_{-\alpha}(x,u,y,v) = k_\alpha^*(x,u,y,v)}$

Proof: Consider

$$k_{-\alpha}(x,u,y,v) = \sqrt{\frac{1-icot(-\alpha)}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)\cot(-\alpha)}{2}} \cos(cosec(-\alpha).ux)\cos(cosec(-\alpha).vy)$$

$$k_{-\alpha}(x,u,y,v) = \sqrt{\frac{1-(-i)cot\alpha}{2\pi}}\, e^{\frac{-i(x^2+y^2+u^2+v^2)\cot(\alpha)}{2}} \cos(-cosec\alpha.ux)\cos(-cosec\alpha.vy)$$

$k_{-\alpha}(x,u,y,v) = k_\alpha^*(x,u,y,v)$, where * denotes the conjugation.

3.4 To prove $\boldsymbol{k_\alpha((-x),u,(-y),v) = k_\alpha(x,-u,y,-v) = k_\alpha(x,u,y,v)}$

Proof: Consider

$$k_\alpha((-x),u,(-y),v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i((-x)^2+(-y)^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.u(-x))\cos(cosec\alpha.v(-y))$$

$$k_\alpha((-x),u,(-y),v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(-cosec\alpha.ux)\cos(-cosec\alpha.vy)$$

$$k_\alpha((-x),u,(-y),v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\cos(cosec\alpha.vy)$$

$$k_\alpha((-x),u,(-y),v) = k_\alpha(x,-u,y,-v) = k_\alpha(x,u,y,v)$$

3.5 To prove $\boldsymbol{k_\alpha(x,0,y,v) = e^{\frac{ix^2cot\alpha}{2}} k_\alpha(y,v)}$

Proof: Consider

$$k_\alpha(x,0,y,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+0^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.0x)\cos(cosec\alpha.vy)$$

$$k_\alpha(x,0,y,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.vy)$$

$$k_\alpha(x,0,y,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2)cot\alpha}{2}} e^{\frac{i(y^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.vy)$$

$$k_\alpha(x,0,y,v) = e^{\frac{i(x^2)cot\alpha}{2}} \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(y^2+v^2)cot\alpha}{2}} \cos(cosec\alpha.vy)$$

$$k_\alpha(x,0,y,v) = e^{\frac{ix^2cot\alpha}{2}} k_\alpha(y,v)$$

3.6 To prove $\boldsymbol{k_\alpha(x,u,y,0) = e^{\frac{iy^2cot\alpha}{2}} k_\alpha(x,y)}$

Proof: Consider

$$k_\alpha(x,u,y,0) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+0^2)cot\alpha}{2}} \cos(cosec\alpha.ux)\cos(cosec\alpha.0y)$$

$$k_\alpha(x,u,y,0) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2)cot\alpha}{2}} \cos(cosec\alpha.ux)$$

$$k_\alpha(x,u,y,0) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(y^2)cot\alpha}{2}} e^{\frac{i(x^2+u^2)cot\alpha}{2}} \cos(cosec\alpha.ux)$$

$$k_\alpha(x,u,y,0) = e^{\frac{iy^2cot\alpha}{2}} k_\alpha(x,y).$$

3.6 To prove $\boldsymbol{k_\alpha\left(\frac{x}{sec\alpha},\frac{y}{sec\alpha},0,0\right) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2)cos^3\alpha}{2sin\alpha}}}$

Proof: $k_\alpha\left(\frac{x}{sec\alpha},\frac{y}{sec\alpha},0,0\right) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}\left(\frac{x^2}{sec^2\alpha}+\frac{y^2}{sec^2\alpha}\right)cot\alpha} \cos(cosec\alpha.0x)\cos(cosec\alpha.0y)$

$$k_\alpha\left(\frac{x}{sec\alpha},\frac{y}{sec\alpha},0,0\right) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i}{2}\left(\frac{x^2}{sec^2\alpha}+\frac{y^2}{sec^2\alpha}\right)cot\alpha}$$

$$k_\alpha\left(\frac{x}{sec\alpha},\frac{y}{sec\alpha},0,0\right) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2)cos^3\alpha}{2sin\alpha}}$$

4. Proposition

4.1 Generalized two dimensional fractional cosine transform reduces to Fourier cosine transform.

Proof we know the generalized two dimensional fractional cosine transform is

$$F_c^{\alpha}(f(x,y))(u,v)=\int_0^{\infty}\int_0^{\infty} f(x,y)\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\cos(cosec\alpha.vy)dxdy$$

