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Plane gravitational waves with mesonic perfect fluid in bimetric relativity

Sulbha R. Suple[1] and S. D. Deo[2]

[1]*Department of Mathematics, Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur, India*
[2]*Department of Mathematics, N. S. Science and Arts College, Bhadrawati, Dist-Chandrapur, (M.S.), India*

ABSTRACT

In this paper, we will study $Z = \left(\frac{t}{\sqrt{x^2 + y^2 + z^2}} \right)$ *type plane gravitational waves with perfectfluid and scalar meson matter field respectively. We observed that massive scalar field coupled with perfect fluid in plane gravitational waves doesnot exist in bimetric theory of gravitation formulated by Rosen.Only a vacuum model can be constructed.*

Keywords: - Plane gravitational waves, Scalar Meson field, Perfect fluid, Bimetric Relativity
AMS Code-83C05 (General relativity)

INTRODUCTION

A new theory of gravitation called the Bimetric theory of gravitation,was proposed by Rosen[1][2]to modify the Einstein's general theory of relativity by assuming two metric tensors,viz.,a Riemannian metric tensor g_{ij} and a background metric tensor γ_{ij} .The metric tensor g_{ij}determines the Riemannian geometry of the curved spacetime which plays the same role as given in the Einstein's general relativity and it interacts with matter. The background metric tensor γ_{ij} refers to the geometry of the empty(free from matter and radiation) universe and describes the inertial forces.This metric tensor γ_{ij} has no direct physical significance but appears in the field equations.Therefore it interacts with g_{ij}but not directly with matter.OneCan regard γ_{ij} as giving the geometry that would exist if there were no matter.In the absence of matter one would have $g_{ij}= \gamma_{ij}$ .Moreover,the bimetric theory also satisfied the covariance and equivalence principles:the formation of general relativity. The theory agrees with the present observational facts pertaining to general relativity.Thus at every point of space-time there are two line elements:

$$ds^2 = g_{ij} dx^i dx^j \quad (1.1)$$

And $$d\sigma^2 = \gamma_{ij} dx^i dx^j \quad (1.2)$$

Where ds is the interval between two neighboring events as measured by means of a clock and a measuringrod.

The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable.

One can regard it as describing the geometry that would exist if no matter were present.

Plane gravitational waves are usually discussed as a special case of the well-established plane fronted gravitational waves with parallel rays , the so called pp- waves .The method of specialization is quite technical , e.g. the curvature tensor must be complex recurrent with a recurrence vector which is collinear with a real null vector . H Takeno (1961) [3] propounded a rigorous discussion of plane gravitational waves, definedvarious terms by formulating a meaningful mathematical version and obtained numerous results.

A fairly general case of "plane" gravitational wave is represented by the metric $ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - dz^2 + dt^2$ (1.3)

both for weak field approximation and for exact solutions of Einstein field equations.Reformulating Takeno's (1961) [3] definition of plane wave, we will use here,

Z = $\left(\frac{t}{\sqrt{x^2 + y^2 + z^2}} \right)$ type plane gravitational waves by using the line elements,

$ds^2 = -\frac{3At^2}{\left(x^2+y^2+z^2\right)^2}\left(x^2dx^2+y^2dy^2+z^2dz^2\right) - Bdu^2 - Cdv^2 + Adt^2$ (1.4)Mohseni, Tucker and Wang [4] have studied the motion of spinning test particles in plane gravitational waves.

S Kessari, D Singh et al [5], analyzed the motion of electrically neutral massive spinningtest particle in theplane gravitational and electromagnetic wave background. The theory of plane gravitational waves have beenstudied by many investigators Takeno [6]; Pandey [7]; Lal and Shafiullah [8];Lu Huiqing [9];

Bondi, H.et.al.[10],Torre,C.G.[11]; Hogan, P.A.[12];Deo and Ronghe[13],[14] ;Deo and Suple [15],[16] ,[17]and they obtained the solutions .

In this paper,we will study Z = $\left(\frac{t}{\sqrt{x^2 + y^2 + z^2}} \right)$ type plane gravitational wave with macro and micro matter fieldcoupledwith perfect fluid and will observe the result in the context of Bimetric theory of relativity.

