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Plate Buckling Solution Based on Pre-Buckling Deflection Factor

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ABSTRACT

By the capacity of Kirchhoff's rectangular plate differentials, buckling and deflection are solved together and since the deflection-factor is readily recognizable, buckling results are partially confirmed. In this way, unsafe buckling results are identified for correction. Deflection factors are needed to eliminate many un-useful harmonic buckling-solutions; a plate with a larger surface area will have a larger factor. The all-round simply-supported plate at an aspect ratio of "s=3.5" is found to be only 30-percent as stiff as that of "s*=1.0"; the two plates cannot have the same critical load. This new capacity-analysis is able to eliminate the discontinuities prevalent in strength versus aspect-ratio correlations, emanating from joining isolated harmonic waves as final. A large number of plates in mixed boundary conditions have been successfully treated by the new method.*

Keywords: Kirchhoff's plate, Differentials capacity, Rectangular plates, Harmonic-deflections, buckling, Mohr's loading-curvature, Unsafe solutions.

INTRODUCTION

Studies in rectangular plates are readily justified by their ever extensive deployment in the assembly of vessels-ships, aircraft, cans, cylinders. George Gerard and Herbert Becker [1] supplied studies in early analytical and supporting works in buckling in a Handbook on Structural Stability. Supporting experiments in the subject are averagely available, Richard Pride and George Heimeri [2]; Nishino et al., Lehigh Preserve [3]; A striking aspect of the latter is that the theoretical prediction of strength exceeded the experiment by 30 to 50 percent in simply supported plates. A very extensive coverage, and perhaps still the most comprehensive, in plate buckling is found in "Buckling of laminated composite plates and shell panels by Arthur W. Leissa, Ohio State University for Wright-Patterson Air Force Base-Flight Dynamics Laboratories" [4]. A recent "MIT-2013" lecture, OpenCourseWare, [5] updates the state of the art in the subject. The trigonometric deflection representation has always been associated with buckling and vibration in exact Eigen-value solutions, but the Rayleigh-Ritz principle also embraces polynomial representations for approximate results. Iwuoha [6] demonstrated good use of polynomials for the complicated plate with three sides clamped and the fourth simply supported; polynomials cannot generally yield exact solutions and are less attractive in use; there is consuming interest in modes since a compressed bar bends in a pure Sine-wave, after Euler.

The aforementioned studies have produced extensive data on buckling but the problem of discontinuities in mode-coupling harmonic solutions has lingered to date. Figure 1, is a typical nature of the 2-way-supported plate; a choice has to be made between the clashing of two meeting waves (as is seen in buckling monographs) or the weaving of the waves as accepted in vibration studies in Dunkerly's resultants. Buckling and vibration are the same!

Existing problems in the use of harmonic buckling monographs, which now hold sway, include (i) rampant discontinuities in load versus aspect-ratio correlations; this disrupts design. (ii) The strength, about the meeting point, of any two solutions-waves (point-i in Figure 1, lacks interpretation; a plate weakens as its surface-area increases but some existing graphs imply strengthening. A case in point, among many others, is the all-round simply supported plate, [4]; the critical load between mode numbers, Figure 1, is yet not known. A design aspect-ratio of, say 1.7, has no direct correct-solution. This is a challenge to safety. Can a harmonic, say "m=3=a", independently yield a safe result? The existing result of "Ncr=4π2" is untenable compared with the case of "m=1=a" which has the same result; it implies the plate is not weakening with larger surface-area; attaching deflection-factors to solutions will cure this problem.

From history, deflection has always been employed to relate buckling and modes; the "Southwell-plot is an ingenious method of extracting first-mode bar-buckling; the method has been extended to plates. The 6th international specialty conference on cold-formed steel structures, [7], gives good coverage.

The present investigation, stressing the capacity of Kirchhoff's plate differentials, aims to detect unsafe results, relying on relative displacements and forces between consecutive points of the aspect-ratio curve; the latter is modeled to provide incremental analysis of the plate. The deflection-factors employed emanate from the capacity of the Kirchhoff's plate differentials which form the basis of all analyses, analytical or finite-elements; these are easily recognizable. A fast approximate spot-solution is also necessary for additional checks; Mohr's loading curvature-circles are devised to meet this need.

