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The light D wave meson spectra in constituent quark models.

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ABSTRACT

The mass spectrum of the D wave mesons has been investigated and compared in the frame work of non-relativistic (NRQM) and relativistic (RHM) quark models. The NRQM Hamiltonian used in the investigation has kinetic energy, confinement potential, one-gluon-exchange potential (OGEP) and instanton induced quark-antiquark interaction (III) whereas RHM Hamiltonian includes the Lorentz scalar plus a vector harmonic-oscillator potential, the confined-one-gluon-exchange potential (COGEP) and III. The calculated D wave meson masses are in agreement with the experimental D wave meson masses. The respective role of III, OGEP and COGEP in the D wave meson spectrum is discussed and compared between two models.

Keywords. Quark Model; Confined One-Gluon-Exchange Potential; Instanton Induced Interaction; D Wave Meson Spectra.

INTRODUCTION

The hadron spectroscopy has received tremendous importance both experiementally and theoreticall since there is a wast experimental data in hadron spectroscopy that would constitute a good testing ground for non perturbative Quantum Chromodynamics (QCD). Within the standard model, hadron is a composite system of quarks and gluons. Since QCD is not exactly solvable in the non-perturbative regime, one has to resort to models which incorporate the basic features of the QCD. As a consequence, our understanding of hadrons continues to rely on insights obtained from the experiments and QCD motivated models in addition to lattice QCD results. The phenomenological models developed to explain observed properties of hadrons are either non-relativistic quark models (NRQM) with suitably chosen potential or relativistic quark models (RQM) [1-9] where the interaction is treated perturbatively. There are successful NRQM and RQM to explain the meson spectra. The NRQM usually contain three main ingredients: the kinetic energy, confinement potential and a hyperfine interaction term which has often been taken as an effective one-gluon-exchange potential (OGEP) [10]. On the other hand, the relativistic models have a confinement potential which is usually taken to be Lorentz scalar plus vector potential. There are models both non-relativistic and relativistic employed to explain meson spectra with OGEP. Other type of interactions have been introduced in the literature from the non-relativistic reduction of the t'Hooft interaction [11-14], termed as instanton induced interaction (III) which has been successfully applied in several studies of the hadron spectra [6,12-13]. The main achievement of the III in hadron spectroscopy is the resolution of the U_A (1) problem, which leads to a good description of the masses of η and η' mesons. In literature there are models which have tried to explain hadron spectroscopy only with OGEP [1-4] and some models only with III [6], ignoring completely the OGEP. It may be an exaggeration to eliminate OGEP completely for light quarks. The OGEP has to be present but with a smaller strength consistent with the asymptotic freedom, since the III vanishes for heavy quarks.

In the present work an attempt has been made to obtain the masses of D wave mesons in the frame work of NRQM and RQM. The basic aim is to obtain the D wave mesons with minimum number of parameters and to investigate the relativistic effects on the mass spectrum. The existence of a gluon self-coupling in QCD states suggest that, in addition to the conventional $q\bar{q}$ states, there may be non- $q\bar{q}$ mesons including gluons and $q\bar{q}$ g hybrids and multiquark states [15]. Since the theoretical guidance on the properties of unusual states is often contradictory, models that agree in the $q\bar{q}$ sector differ in their predictions about new states. In our work we have investigated the meson nonets which have the $q\bar{q}$ quark model assignments, according to the most recent review of Particle Physics [15].

Hence, to study the D wave light meson spectra we have developed two models: the non-relativistic (M1) and relativistic (M2) models. The non-relativistic model has kinetic energy, confinement potential, OGEP and III. In the relativistic model (M2), we have made use of the successful relativistic harmonic model (RHM) [16-20] in which the confinement potential is a Lorentz scalar plus vector potential. Both scalar and vector potential are harmonic oscillator potentials. In M2, the effect of confinement of gluons also has been taken into account. In the existing models though the effect of confinement of quarks has been taken into account the effect of confinement of gluons has not been taken into account. In M2, a consistent scheme has been employed for the confinement of gluons. For the confinement of gluons, we have made use of the current confinement model (CCM) [19-20]. The confined gluon propagators (CGP) derived in CCM has been used to obtain the confined one gluon exchange potential (COGEP). In M2, the total Hamiltonian has Lorentz scalar plus vector potential along with COGEP (instead of OGEP in M1). The M1 and M2 models along with III have been successful in obtaining the mass spectra of S and P wave light mesons [21-24]. The full discussion of the Hamiltonian of M1 and M2 are given in section 2. The results of the calculation are presented in section 3 and the conclusions are given in section 4.

