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Extended Abstract

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Subspace approximations to the Fokker-Planck equation

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In 1909 Theodor Wulf made the first detections of cosmic rays. Over a century later, the origin of cosmic rays is still an open problem for physicists. This problem is due to the difficulty of determining the trajectories these particles made on their way to Earth. This is why the study of cosmic ray trajectories is such an interesting and important field. In the early 1900's Adriaan Fokker and Max Planck developed a partial differential equation to describe the probability distribution function of particles experiencing both Brownian motion as well as outside forces. This equation can be applied to cosmic rays trajectories through turbulent magnetic fields. When applied to this situation the equation does not yet have a known simple analytical solution. Usually to solve this problem numerical methods are used, however, numerical methods are not practical solutions for everyone who needs to use these solutions in their work. We have therefore used Legendre polynomials to approximate a solution to the Fokker-Plank equation. The first, second and third order solutions were found as well as some of the solutions' expectation values. These expectation values are useful both in future analytic work as well as in experimental work. These analytic solutions were compared to a numerical solution to show that for very early times the third order approximation is significantly better than the second order approximation, but at late times only a second order Legendre expansion is needed to achieve good results. This solution is so close to the numerical solution will allow people to more easily track the trajectories of cosmic rays.

In the theory of cosmic rays we use diffusive transport equations to model the motion of energetic particles through space. A more fundamental description can be achieved if the cosmic ray Fokker–Planck equation is used instead of the usual diffusive transport equation. The Fokker–Planck equation allows for a pitch-angle dependent description of the transport. An analytical solution of this equation is desired in several applications ranging from solar modulation studies to investigations of particle acceleration at astrophysical shocks such as supernova remnants. Furthermore, analytical solutions of the Fokker–Planck equation are required in the development of theories for particle transport across a mean magnetic field. Previous solutions were either pure numerical solutions or methods which are half numerical and half analytical. For some applications, however, one needs simple and pure analytical solutions. In this paper we employ a subspace approximation method to the Fokker–Planck equation. In particular the second-order solution derived here, will be very useful for applications and agrees well with pure numerical solutions for later times.

Two methods to approximate the probability density function for the molecular copy numbers in biochemical reactions with a few molecular species have been derived and compared. The steady state solution and the time-dependent solution of the Fokker-Planck equation (FPE) have been computed and the same solutions have been obtained by the Stochastic Simulation Algorithm (SSA) [16]. A bound on the difference between the solutions is proved by the maximum principle for parabolic equations. The errors in the numerical methods are estimated and the execution times for equal errors are compared. The FPE approach is much more efficient for the two dimensional test problems while SSA is the preferred choice in higher dimensions. However, the results depend on the properties of the problem: the size of the computational domain, the stiffness of the chemical reactions, and the chosen error tolerance. The chemical master equation (CME) describes the probability for each internal state of the cell or rather the states of a model of the cell. The number of states grows exponentially with the number of chemical species in the model, since each species corresponds to one dimension in the state space. The CME can be approximated by a Fokker-Planck equation (FPE), which can be solved numerically cheaper than the CME. The FPE approximation of the full CME is not always appropriate, while it can be suitable for a subspace in the state space. In order to exploit the lower cost of the FPE approximation a method for splitting the state space in two subspaces where one is approximated by the FPE and one remains unapproximated is presented. A biologically relevant problem in four dimensions is solved as an example. Chemical reactions are often accurately described with a system of ordinary differential equations that is called the reaction-rate equations. Each equation accounts for the change in concentration for a chemical species. This system of equations describes a completely deterministic evolution of the chemical reactor.

Bottom Note: *This work is partly presented at International Conference on Applied Physics & Laser, Optics and Photonics , April 15-16, 2019, Frankfurt, Germany*