2. FIELD EQUATIONS IN BIMETRIC RELATIVITY:

Rosen N. has proposed the field equations of Bimetric Relativity from variationprinciple as

$$K_i^{\ j} = N_i^{\ j} - \frac{1}{2} N g_i^{\ j} = -8\pi\kappa T_i^{\ j} \quad (2.1)$$

Where $N_i^{\ j} = \frac{1}{2}\gamma^{\alpha\beta}\left[g^{hj} g_{hi} |_\alpha \right] |_\beta$ (2.2) $\quad N = N_\alpha^\alpha, \quad \kappa = \sqrt{\frac{g}{\gamma}}$ (2.3)

and $g = \mathbf{det}(\mathbf{g}_{ij}) \quad , \gamma = \mathbf{det}(\gamma_{ij})$ (2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij} .

And, $T_i^{\ j}$ the energy momentum tensor for macro matter field like Perfect fluidis given by

$T_i^j = T_i^{jp} = (\rho + p) u_i u^j - p g_i^j$ (2.5) together with $g_i^j u_i u^j = 1$ where u_i is the flow vector of the fluid having p and ρ asproper pressure and energy density respectively.

3. Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ **type plane gravitational wave with Perfect Fluid:**

For Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ plane gravitational waves, we have the line element as

$$ds^2 = -\frac{3At^2}{(x^2+y^2+z^2)^2}(x^2dx^2 + y^2dy^2 + z^2dz^2) - Bdu^2 - Cdv^2 + Adt^2 \quad (3.1)$$

Where A = A(Z), B = B(Z), C = C(Z) and

Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$

Corresponding to the equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -(dx^2 + dy^2 + dz^2 + du^2 + dv^2) + dt^2 \quad (3.2).$$

Using equations (2.1) to (2.5) with (3.1) and (3.2),
We get the field equations as

$$D\left\{\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) + \frac{1}{2}\left(\frac{\overline{B}^2}{B^2} - \frac{\overline{\overline{B}}}{B}\right) + \frac{1}{2}\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right)\right\} = -16\pi\kappa p \quad (3.3)$$

$$D\left\{2\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) - \frac{1}{2}\left(\frac{\overline{B}^2}{B^2} - \frac{\overline{\overline{B}}}{B}\right) + \frac{1}{2}\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right)\right\} = -16\pi\kappa p \quad (3.4)$$

$$D\left\{2\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) + \frac{1}{2}\left(\frac{\overline{B}^2}{B^2} - \frac{\overline{\overline{B}}}{B}\right) - \frac{1}{2}\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right)\right\} = -16\pi\kappa p \quad (3.5)$$

$$D\left\{\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) + \frac{1}{2}\left(\frac{\overline{B}^2}{B^2} - \frac{\overline{\overline{B}}}{B}\right) + \frac{1}{2}\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right)\right\} = 16\pi\kappa\rho \quad (3.6)$$

where $D = \left[\frac{t^2 - (x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right]$ and

$$\overline{A} = \frac{\partial A}{\partial Z}, \quad \overline{\overline{A}} = \frac{\partial^2 A}{\partial Z^2}, \quad \overline{B} = \frac{\partial B}{\partial Z}, \quad \overline{\overline{B}} = \frac{\partial^2 B}{\partial Z^2}, \quad \overline{C} = \frac{\partial C}{\partial Z}, \quad \overline{\overline{C}} = \frac{\partial^2 C}{\partial Z^2}$$

Using equation (3.3) to (3.6), we get

$$p + \rho = 0 \quad (3.7)$$

This equation of state is known as false vacuum.In view of reality conditions $p > 0, \rho > 0$

Equation (3.7) immediately implies that $p = 0, \rho = 0$ i.ematter field like perfect fluid does not exist in Z = $\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ plane gravitational waves in Rosen's Bimetric theory of relativity.

Hence for vacuum case $p = 0 = \rho$, the field equation reduced to

$$D\left\{\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) + \frac{1}{2}\left(\frac{\overline{B}^2}{B^2} - \frac{\overline{\overline{B}}}{B}\right) + \frac{1}{2}\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right)\right\} = 0 \quad (3.8)$$

$$D\left\{2\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) - \frac{1}{2}\left(\frac{\overline{B}^2}{B^2} - \frac{\overline{\overline{B}}}{B}\right) + \frac{1}{2}\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right)\right\} = 0 \quad (3.9)$$

$$D\left\{2\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)-\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=0 \quad (3.10)$$