2. The constituent quark Models M1 and M2

2.1 Non-relativistic quark model (M1)

In NRQM the full Hamiltonian is,

$$H = K + V_{OGEP}(\vec{r}_{ij}) + V_{CONF}(\vec{r}_{ij}) + V_{III}(\vec{r}_{ij}) \tag{1}$$

where

$$K = \sum_{i=1}^{2}\left(M_i + \frac{P_i^2}{2M_i}\right) - K_{CM} \tag{2}$$

here M_i and P_i are the mass and momentum of the i^th quark. The K is the sum of the kinetic energies including the rest mass minus the kinetic energy of the centre of mass motion (CM) of the total system. The potential energy part consists of confinement term V_{CONF}, the residual interaction V_{OGEP} and the instanton induced interaction V_{III}.

The confinement term represents the non-perturbative effect of QCD that confines quarks within the colour singlet system, and is taken to be linear.

$$V_{CONF}(\vec{r}_{ij}) = -a_c \mathrm{r}_{ij}\left(\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\right) \tag{3}$$

where a_c is the confinement strength and r_{ij} here and elsewhere in the paper stands for the relative distance between the two quarks. The $\boldsymbol{\lambda}_i$ and $\boldsymbol{\lambda}_j$ are the generators of the color SU(3) group for the i^{th} and j^{th} quark.
The following central part of two-body potential due to OGEP is usually employed [10],

$$V_{OGEP}^{cent}(\vec{r}_{ij}) = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\left[\frac{1}{r_{ij}} - \frac{\pi}{M_iM_j}\left(1+\frac{2}{3}\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\right)\delta(\vec{r}_{ij})\right] \tag{4}$$

where the first term represents the residual Coulomb energy and the second term the chromo-magnetic interaction leading to the hyperfine splitting. The $\boldsymbol{\sigma}_i$ is the Pauli spin operator and α_s the quark-gluon coupling constant.

The non-central part of OGEP has the spin-orbit interaction $V_{OGEP}^{SO}(\vec{r}_{ij})$ and the tensor term $V_{OGEP}^{TEN}(\vec{r}_{ij})$. The spin-orbit interaction of OGEP is,

$$V_{OGEP}^{SO}(\vec{r}_{ij}) = -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\left[\frac{3}{8M_iM_j}\frac{1}{r_{ij}^3}\left(\vec{r}_{ij}\times\vec{P}_{ij}\right)\cdot\left(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j\right)\right] \tag{5}$$

where the relative angular momentum is defined as usual in terms of relative position $\vec{r}_{ij}$ and the relative momentum $\vec{P}_{ij}$. There are several versions of the tensor term in literature. We have used the expression derived in [10] from the QCD lagrangian in the non-relativistic limit and used subsequently by many authors [25]

$$V_{OGEP}^{TEN}(\vec{r}_{ij}) = -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i.\boldsymbol{\lambda}_j\left[\frac{1}{4M_iM_j}\frac{1}{r_{ij}^3}\right]\hat{S}_{ij} \tag{6}$$

where,

$$S_{ij} = [3(\vec{\sigma}_i.\hat{r})(\vec{\sigma}_j.\hat{r}) - \vec{\sigma}_i.\vec{\sigma}_j].$$

The tensor potential is a scalar which is obtained by contracting two second rank tensors. Here, $\hat{r} = \hat{r}_i - \hat{r}_j$ is the unit vector in the direction of $\vec{r}$. In the presence of the tensor interaction, $\vec{L}$ is no longer a good quantum number. The central part of III potential is given by [7, 12-13],

$$V_{III} = \begin{cases} -8g\delta(r_{ij})\,\delta_{S,0}\,\delta_{L,0}\,,\ for\ I=1, \\ -8g'\delta(r_{ij})\,\delta_{S,0}\,\delta_{L,0}\,,\ for\ I=1/2, \\ 8\begin{pmatrix} g & \sqrt{2}g' \\ \sqrt{2}g' & 0\end{pmatrix}\delta(r_{ij})\,\delta_{S,0}\delta_{L,0}\,,\ for\ I=0 \end{cases} \tag{7}$$