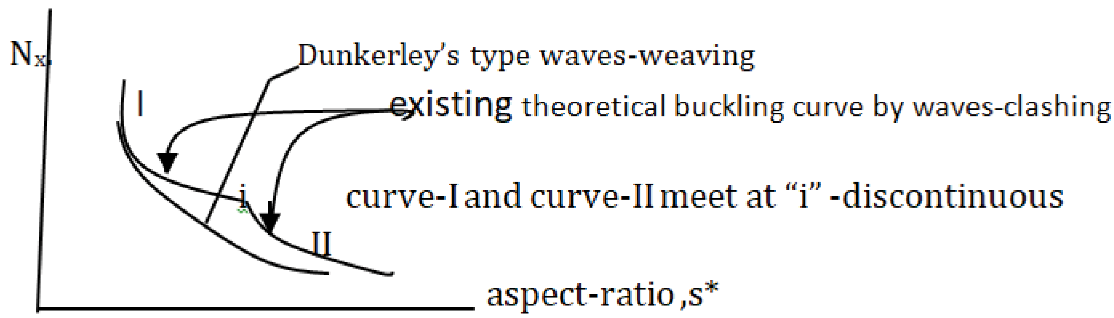

Figure 1: Discontinuous buckling curve (two waves meeting at "i"; no existing solution about "i")

NOMENCLATURE

- a, b: Rectangular plate dimensions in X,Y
- s*: Aspect ratio,
- E: Young's modulus of elasticity
- $\rho=D\pi^2/b^2$
- T: Thickness of plate
- D: Flexural rigidity of plate, (isotropic); $D = Et^3 / 12(1-\mu)^2$
- μ: Poisson's ratio
- W: Deflection symbol; δΔ=small change in deflection
- w_{xx-r}; w_{yy-r}; Relative curvature in X-direction; Y-direction
- w_{xx-r}/w: Relative-curvature/deflection ratio; must be a scalar for any solution
- XX-SC; YY-CC: Plate simply and clamped on X-X; and clamped-clamped on Y-Y
- rcap: the capacity ratio of axes as, $(\partial^4 w/\partial x^4) / (\partial^4 w/\partial y^4)$
- $H = (\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4) = H_{xx} + H_{xy} + H_{yy}$
- $\mathcal{X}$: Curvature; $\delta\mathcal{X}$=small change in curvature
- m, n: Wave numbers
- σ_{cr}; N_{cr}: Critical stress symbol; critical buckling load symbol

APPLICABLE EQUATIONS

The existing uni-axial buckling equation, Eq. 1, is first stated. The uni-axial analysis equation,

$$(\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4) = (N_x / D)(\partial^2 w/\partial x^2) \quad (1)$$

$$D\{\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4\} = [N_x\partial^2 w/\partial x^2 + N_y\partial^2 w/\partial y^2] \quad (2)$$

Also the shear loading equation, Eq. 3 is balanced.

$$\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4 = \left(N_{xy}/D\right)(2\partial^2 w/\partial x\partial y) \quad (3)$$

Under uniformly distributed transverse loading, q, Eq. 4 ensues

$$\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4 = q/D \quad (4)$$

Eq. 1-4 are summarized as,

$$H_{xx} + H_{xy} + H_{yy} = H = RHS \text{ (loading)} \quad (5)$$

"H" is the invariant Capacity (in bending, buckling, vibration) for a given deflection shape function.