Putting $\theta=\frac{\pi}{2}$

$$F_c^{\frac{\pi}{2}}(f(x,y))(u,v)=\int_0^{\infty}\int_0^{\infty} f(x,y)\sqrt{\frac{1-icot\frac{\pi}{2}}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\frac{\pi}{2}}{2}}\cos\left(cosec\frac{\pi}{2}.ux\right)\cos\left(cosec\frac{\pi}{2}.vy\right)dxdy$$

$$F_c^{\frac{\pi}{2}}(f(x,y))(u,v)=\int_0^{\infty}\int_0^{\infty} f(x,y)\sqrt{\frac{1}{2\pi}}\cos(ux)\cos(vy)dxdy$$

$$F_c^{\frac{\pi}{2}}(f(x,y))(u,v)=\sqrt{\frac{1}{2\pi}}\int_0^{\infty}\int_0^{\infty} f(x,y)\cos(ux)\cos(vy)dxdy$$

$F_c^{\frac{\pi}{2}}(f(x,y))(u,v)=F_c\{f(x,y)\}(u,v)$, where $F_c\{f(x,y)\}(u,v)$ denote Fourier cosine transform of $f(x,y)$

5. Scaling property:

If $F_c^{\alpha}(f(x,y))(u,v)$ is generalized two dimensional fractional cosine transform of $f(x,y)$ then

$$F_c^{\alpha}(f(ax,by))(u,v)=$$
$$\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}((u^2+v^2)cot\theta-\left(\left(\frac{u}{a}\right)^2+\left(\frac{v}{b}\right)^2\right)\frac{csc^2\alpha.sin^2\theta}{2})}F_c^{\alpha}\left(e^{\frac{i}{2}((b^2-1)(ax)^2+(a^2-1)(by)^2)cot\theta}f(ax,by)\right)(P,Q)$$

Proof: consider

$$F_c^{\alpha}(f(ax,by))(u,v)=\int_0^{\infty}\int_0^{\infty} f(ax,by)\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\cos(cosec\alpha.vy)dxdy$$

$$F_c^{\alpha}(f(ax,by))(u,v)=AB\int_0^{\infty}\int_0^{\infty} f(ax,by)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\cos(cosec\alpha.vy)dxdy$$

$A=\sqrt{\frac{1-icot\alpha}{2\pi}}$, B= $e^{\frac{i(u^2+v)cot\alpha}{2}}$

Let $ax=T$, $by=S$ $x=\frac{T}{a}$, $y=\frac{S}{b}$, $dx=\frac{dT}{a}$, $dy=\frac{dS}{db}$

When $x=0$, $T=0$ when y=0, S=0 when $x=\infty$, $T=\infty$ when y=∞ , S=∞

$$F_c^{\alpha}(f(ax,by))(u,v)=AB\int_0^{\infty}\int_0^{\infty} f(T,S)e^{\frac{i}{2}\left(\frac{T^2}{a^2}+\frac{S^2}{b^2}\right)cot\alpha}\cos\left(cosec\alpha.u\frac{T}{a}\right)\cos\left(cosec\alpha.v\frac{S}{b}\right)\frac{dT}{a}\frac{dS}{b}$$

$$F_c^{\alpha}(f(ax,by))(u,v)=\frac{AB}{ab}\int_0^{\infty}\int_0^{\infty} f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)\frac{cot\alpha}{a^2b^2}}\cos\left(cosec\alpha.u\frac{T}{a}\right)\cos\left(cosec\alpha.v\frac{S}{b}\right)dTdS$$

Let $\frac{cot\alpha}{a^2b^2}=cot\theta$

$$F_c^{\alpha}(f(x,y))(u,v)=\frac{AB}{ab}\int_0^{\infty}\int_0^{\infty} f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)cot\theta}\cos\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{u}{a}\right)T\right)\cos\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{v}{b}\right)S\right)dTdS$$

$P=\frac{csc\alpha}{csc\theta}\frac{u}{a}$ $P=\frac{sin\theta}{sin\alpha}\frac{u}{a}$ $Q=\frac{csc\alpha}{csc\theta}\frac{v}{b}$ $Q=\frac{sin\theta}{sin\alpha}\frac{v}{b}$

$$F_c^{\alpha}(f(ax,by))(u,v)=\frac{AB}{ab}\int_0^{\infty}\int_0^{\infty} f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)cot\theta}\cos\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{u}{a}\right)T\right)\cos\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{v}{b}\right)S\right)dTdS$$

$$F_c^{\alpha}(f(ax,by))(u,v)=\frac{AB}{ab}\int_0^{\infty}\int_0^{\infty} f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)cot\theta}\cos(csc\theta\,PT)\cos(csc\theta\,QS)\,dTdS$$

$$F_c^{\alpha}(f(ax,by))(u,v)$$
$$=\frac{AB}{ab}\int_0^{\infty}\int_0^{\infty} f(T,S)e^{\frac{i}{2}((bT)^2+(aS)^2+P^2+Q^2)cot\theta}e^{\frac{-i}{2}(P^2+Q^2)cot\theta}\cos(csc\theta\,PT)\cos(csc\theta\,QS)\,dTdS$$