The symbols S, L and I are respectively the spin, the relative angular momentum and the iso-spin of the system. The g and g' are the coupling constants of the interaction. The Dirac delta-function appearing has been regularized and replaced by a Gaussian- like function:

$$\delta_{ij} \rightarrow \frac{1}{(\Lambda\sqrt{\pi})^3}\exp\left[-\frac{r_{ij}^2}{\Lambda^2}\right] \tag{8}$$

where Λ is the size parameter.
The non-central part of III has contributions from both spin- orbit and tensor terms. The spin-orbit contribution comes from relativistic corrections to the central potential of III. It is given by [13],

$$V_{III}^{SO}(\vec{r}_{ij}) = V_{LS}(\vec{r}_{ij})\vec{L}\cdot\vec{S} + V_{L\Delta}(\vec{r}_{ij})\vec{L}\cdot\vec{\Delta} \tag{9}$$

The first term in Eqn. (9) is the traditional symmetric spin-orbit term proportional to the operator $\vec{L}\cdot\vec{S}$. The other term is the anti-symmetric spin-orbit term proportional to $\vec{L}\cdot\vec{\Delta}$ where $\vec{\Delta} = \frac{1}{2}\left(\vec{\sigma}_1 - \vec{\sigma}_2\right)$. The radial functions of Eqn. (9) [12],

$$V_{LS}(\vec{r}_{ij}) = \left(\frac{1}{M_i^2}+\frac{1}{M_j^2}\right)\sum_{k=1}^{2}\kappa_k\frac{\exp(-r_{ij}^2/\eta_k^2)}{(\eta_k\sqrt{\pi})^3} + \left(\frac{1}{M_iM_j}\right)\sum_{k=3}^{4}\kappa_k\frac{\exp(-r_{ij}^2/\eta_{k-2}^2)}{(\eta_{k-2}\sqrt{\pi})^3} \tag{10}$$

and

$$V_{L\Delta}(\vec{r}_{ij}) = \left(\frac{1}{M_i^2}-\frac{1}{M_j^2}\right)\sum_{k=5}^{6}\kappa_k\frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3} \tag{11}$$

The term $V_{LS}(\vec{r})$ is responsible for the splitting of the ^3L_J states with $J = L-1$, L, L+1. With such a term L is still good quantum numbers but S is not. The term $V_{L\Delta}(\vec{r})$ couples states $^1L_{J=L}$ and $^3L_{J=L}$. Due to the mass dependence in Eqn. (11), it is clear that this term is inoperative when the quarks are identical. In practice the antisymmetric spin orbit term is important only in the K-sector. The κ_i and η_i are free parameters in the theory [12,23,24]. The M_i corresponds to the mass of the strange quark (s) and M_j corresponds to mass of (u/d) quark. This term accounts for the splitting between 1^1D_2 and 1^3D_2 states in the K sector.

The tensor interaction of III is [13],

$$V_{III}^{TEN}(\vec{r}_{\rm ij}) = \frac{\hat{S}_{ij}}{M_i M_j}\sum_{k=7}^{8}\kappa_k \frac{\exp(-{\rm r}_{\rm ij}^2/\eta_{k\text{-}4}^2)}{(\eta_{k\text{-}4}\sqrt{\pi})^3} \tag{12}$$

With the tensor interaction, L is no longer a good quantum number since this term couples the states $^3L_{J=L+1}$ and $^3(L+2)_{J=L+1}$. It is to be noted that III and OGEP have the same spin dependence except for $V_{L\Delta}(\vec{r})$ term. The equations (9)-(12) have been used by a number of authors and are obtained from the non-relativistic reduction of the 't Hooft interaction[11-14].

2.2 The relativistic harmonic model (M2)

In RHM [16-20], quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic-oscillator potential

$$V_{conf}(r) = \frac{1}{2}(1+\gamma_0)A^2r^2 + M \tag{13}$$

where γ_0 is the Dirac matrix:

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{14}$$

M is the quark mass and A^2 is the confinement strength. They have a different value for each quark flavour. In RHM, the confined single quark wave function (ψ) is given by:

$$\psi = N\begin{pmatrix} \phi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M}\phi \end{pmatrix} \tag{15}$$

with the normalization

$$N = \left(\frac{2(E+M)}{3E+M}\right)^{1/2} \tag{16}$$

where E is an eigenvalue of the single particle Dirac equation with the interaction potential given in (13). The lower component is eliminated by performing the similarity transformation,

$$U\psi = \phi \tag{17}$$

Where U is given by,

$$\frac{1}{N\left[1+\dfrac{\mathbf{P}^2}{(E+M)^2}\right]}\begin{pmatrix} \mathbf{1} & \dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} \\ -\dfrac{\boldsymbol{\sigma}\cdot\mathbf{P}}{E+M} & \mathbf{1} \end{pmatrix} \tag{18}$$