Capacity of the Kirchhoff's plate differentials, "H"

By giving finite values of the left-hand differentials, the capacity of a Kirchhoff's plate is found; this is achieved through valid deflection functions, "W".

$$\frac{\partial^4 \boldsymbol{w}/(\partial \boldsymbol{x}4)}{(1)} = \frac{\iint \boldsymbol{w}\left(\partial^4 \boldsymbol{w}/\partial \boldsymbol{x}^4\right)\partial \boldsymbol{x}\partial \boldsymbol{y}}{\iint \boldsymbol{w}\partial \boldsymbol{x}\partial \boldsymbol{y}} = H_{xx} \quad (6)$$

$$\frac{(\partial^4 \boldsymbol{w})/(\partial \boldsymbol{y}^4)}{(1)} = \frac{\iint \boldsymbol{w}\left(\partial^4 \boldsymbol{w}/\partial \boldsymbol{y}^4\right)\partial \boldsymbol{x}\partial \boldsymbol{y}}{\iint \boldsymbol{w}\partial \boldsymbol{x}\partial \boldsymbol{y}} = H_{yy} \quad (7)$$

$$\frac{(2\partial^4 \boldsymbol{w})/(\partial \boldsymbol{x}^2\partial \boldsymbol{y}^2)}{(1)} = \frac{\iint 2\boldsymbol{w}\left(\partial^4 \boldsymbol{w}/\partial \boldsymbol{x}^2\partial \boldsymbol{y}^2\right)\partial \boldsymbol{x}\partial \boldsymbol{y}}{\iint \boldsymbol{w}\partial \boldsymbol{x}\partial \boldsymbol{y}} = H_{xy} \quad (8)$$

$$\frac{(\partial^2 \boldsymbol{w})/(\partial \boldsymbol{x}^2)}{(1)} = \frac{\iint \boldsymbol{w}\left(\partial^2 \boldsymbol{w}/\partial \boldsymbol{x}^2\right)\partial \boldsymbol{x}\partial \boldsymbol{y}}{\iint \boldsymbol{w}\partial \boldsymbol{x}\partial \boldsymbol{y}} = \chi_x \quad (9)$$

$$\frac{(\partial^2 \boldsymbol{w})/(\partial \boldsymbol{y}^2)}{(1)} = \frac{\iint \boldsymbol{w}\left(\partial^2 \boldsymbol{w}/\partial \boldsymbol{y}^2\right)\partial \boldsymbol{x}\partial \boldsymbol{y}}{\iint \boldsymbol{w}\partial \boldsymbol{x}\partial \boldsymbol{y}} = \chi_y \quad (10)$$

$$\chi_{1,2} = (\chi_x + \chi_y)/2 \pm \sqrt{\left[\left\{(\chi_x - \chi_y)^2/2\right\}^2 + \left\{(\chi_{xy})/2\right\}^2\right]}\text{; principal curvatures;} \quad (11a)$$

Or by reference to the Mohr's circle, $\chi_{1,2} = (\chi_x + \chi_y)/2 \pm R$; R=Mohr's circle radius (11b)

The average curvature, $(\chi_x + \chi_y)/2$, is found significant as an intermediate loading curvature when "$\chi_x < \chi_y$" over the range of "$\chi_y > \chi_x$" in uni-axial X-loading. For the bi-axial case, $\chi_{biaxial} = (\chi_x + \chi_y)$.

These integrals are the outcomes of the criterion of buckling as relative-curvature/deflection resonance. A typical buckling resistance integral is,

$$\frac{(\partial^4 \boldsymbol{w})/(\partial \boldsymbol{x}^4)}{(1)} = C_{xd4}(w,_{xx-r}/w) = C_{xd4}(R_{xcd}) \quad (12)$$

The ratio, "$(w,_{xx-r}/w) = (R_{xcd})$" must always be a scalar or else the function-w is inadmissible. The function-w is chosen as to make the ratio, $(w,_{xx-r}/w)$, a scalar. The domain compliant factor at resonance, C_{xd4}, is what is left to be found. Multiply both sides of Eq. 12 and integrate them to find it

$$C_{xd4}.R_{xcd} = [\frac{(\partial^4 w / \partial x^4)}{1}] = [\frac{\iint w(\partial^4 w / \partial x^4)\partial x \partial y}{\iint w \partial x \partial y}] \quad (13)$$

Buckling potential limits

Three possibilities are identified relative to X- and Y-axes in emulating the reactive potentials "$\partial^4 w / \partial x^4 ; 2\partial^4 w / \partial x^2 \partial y^2 ; \partial^4 w / \partial y^4$"

(i) $\sigma_x \chi_x$

This is first in contention in uni-axial X-compression; this case easily solves Eq. 1.