Solving equations (3.8) to (3.10), we have

$$A = K_1 e^{K_2 Z} \quad (3.11)$$

$$B = K_3 e^{K_4 Z} \quad (3.12)$$

$$C = K_5 e^{K_6 Z} \quad (3.13)$$

where K_1 , K_2, K_3, K_4, $K_5$and K_6 are the constants of integration.Thus substituting the value of (3.11) and (3.13) in (3.1), we get the vacuum line element as

$$ds^2=-\frac{3K_1e^{K_2Z}t^2}{\left(x^2+y^2+z^2\right)^2}\left(x^2dx^2+y^2dy^2+z^2dz^2\right)-K_3e^{K_4Z}du^2-K_5e^{K_6Z}dv^2+K_1e^{K_2Z}dt^2$$

(3.14)

Thus, it is found that in plane gravitational waveZ $=\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$, the macro matter field like perfect fluid does not survive in Bimetric theory of relativity and only vacuum model can be constructed.

4.Z $=\left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ **type plane gravitational wave with Scalar Meson Field:**

In this section, we consider the region of the space-time filled with massive scalar field whose energy momentum tensor is given by

$$T_i^j=T_i^{j^s}=V_{,i}V^{,j}-\frac{1}{2}g_i^j\left(V_{,k}V^{,k}-m^2V^2\right), \quad (4.1)$$

together with $\sigma=g_i^jV_{;i}^j+m^2V$, where m is the mass parameter and σ is the source density of the meson field.Here afterwards the suffix(,) and semicolon (;) after a field variable represent ordinary and covariant differentiation with respect to x^i and g_i^j resp.

Using equations (2.1) to (2.5) with (3.1) and (3.2) with energy momentum tensor (4.1) are obtained as

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)+\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=-8\pi\kappa\left(V_6^2-m^2V^2\right) \quad (4.2)$$

$$D\left\{2\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)-\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)+\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=-8\pi\kappa\left(V_6^2-m^2V^2\right) \quad (4.3)$$

$$D\left\{2\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)-\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=-8\pi\kappa\left(V_6^2-m^2V^2\right) \quad (4.4)$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)+\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=8\pi\kappa\left(V_6^2+m^2V^2\right) \quad (4.5)$$

Using (4.2) and (4.5), we get

$$16\pi\kappa V_6^2 = 0$$

ie $V_6 = 0$ ie V = constant (4.6)

Thus for the space-time(3.1) the Scalar Meson field with or without mass parameter does not survive in Bimetric theory of relativity.In both cases source density becomes constant.

5.Coupling of Scalar Meson Field with Perfect Fluid:

The energy momentum tensor for a mixture of perfect fluid and scalar meson field together is given by

$$T_i^{\ j} = T_i^{\ j^p} + T_i^{\ j^s} \quad (5.1)$$

By the use of co-moving co-ordinate system,the field equation (2.1) to (2.4) for the metric (3.1) and (3.2) corresponding to the energy momentum tensor (5.1) can be written as

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)+\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=-8\pi\kappa\left(p+\frac{1}{2}(V_6^{\ 2}-m^2V^2)\right) \quad (5.2)$$

$$D\left\{2\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)-\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)+\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=-8\pi\kappa\left(p+\frac{1}{2}(V_6^{\ 2}-m^2V^2)\right) \quad (5.3)$$

$$D\left\{2\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)-\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=-8\pi\kappa\left(p+\frac{1}{2}(V_6^{\ 2}-m^2V^2)\right) \quad (5.4)$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{1}{2}\left(\frac{\bar{B}^2}{B^2}-\frac{\bar{\bar{B}}}{B}\right)+\frac{1}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=8\pi\kappa\left(\rho+\frac{1}{2}(V_6^{\ 2}+m^2V^2)\right) \quad (5.5)$$

Using (5.2) and (5.5),we obtain

$$(\rho + p) + V_6^{\ 2} = 0 \quad (5.6)$$

In view of the reality conditions i.e. $p > 0, \rho > 0$, the above equation implies that $p = 0, \rho = 0$ and V= constant.

CONCLUSION

In the study of **Z** = $\left(\frac{t}{\sqrt{x^2 + y^2 + z^2}}\right)$ type plane gravitational waves, there is nil contribution of MesonicPerfect fluid in Bimetric theory of relativity respectively.It is observed that the matter fields either massive scalar field or perfect fluid cannot be a source of gravitational field in the Rosen's Bimetric theory but only vacuum model exists.The conclusion arrived at viz.,V = constant, σ = constant, p = 0 , ρ = 0 are invariant statements and hold in all coordinate systems even though we have derived these in co-moving coordinate system.

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