Here, U is a momentum and state (E) dependent transformation operator. With this transformation, the upper component ϕ satisfies the harmonic oscillator wave equation.

$$\left[\frac{\mathbf{P}^2}{E+M}+A^2r^2\right]\phi=(E-M)\phi, \qquad (19)$$

which is like the three dimensional harmonic oscillator equation with an energy-dependent parameter Ω_n^2 :

$$\Omega_n = A\left(E_n+M\right)^{1/2} \qquad (20)$$

The eigenvalue of (19) is given by,

$$E_n^2 = M^2+(2n+1)\Omega_n^2 . \qquad (21)$$

Note that eqn.(19) can also be derived by eliminating the lower component of the wave function using the Foldy-Wouthuysen transformation as it has been done in [17-18].

Adding the individual contributions of the quarks we obtain the total mass of the hadron. The spurious centre of mass (CM) is corrected [25] by using intrinsic operators for the $\sum_i r_i^{\ 2}$ and $\sum_i \nabla_i^2$ terms appearing in the Hamiltonian. This amounts to just subtracting the CM motion zero point contribution from the E^2 expression. It should be noted that this method is exact for the 0S-state quarks as the CM motion is also in the 0S state.

The COGEP is obtained from the scattering amplitude [17-19]

$$M_{fi} = \frac{g_s^2}{4\pi}\bar{\psi}_i\gamma^\mu\frac{\lambda_i^a}{2}\psi_i D_{\mu\nu}^{ab}(q)\bar{\psi}_j\gamma^\nu\frac{\lambda_j^b}{2}\psi_j , \qquad (23)$$

where, $\bar{\psi}=\psi^+\gamma_0$, $\psi_{i/j}$ are the wave functions of the quarks in the RHM, $D_{\mu\upsilon}^{ab}=\partial_{ab}D_{\mu\upsilon}$ are the CCM gluon propagators in momentum representation, $g_s^2/4\pi$ (= α_s) is the quark-gluon coupling constant and λ_i is the color $SU(3)_c$ generator of the i^{th} quark. The details can be found in references [17-19]. Below we give the expressions for the central part of the COGEP.

The central part of COGEP is [19],

$$V_{COGEP}^{cent}(\vec{r}_{ij})= \frac{\alpha_s N^4}{4}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\left[D_0(\vec{r}_{ij})+\frac{1}{(E+M)^2}\left[4\pi\delta^3(\vec{r}_{ij})-c^4r^2D_1(\vec{r}_{ij})\right]\left[1-2/3\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right]\right] \qquad (24)$$

To calculate the matrix elements (ME) of COGEP, we have fitted the exact expressions of $D_0(\vec{r})$ and $D_1(\vec{r})$ by Gaussian functions. It is to be noted that the $D_0(\vec{r})$ and $D_1(\vec{r})$ are different from the usual Coulombic propagators. However, in the asymptotic limit ($\vec{r}\to 0$) they are similar to Columbic propagators and in the infra-red limit ($\vec{r}\to\infty$) they fall like Gaussian. In the above expression the c (fm^{-1}) gives the range of propagation of gluons. The D_0(r) and D_1(r) are given by,

$$D_0(\vec{r}) = \left(\frac{\alpha_1}{r}+\alpha_2\right)\exp\left[\frac{-r^2c_0^{\ 2}}{2}\right]; \qquad D_1(\vec{r}) = \frac{\gamma}{r}\exp\left[\frac{-r^2c_2^{\ 2}}{2}\right]$$

Where α_1 = 1.035994, α_2 = 2.016150 fm^{-1}, c_0 = $(3.001453)^{1/2}$ fm^{-1}, γ = 0.8639336 and c_2 = $(4.367436)^{1/2}$ fm^{-1}. It should be noted that in the limit c $\to$ 0, the central part of the COGEP goes over to the corresponding potential OGEP of the NRQM [19].