(ii) $\sigma_y \chi_y$

This is out of contention when no load is applied in the Y-axis, whatever the value of "χ_y"

(iii) $\sigma_x \chi_{av}$

This "average loading-curvature" situation will always happen and also in contention. (i) and (iii) are identified in Mohr's diagram (Figure 2).

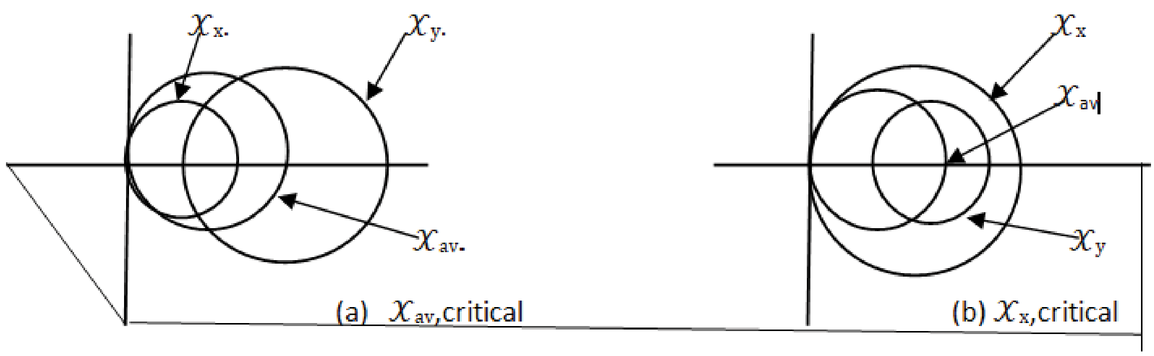

Figure 2: Buckling curvature-loading-circles; X-compression (useful for spot-solution)

In effect, two curvature-loading circles (χ_x, χ_{av}) are operative and the larger circle gives the required solution "N_x". This process softens the stiff constraint that the wave numbers "m, n", must be whole.

a. Importance of curvature, "$(\partial^2 \mathbf{w} / \partial \mathbf{x}^2)_{cr}$" ***, in X-compression***

The solution of Eq. 1 is easy when

$(\partial^2 w / \partial x^2) \geq (\partial^2 w / \partial y^2)$; **and** $\chi_x = \chi_{cr} = (\partial^2 w / \partial x^2)$

When

$(\partial^2 w / \partial x^2) < (\partial^2 w / \partial y^2)$, "$\chi_1$" may be interpreted as "effective principal loading curvature."

So, Figure 1, explains Mohr's loading-curvature, supplying the critical curvatures, exact or near-exact.

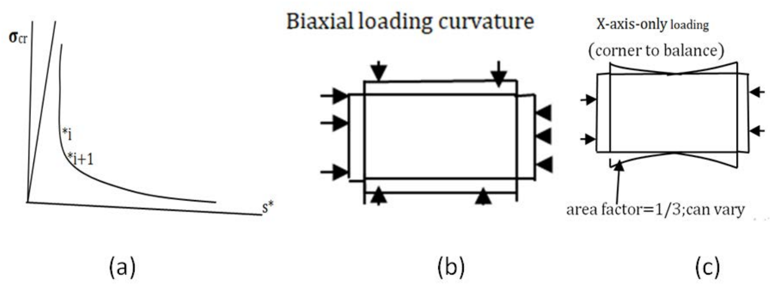

Figure 3: (a): Expected buckling stress-aspect ratio curve; (b): Biaxial loading curvature; (c): stressed boundary lengths-ratio representing side areas

b. Alternate critical X-curvature, from deflection

Relying on the deflection coefficients, Δ_1, Δ_2 at two consecutive locations, "i" and"i+1", Figure 3a, the curvature at the second location may be found from Eq. 14

$$(\Delta_1 / A_1)(\chi_{x,1}) = (\Delta_2 / A_2)(\chi_{x,2}) \tag{14}$$

$A_2/ A_1 = C_A$, stressed boundary lengths-ratio representing side areas; Figure 3b and 3c.

Aspect ratio, "s*" gap of 25-percent can be tolerated.