$$F_c^{\alpha}(f(ax,by))(u,v)=\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\sqrt{\frac{1-icot\theta}{2\pi}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{csc\alpha}{csc\theta}\frac{u}{a}\right)^2+\left(\frac{csc\alpha v}{csc\theta b}\right)^2\right)cot\theta}$$
$$\int_0^{\infty}\int_0^{\infty} f(T,S)e^{\frac{i}{2}((bT)^2+(aS)^2-T^2-S^2)cot\theta}e^{\frac{i}{2}(P^2+Q^2+T^2+S^2)cot\theta}\cos(csc\theta\,PT)\cos(csc\theta\,QS)\,dTdS$$

$$F_c^{\alpha}\big(f(ax,by)\big)(u,v) = \sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}(P^2+Q^2)cot\theta}$$

$$\int_0^{\infty}\int_0^{\infty}\sqrt{\frac{1-icot\theta}{2\pi}}f(T,S)e^{\frac{i}{2}((b^2-1)T^2+(a^2-1)S^2)cot\theta}\,e^{\frac{i}{2}(P^2+Q^2+T^2+S^2)cot\theta}\cos(csc\theta\,PT)\cos(csc\theta\,QS)\,dTdS$$

$$F_c^{\alpha}\big(f(ax,by)\big)(u,v) =$$

$$\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{csc\alpha}{csc\theta}\frac{u}{a}\right)^2+\left(\frac{csc\alpha v}{csc\theta b}\right)^2\right)cot\theta}F_c^{\alpha}\left(e^{\frac{i}{2}((b^2-1)T^2+(a^2-1)S^2)cot\theta}f(T,S)\right)(P,Q)$$

$$F_c^{\alpha}\big(f(ax,by)\big)(u,v) =$$

$$\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{csc\alpha}{csc\theta}\frac{u}{a}\right)^2+\left(\frac{csc\alpha v}{csc\theta b}\right)^2\right)cot\theta}F_c^{\alpha}\left(e^{\frac{i}{2}((b^2-1)(ax)^2+(a^2-1)(by)^2)cot\theta}f(ax,by)\right)(P,Q)$$

$$F_c^{\alpha}\big(f(ax,by)\big)(u,v) =$$

$$\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{u}{a}\right)^2+\left(\frac{v}{b}\right)^2\right)\frac{csc^2\alpha.sin^2\theta}{2}}F_c^{\alpha}\left(e^{\frac{i}{2}((b^2-1)(ax)^2+(a^2-1)(by)^2)cot\theta}f(ax,by)\right)(P,Q)$$

$$F_c^{\alpha}\big(f(ax,by)\big)(u,v) =$$

$$\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}\left((u^2+v^2)cot\theta-\left(\left(\frac{u}{a}\right)^2+\left(\frac{v}{b}\right)^2\right)\frac{csc^2\alpha.sin^2\theta}{2}\right)}F_c^{\alpha}\left(e^{\frac{i}{2}((b^2-1)(ax)^2+(a^2-1)(by)^2)cot\theta}f(ax,by)\right)(P,Q)$$

CONCLUSION

We have extended two-dimensional fractional Cosine transform in the distributional generalized sense and some operation transform formulae are proved.

REFERENCES

[1] Yip, P. "Sine and Cosine Transforms."*The Transforms and Applications Handbook: Second Edition.* Ed. Alexander D. Poularikas Boca Raton: CRC Press LLC, **2000**
[2] N. Wiener, *Math, Phys*., 8:70-73, **1929**.
[3] E.U. Condon. *Proc. National Academy Sciences*, 23:158-164, **1937**
[4] H. Kober. *Quart, J. Math. Oxford Ser*., 10:45-49, **1939**
[5] V. Namias. *J. Inst. Math. Appl*., 25:241-265, **1980**.
[6] A.C. McBride and F.H. Kerr. *IMA J. Appl. Math*., 39:159{175, **1987**.
[7] D. Mendlovic and H.M. Ozaktas. *I. J. Opt. Soc. Amer. A*, 10:1875-1881, **1993**.
[8] H.M. Ozaktas and D. Mendlovic. *J. Opt. Soc. Amer. A*, 10:2522-2531, **1993**.
[9]T. Alieva, V. Lopez, F. Agullo-Lopez, and L.B. Almeida. *J. Mod. Opt*., 41:1037-1044, **1994**.
[10] L.B. Almeida. *IEEE Trans. Sig. Proc*., 42:3084-3091, **1994**.
[11] H.M.Ozaktas, Z. Zalevsky, and M.A. Kutay. The fractional Fourier transform. Wiley, Chichester, **2001**.
[12] G.S. Khan, N.K. Nishchal, J. Joseph, and K. Singh, *Asian Journal of Physics*, 10(2):251-299, **2001**.
[13] A.H. Zemanian, **1996**, "distribution Theory and Transform Analysis, "McGraw Hill, New York.
[14] V. D. Sharma, S.A. Khapre; "Analyticity of the generalized two dimensional fractional Cosines transforms, "*J. Math. Computer Sci.* ISSN. **1927**-5307.