Tensor part of COGEP is [19],

$$V_{COGEP}^{TEN}(\vec{r}_{ij}) = -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{N^4}{(E+M)^2}\left(\frac{D_1''(\vec{r}_{ij})}{3} - \frac{D_1'(\vec{r}_{ij})}{3r}\right)\hat{S}_{ij} \tag{25}$$

Where $\hat{S}_{ij} = [3(\boldsymbol{\sigma}_i.\hat{r})(\boldsymbol{\sigma}_j.\hat{r}) - \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j]$

Where $\hat{\mathbf{r}} = \hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j$ is the unit vector in the direction of $\vec{r}$. In the above expression primes and double primes correspond to first and second derivatives of $D_1(\vec{r})$. The derivatives of $D_1(\vec{r})$ were fitted to Gaussian functions.

$$D_1'(\vec{r}_{ij}) = \frac{1}{r}\varepsilon \exp\left[\frac{-r^2 c_3^{\,2}}{2}\right] - \frac{1}{r^2}\gamma \exp\left[\frac{-r^2 c_2^{\,2}}{2}\right]$$

$$D_1''(\vec{r}_{ij}) = \frac{2}{r^3}\gamma \exp\left[\frac{-r^2 c_2^{\,2}}{2}\right] - \frac{2}{r^2}\varepsilon \exp\left[\frac{-r^2 c_3^{\,2}}{2}\right] + \frac{1}{r}\kappa r^2 \exp\left[-\frac{r^2 c_4^{\,2}}{2}\right]$$

ε = -1.176029 fm^{-1}, κ=5.118019 fm^{-4}, $c_3 = (2.117112)^{1/2}$ fm^{-1}, $c_4 = (3.255009)^{1/2}$ fm^{-1}

The spin-orbit part of COGEP is [19],

$$V_{COGEP}^{LS}(\vec{r}_{ij}) = -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{N^4}{(E+M)^2}\frac{1}{2r}\left[\left(\left[\vec{r}_{ij} \times (\vec{p}_i - \vec{p}_j)\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\right]\left[D_0'(\vec{r}_{ij}) + 2D_1'(\vec{r}_{ij})\right]\right)\right. \tag{26}$$

Where

$$D_0'(\vec{r}_{ij}) = \frac{1}{r}[\beta_1 + r\,\beta_2]\exp\left[\frac{-r^2 c_1^{\,2}}{2}\right] - \frac{1}{r^2}[\alpha_1 + r\,\alpha_2]\exp\left[\frac{-r^2 c_0^{\,2}}{2}\right]$$

$$2D_1'(\vec{r}_{ij}) = 2\left(\frac{1}{r}\varepsilon \exp\left[\frac{-r^2 c_3^{\,2}}{2}\right] - \frac{1}{r^2}\gamma \exp\left[\frac{-r^2 c_2^{\,2}}{2}\right]\right)$$

Where β_1=2.680358 fm^{-1}, β_2=-7.598860 fm^{-2} and $c_1 = (2.373588)^{1/2}$ fm^{-1}

It should be noted that in the limit c $\rightarrow$ 0, the central, tensor and spin-orbit part of the COGEP goes over to the corresponding potentials of the OGEP [19].

3. Results of D wave Meson Spectra in M1 and M2

In our investigation, we have expressed the product of quark-antiquark oscillator wave functions in terms of oscillator wave functions corresponding to the relative and centre-of-mass coordinates (CM). The normalised relative radial wave function for 0D state is,

$$\psi_{0D}(r_{ij}) = \frac{4}{\sqrt{15}}\frac{\exp\left(\frac{-r_{ij}^2}{2b^2}\right)}{b^{\frac{7}{2}}}\frac{r_{ij}^2}{\pi^{\frac{1}{4}}}$$

where b is the oscillator size parameter. There are seven parameters associated with the central parts of the potential. The masses of up (M_u), down (M_d), strange (M_s) quarks which are taken as free parameters in both M1 and M2. The other parameters are confinement strength a_c, the oscillator size parameter b and the strong coupling constant α_s. The value of b is fixed by minimizing the expectation value of the Hamiltonian for the pseudo scalar mesons. The confinement strength a_c is fixed by the stability for variation of mass of the mesons against the size parameter b. The