This equation is similar to Eq. 1 as "(Force/Area) (curvature)=Constant"=Kirchhoff's plate-capacity

Deflection-factor as part of buckling solution

From Eq. 1,

$$D(\partial^4 w / \partial x^4 + 2\partial^4 w / \partial x^2 \partial y^2 + \partial^4 w / \partial y^4) = H = (N_x)(\partial^2 w / \partial x^2) = q*$$

For a given plate-function the "LHS" is invariant and once computed can be used for bending, buckling, and vibration; $q*$it is equivalent uniform transverse pressure. That is, $\Delta = (1/H)(w_{-shape})$; the primitive value is sufficient. The familiarity of "Δ-value" gives confidence the solution is on track.

ILLUSTRATIONS

The "SCCC" plate, Figure 4

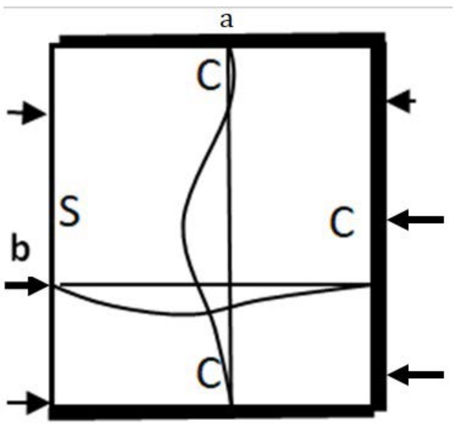

Figure 4: "SCCC" plate in details

$W = (Sin\ Kx/a + Ax/a)((Cos\ n\pi y/b) - 1)$; n=2,4,6…; K=4.5;A=0.977

(i) $\iint w \partial x \partial y = 0.757$; b=1

(ii) $\dfrac{\iint w(\partial^4 w / \partial x^4)\partial x \partial y}{1} = (0.675K^4 / a^4)$; (ii)/(i)=365.74/a^4.

(iii) $\dfrac{\iint 2w(\partial^4 w / \partial x^2 \partial y^2)\partial x \partial y}{1} = (0.45n^2.\pi^2.K^2 / a^2.b^2)$; (iii)/(i)=475.24/a^2.

(iv) $\dfrac{\iint w(\partial^4 w / \partial y^4)\partial x \partial y}{1} = (0.383n^4.\pi^4 / b^4)$; (iv/i)=788.46

(v) $\dfrac{\iint w(\partial^2 w / \partial x^2)\partial x \partial y}{1} = (0.675K^2 / a^2)$; (v/i)=18.056/a^2.

(vi) $\dfrac{\iint w(\partial^2 w / \partial y^2)\partial x \partial y}{1} = (0.383n^2.\pi^2 / b^2)$; (vi/i) ≅20/b^2.

a/b=0.95; $\mathcal{X}$x=(18.06/0.952)=20.0; $\mathcal{X}$y=20.0; Hxx=448.49; Hxy=526.58; Hyy=788.46; H=1763.53; **Δ**s*=0.95=wc/H=0.00143; Nx,s*=0.95=H/$\mathcal{X}$=8.93ρ (expected to be exact), cf ≅ 8.98 [4]; cf 8.98 (approx.) [4]

Continuing, “SCCC”; “a/b=1”, by Eq. 14, knowing, “s*cr=0.95” from,

“$\chi x=\chi y$”: Hxx=365.3; Hxy=475.46; Hyy=788.46; H=1629.0; Δs*=1=wc/H=0.001554; cf 0.00161, Oba et al. [8].