α_s is fixed by the π-ρ mass splitting. The mass difference arises from the colour magnetic term of OGEP/COGEP. In M2, A^2 is the confinement strength parameter and Ω (=1/b) is the oscillation parameter are fixed in the same way as in M1. In M2 there is an additional parameter c, termed CCM parameter which was fitted to iota (1440 MeV),$J^{pc} = 0^{-+}$ (the oldest glue ball candidate) as a digluon glue ball [19]. The values of the parameters used in our calculation in M1 and M2 are listed in table 1. The strength parameters of III, namely, g, g' and the scale parameter Λ were fixed to obtain S wave meson spectra. In the current work, these parameters values are fixed at the values chosen in [23-24] and are given in table 1. Among the non-central parts of the potentials, the hyperfine terms of III has 12 additional strength (κ) and size (η) parameters. These have been fixed as explained in our previous works [23-24]. We were able to reproduce the light P-wave meson masses with all η's and κ's held fixed and by only varying the κ_7 and κ_8 parameters. The values of κ_7 and κ_8 parameters used in our model are listed in tables 2. The oscillator quantum number for the CM wave functions is restricted to N_{cm}= 0. The Hilbert space of relative wave functions is truncated at radial quantum number n_{max} = 4. The Hamiltonian matrix is constructed for each meson separately in the basis states of $\left| N_{CM} = 0, L_{CM} = 0;\ {}^{2S+1}L_J \right\rangle$ and diagonalised.

We have investigated two singlet light D wave mesons and six triplet light D wave mesons namely π_2 (1670) (1^1D_2) , K_2(1770) (1^1D_2), ω(1650) (1^3D_1), K*(1680) (1^3D_1), K_2(1820) (1^3D_2), ω_3 (1670) (1^3D_3), K*(1780) (1^3D_3), ϕ_3 (1850) (1^3D_3) [15] in the frame work of M1 and M2. Table 4, gives the diagonal contributions to the masses of D wave mesons by linear confinement, kinetic energy, colour-electric (CE), colour-magnetic (CM), spin-orbit, tensor terms of OGEP and spin-orbit, tensor terms of III (in MeV) in M1. Table 5, gives the diagonal contributions to the masses of mesons by V_{conf}, color-electric (CE), color-magnetic (CM), spin-orbit, tensor terms of COGEP and spin-orbit, tensor terms of III (in MeV) in M2. The dominant contribution to the masses comes from the kinetic energy and linear confinement potential in M1 and from the Lorentz scalar plus a vector harmonic-oscillator potential in M2

Table 1. Values of parameters used in M1 and M2.

b	0.62 fm
$M_{u,d}$	380 MeV
M_s	560 MeV
a_c	10 MeV fm^{-1}
α_s	0.2
η_1	0.2 fm
η_2	0.29 fm
η_3	1.4 fm
η_4	1.3 fm
κ_1	1.8
κ_2	1.7
κ_3	1.9
κ_4	2.1
κ_5	-22.0
κ_6	-24.5

Table 2. Values of κ_7 and κ_8 parameters in M1 and M2.

Meson	κ_7	κ_8
ω(1650)	28.0	39.0
K*(1680)	40.0	50.0
K_2(1820)	37.0	45.5
ω_3 (1670)	-5.0	-8.0
K*(1780)	1.5	2.0
ϕ_3 (1850)	10.0	13.0

Table 3. Masses of D wave mesons (MeV)

$N^{2S+1}L_J$	Meson	Experimental Mass	Calculated Mass in M1	Calculated Mass in M2
1^1D_2	π_2 (1670)	1670±20	1696.6	1673.8
	K_2 (1770)	1773±8	1727.4	1768.8
1^3D_1	ω(1650)	1649 ± 24	1672.2	1622.9
	K*(1680)	1717 ± 27	1707.4	1734.9
1^3D_2	K_2(1820)	1816 ±13	1812.0	1824.0
1^3D_3	ω_3 (1670)	1667 ± 4	1719.2	1653.9
	K*(1780)	1776 ± 7	1776.7	1787.5
	ϕ_3 (1850)	1854 ± 7	1816.9	1899.7

The important role played by III in obtaining the masses of D mesons can be understood by examining table 7. In table 7, we have listed the calculated masses of triplet D wave mesons in M1 and M2 without the inclusion of III potential. The role of III is crucial in explaining the mass differences of D wave K mesons in both the models. It is interesting to note that the calculated masses without III contribution of triplet D wave mesons involving only u/d quarks are higher than the experimental masses. By comparing these values in both the models we conclude that the calculated masses of D wave mesons involving only u/d quarks are higher in case of M1. The inclusion of III lowers the masses of D mesons in u/d sector. In case of other triplet D wave mesons III has attractive contribution and the calculated masses are higher in M2 when compared with the values of M1. Table 6 gives the comparison between the diagonal OGEP and COGEP contributions to the masses of triplet mesons. In both the models the contribution is attractive. The contribution due to COGEP in M2 is less compared to that of OGEP in M1. In table 3 we have listed the calculated masses of D wave mesons in M1 and M2 which are in good agreement with the experimental masses
It has been reported in literature [8] that there is a common mass degeneracy of 1^3D_1 and 1^3D_3 states. But our results in both M1 and M2, for 1^3D_1 and 1^3D_3 states do not exhibit this degeneracy as the strength of tensor and spin-orbit interactions are different (table 4 and 5).