Find “𝒳x,s*=1” from: “𝒳x,s*=0.95”

by Eq. 14

(𝒳x,s*=1)=(CA)(Δs*=0.95/Δs*=1)(𝒳x,s*=0.95);

from Figure 3(c),

(CA)=2.5/2.475=1.01; (𝒳x,s*=1)=18.64;

Nx,s*=1=87.41=8.85ρ

Continuing, “SCCC-case”; Check competing spot solution (Moh’r circles, Figure 2)

at “s*=1”: Hxx=365.3; Hxy=475.24; Hyy=788.46; H=1629.0; 𝒳x=18.06; 𝒳y=20; 𝒳cr=𝒳av=19.03. Figure 2(a)

(Nx,s*= 1)=8.67; close to 8.85ρ above.

a The “CCCC”; (Δ-method), Eq. 14, for “χcr”; “a/b”=1→1.25→1.5, etc

$W = (1 - Cos\ m\pi x./a)(1 - Cos\ n\pi y./b)$; m, n =2,4,6…

(i) a/b=1; Hxx=1168.8; Hxy=779.2; 𝒳x=𝒳y=𝒳cr=29.61; Hyy=1168.8; H=3116.8 Ns*=1=31168.8/29.61=10.66ρ; Δc=0.00128;

cf, 0.00126 [9]; the nearness of the primitive-Δ to the final-Δ, (0.00128 to 0.00126), confirms “H” on which “Nx” depends. Further, the exactness of the w-function is verified.

Continuing, “CCCC”: {a/b=1.5; H=1746; Δ=0.00229; 𝒳x,s*=1.5=19.0;Nx,s*=1.5=9.3ρ}.

(ii) “a/b=2”; For comparison a spot-solution by Figure 2a, Mohr’s-circle, for waves, m=n=2

(𝒳x)=7.4, (𝒳y)=29.608, (𝒳av)=18.5 and Ns*=2,mohr=1436.65/18.5=7.87ρ; cf, 8.0 [4]

b. The “CCCC” plate is summarized in Table 1 for the “Δ-method”:

Table 1: “CCCC” plate and s*; Nx=DKcr. ;“𝒳” from “Δ”; (Δi=1/H)

s*	Δs*	CA.	χΔ.	H	Kcr; [Ref.-4]	(δx/δΔ)=monitor
1	0.00128		29.608	3116.8	10.66ρ, [10.4]	-
1.25	0.00186	1.05	21.72	2146.2	10.01ρ, [9.9]	-13,600.0
1.5	0.00229	1.071	19.00	1746.0	9.3ρ, [9.3]	- 6,325.6
1.75	0.00258	1.055	18.06	1547.85	8.68ρ, [8.6]	- 3,241.4
2.0	0.002784	1.053	17.88	1436.65	8.14ρ, [8.0]	-900.0
2.25	0.002923	1.051	17.88	1368.3	7.75ρ, [7.7]	0
2.35	0.002967	1.019	17.88	1348.2	7.64ρ	0
2.45	0.003005	1.019	17.88	1331.05	7.54ρ	0
2.5	0.003022	1.009	17.88	1323.39	7.50ρ, [7.5]	0
2.75	0.00309 (0.00302)	1.045	18.24 (17.88)	1292.5	7.32ρ	+493.2, δχ/δΔ=min,so,δΔ=0
3	0.00302	1.0526	18.70 (17.88)	1292.5	7.32ρ, [7.35]	(“H, χ”, move together)

The “SSSS” plate, (Nx From Δ) is compiled in Table 2

The trend is similar to the “CCCC” in Table 1; discontinuities are now overcome.

Table 2: “SSSS” plate and s*; Nx= DKcr.; “𝒳” From “Δ”; (Δ i=1/H)

s*	Δs*	CA.	χΔ.	H	Kcr; [Ref. 4]	(δχ/δΔ)=monitor
1	0.00416	-	6.0875	240.34	4.0ρ [4.0]	-

1.25	0.00619	1.06	4.337	161.6	3.78 [>4]	-862
1.5	0.00798	1.06	3.566	125.36	3.56 [>4]	-431
1.75	0.00948	1.055	3.174	105.73	3.375 [>4]	-265
2	0.0106	1.051	2.977	93.73	3.195 [4]	-173
2.25	0.0116	1.051	2.859	86.11	3.05 [>4]	-118
2.5	0.01236	1.0048	2.812	80.85	2.91ρ	-61.8
2.75	0.013	1.046	2.797	77.02	2.79 ρ	-23.4
3	0.0135	1.043	2.797	74.18	2.69 [4]	0
3.25	0.0139 (0.0135)	1.04	2.83 (2.797)	72	2.60$\rho\Delta$	+82.5,(δΔ/ δΔ=min,so,δΔ=0)
3.5	0.0135				2.6	(“H, $\mathcal{X}$” move together)