Table 4. The diagonal contributions to the masses of mesons by kinetic energy, color-electric (CE), color-magnetic (CM), spin-orbit, tensor terms of OGEP and spin-orbit, tensor terms of III (MeV) in M1.

Meson	V_{conf}	V_{kin}	V^{CE}_{OGEP}	V^{CM}_{OGEP}	V^{LS}_{OGEP}	V^{TEN}_{OGEP}	V^{LS}_{III}	V^{TEN}_{III}
π_2 (1670)	59.699	1693.11	-47.96	-6.24	...	...	...	...
K_2 (1770)	59.699	1723.14	-48.96	-4.23	...	...	...	...
ω(1650)	59.699	1693.11	-47.96	2.08	-40.32	-8.96	-41.83	-269.83
K*(1680)	59.699	1723.14	-48.96	1.41	-27.36	-6.08	-29.74	-245.11
K_2(1820)	59.699	1723.14	-48.96	1.41	-9.12	6.08	-9.91	224.56
ω_3 (1670)	59.699	1693.11	-47.96	2.08	26.88	-2.56	27.89	15.02
K*(1780)	59.699	1723.14	-48.96	1.41	18.24	-1.74	19.83	-2.73
ϕ_3 (1850)	59.699	1753.18	-49.64	0.96	12.38	-1.18	12.84	-12.16

Table 5. The diagonal contributions to the masses of mesons by V_{conf}, color-electric (CE), color-magnetic (CM), spin-orbit, tensor terms of COGEP and spin-orbit, tensor terms of III (MeV) in M2.

Meson	V_{conf}	V^{CE}_{COGEP}	V^{CM}_{COGEP}	V^{LS}_{COGEP}	V^{TEN}_{COGEP}	V^{LS}_{III}	V^{TEN}_{III}
π_2 (1670)	1675.26	-2.83	1.43	...	...	...	...
K_2 (1770)	1770.79	-3.13	1.22	...	...	...	...
ω(1650)	1675.26	-2.83	-0.48	2.12	0.44	-41.83	-269.83
K*(1680)	1770.79	-3.13	-0.41	1.81	-0.38	-29.74	-245.11
K_2(1820)	1770.79	-3.13	-0.41	0.60	0.38	-9.91	224.56
ω_3 (1670)	1675.26	-2.83	-0.48	-1.41	-0.13	27.89	15.02
K*(1780)	1770.79	-3.13	-0.41	-1.21	-0.11	19.83	-2.73
ϕ_3 (1850)	1866.32	-3.42	-0.35	-1.05	-0.09	12.84	-12.16

Table 6. Comparison between the diagonal OGEP and COGEP contributions to the masses of triplet mesons (MeV).

Meson	V_{OGEP}	V_{COGEP}
π_2 (1670)	-54.20	-1.40
K_2 (1770)	-53.19	-1.91
ω(1650)	-95.16	-0.75
K*(1680)	-80.99	-2.11
K_2(1820)	-50.59	-2.56
ω_3 (1670)	-21.56	-4.85
K*(1780)	-31.05	-4.86
ϕ_3 (1850)	-37.48	-4.91

Table 7. Comparison of masses of triplet D mesons in M1 and M2 (MeV) without III

Meson	Experimental Mass	Calculated Mass in NRQM	Calculated Mass in RHM
ω(1650)	1649 ± 24	1679.9	1676.3
K*(1680)	1717 ± 27	1716.1	1770.9
K_2(1820)	1816 ±13	1730.1	1768.2
ω_3 (1670)	1667 ± 4	1737.4	1670.4
K*(1780)	1776 ± 7	1755.9	1765.9
ϕ_3 (1850)	1854 ± 7	1777.7	1861.4

CONCLUSION

We have investigated the effect of the III on the masses of the D wave mesons in the frame work of NRQM and RHM. We have shown that the computation of the masses using only OGEP and COGEP is inadequate in case of NRQM and RHM respectively. The contribution of the III is found to be significant in both the models. To obtain the masses of D wave mesons, 5x5 Hamiltonian matrix was diagonalised. The contribution from the tensor and spin-orbit part of the III is found to be significant in case of triplet D wave mesons. To obtain the physical masses of the mesons in the K sector it is necessary to include the anti-symmetric part of III. There is a good agreement between the calculated and experimental masses of D wave mesons in both M1 and M2.