Deflection limit

In Tables 1 and 2, the factor, $\delta\mathcal{X}/\delta\Delta$, changes sign at a critical point, Figure 5, where the deflection limit is reached. The minimum buckling load is indicated at that point, whatever the values of “m, n or s*”. The relative weakness of a plate is indicated in buckling-vibration-bending analyses [10-12]. The plate analysis starts with beam-strip solutions that are already fully known; for simply supported beam, Δ=5/384=0.01302 and the “SSSS” plate ends in this value when it is very long or very short; for example, check the Δ=0.0135 in Table 2 in the “SSSS” at s*=3.5; so the size of “Δ” can, also, be used to terminate solutions.

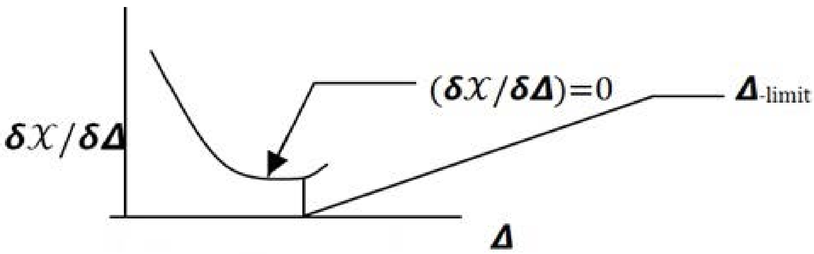

Figure 5: Curvature-displacement-rate (δ$\mathcal{X}$/δΔ) Versus Displacement (Δ)

Higher modes in buckling; The “SSSS”

Table 2 has already solved the problem relying only on the fundamental wave, m=1, but the question may be posed: what is the failure mode for a given aspect ratio? In combining two neighboring symmetrical waves, the tried Dunkerley’s approximate resultant is used to study this question.

For example; s*=2.5, try two waves placed between the actions,

- m=1, n=1 ≡ Cs*=2.5, m=1, or C 2.5, 1, and
- m=3, n=1 ≡ C 2.5, 3

In details

- C 2.5, 1: Hxx=1.54; Hxy=19.2; Hyy=60.1; H=80.8; $\mathcal{X}$=0.94; Nxx=82.95=8.4ρ
- C 2. 5, 3: Hxx=374; Hxy=519; Hyy=180.2; H=1073; $\mathcal{X}$x=26.3; Nxx=4.13

Combining by Dunkerley’s; Ncr=8.4 (4.13)/(8.4+4.13)=2.77, cf, 2.91 in Table 2 above. This is a fail-safe combination.

So, it can be said that the waves “m=1 and m=3” combine for the aspect ratio, s*=2.5; N 2.5, 1, 3=2.77ρ This result is very different from the reference value of “4ρ”, [4,5]. In this way, the complementary question of failure-mode is answered after the strength-solution.

CONCLUSION

- The “Capacity” of the Kirchhoff’s plate differentials that produces buckling results also yields known primitive deflection coefficients for design purposes

- Deflection coefficients are found at each aspect ratio, with these an alternate curvature of the new point is also found from a relation connecting the former; these curvatures yield critical buckling loads. The only input mode needed is the exact fundamental
- Every plate has a critical aspect ratio, s*cr, where "$\mathcal{X}$x=$\mathcal{X}$y". If a buckling solution is found at si* >scr* then "Δi ≥Δcr" otherwise abandon the solution. With this, many results will fail. Where the solution does not, expectedly, rely on deflection, the latter can still be found from references for the purpose of eliminating suspect unsafe cases
- The meaning of infinitely long plate is the aspect-ratio when there is no significant change in deflection in the incremental analysis as demonstrated in the results for the "CCCC and SSSS"
- By this study the "SSSS" plate, at aspect-ratio of "3.5", is only 30-percent as stiff (≡1/Δ) as that of an aspect ratio of "1.0; the two plates cannot, therefore, have the same buckling strength. The corresponding ratio in the "CCCC" is 41-percent the presentation.

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