From our work, we cannot conclude that one of the models considered here is preferable, but it is generally recognized that models with relativistic dynamics are more rigorous from the theoretical point of view. Also, the models should include the confinement of gluons. Hence, M2 seems to be a better approach to investigate the light meson spectra. Our hope is that the good equivalence found between relativistic and non-relativistic spectra for two-quark systems persists for multi quark systems.

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REFERENCES

[1] D. Gromes, *Nucl. Phys*, **1977**, D1, 2445-2478.
[2] O. W. Greenberg, *Ann. Rev. Nucl. Part. Sci*, **1978**, 28, 327 -358.
[3] R. K. Bhaduri, L. E. Cohler, and Y. Nogami; Nuovo Cimento, **1981**, A65, 376-390.
[4] S. Godfrey, and N. Isgur, *Phys. Rev*, **1985**, D32, 189 **-**231.
[5] B. Silvestre-Brac, and C. Gignoux, *Phys. Rev*, **1985**, D32, 743-758.
[6] W. H. Blask, U. Bohn, M. G. Huber, B. Ch. Metsch, and H.R. Petry, *Z. Phys*, **1990**, A337, 327-335.
[7] C. Semay, and B. Silvestre- Brac,, *Nucl. Phys*, **1992**, A618, 455-472.
[8] L. Burakovsky, and T. Goldman, *Nucl. Phys*, **1997**, A625, 220-230.
[9] P. V. Chliapnikov, *Phys. Lett*, **2000**, B496, 129-136.
[10] A. De Rujula, H. Georgi, and S. L. Glashow, *Phys. Rev*, **1975**, D12, 147-162.
[11] G. ' t. Hooft, *Phys. Rev*. **1976**, D14, 3432-3448.
[12] C. Semay, and B. Silvestre- Brac, *Nucl. Phys*, **1999**, A 647, 72-96.
[13] F. Brau, C. Semay, and B. Silvestre- Brac, *Phys. Rev*, **2000**,D62, 1175011-1175014.
[14] R. Oda, K.Yamada, S. Ishida, M. Sekiguschi, and H. Wada, *Prog. Theor. Phys*, **1999**, 102, 102-124.
[15] C. Amsler et al.,(PDG), *Phys. Lett*, **2008**, B667, 1-42.
[16] S. B. Khadkikar, and S. K. Gupta, *Phys.Lett*, **1983**, B124, 523-526.

[17] S. B. Khadkikar, and K. B. Vijaya Kumar, *Phys. Lett*, **1991**, B254, 320-324.
[18] K. B. Vijaya Kumar, and S. B. Khadkikar, *Nucl. Phys*, **1993**, A556, 396-408.
[19] P. C. Vinodkumar, K. B.Vijaya Kumar, and S. B. Khadkikar, *Pramana-J. Phys*, **1992**, 39, 47 -70.
[20] S. B. Khadkikar, *Pramana-J. Phys*, **1985**, 24, 63-72.
[21] K. B. Vijaya Kumar, B. Hanumaiah, and S. Pepin, *Eur. Phys. J*, **2004**, A19, 247-250.
[22] Bhavyashri, K. B. Vijaya Kumar, B. Hanumaiah, S. Sarangi, and Shan-Gui Zhou: *J. Phys.G Nucl. Part. Phys*, **2005**, 31, 981 -986.
[23] Bhavyashri, S. Sarangi, Godfrey Saldanha, and K.B. Vijaya Kumar, *Pramana-J. Phys*, **2008**, 70, 75-85.
[24] K.B. Vijaya Kumar, Bhavyashri, Y.L. Ma, Antony Prakash Monteiro, *Int. J. of Mod.Phys*, **2009**, A24, 4209 -4220.
[25] Bhaghyesh, K.B. Vijaya Kumar and Antony Prakash Monteiro: *Archives of Physics research*, **2010**, 1(4), 200-204.
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