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Extended Abstract

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Tomorrow's science: Fractional calculus view of complexity

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Leonardo da Vinci was the last artist/scientist to make lasting contributions to scientific knowledge, before science broke away from Natural Philosophy. The scientific method, introduced in this breakup, was a strategy for a new way of knowing, involving quantification through the synthesis of simple phenomenological models and measurement. After 300 years of scientific success, we have run out of simple models, and are back with da Vinci at recognizing the importance of the qualitative in addition to the quantitative. Stock market crashes, flash mobs, power grid failures, earthquakes, forest fires, heart attacks, urban growth and even citations, are all exemplars of the ubiquitous complexity that characterize the signature events in our lives. The simplifying assumptions of normalcy, linearity, continuity, stability, ergodicity, and many others, central to modern science, are no longer tenable and require re-examination. A number of attempts have been made to develop new ways of doing science, which are respectful of the complexity of the phenomena being studied. Examples of such efforts that come to mind include Cybernetics, Systems Theory, Catastrophe Theory, Complexity Theory, Nonlinear Dynamics and their subsequent generalizations. The common element of these and other such efforts is the recognition that complex phenomena, whether natural or artificially constructed, ought to be treated as a whole and not selectively dissected and once understood, stitched back together. This talk does not seek to accomplish this Herculean task, but has the more modest goal of juxtaposing a few of the disparate contributions, made by a number of gifted scientists, into a single strategy for gaining understanding and acquiring a new kind of knowledge; one in which the qualitative can be, and often is, as important as the quantitative. This strategy is an application of da Vinci's approach to understanding and it forms the basis of Tomorrow's Science (a new book by the speaker), which in reality is five centuries old. Just as Newton's calculus replaced the geometric description of mechanical phenomena, a more general calculus is necessary to replace the fractal geometry of complex phenomena and this requires a new way of thinking. The fractal trajectories of complex dynamics are non-differentiable, and averages over ensembles of such trajectories are described by fractional derivatives of probability densities, in space, in time, or both.

The fractional calculus has developed in a number of significant ways in the recent past. Sokolov et al maintain that this calculus was restricted to the field of mathematics until the last decade of the twentieth century, when it became very popular among physicists as a powerful way to describe the dynamics of a variety of complex physical phenomena. For example, anomalous diffusion was described using fractional diffusion equations; viscoelastic materials were modeled using fractional Langevin equations; and complex dynamic systems could be governed using fractional control. In the last decade the concept of fractional dynamics has gained further attention in the statistical and chemical physics communities. Fractional differential equations have also been successfully applied to neural dynamics and ecology as well as to traditional fields of engineering.

Herein we provide an alternative interpretation of these extensions that involves the notion of a fluctuating trajectory and interpreting the fractional models as averages over an ensemble of these trajectories on the strange attractor. This view is consistent with one proposed as an extension of conservative Hamiltonian systems to fractional systems. This generalization of classical mechanics is based on a randomization of chronological or clock time in the traditional phase space using the notion of operational time and subordination. Without going into the details of the extension of Hamilton's equations of motion to fractional form it suffices to note that fractional derivatives in chronological time are interpreted as averages of a particle's displacement and momentum over the fluctuating operational time. Consequently, the dynamics of a single fractional harmonic oscillator, for example, is considered to be an average over an ensemble of harmonic oscillators. Stanislavsky emphasizes that each oscillator differs slightly from every other oscillator in frequency because of subordination. The phase space trajectories rather than being level energy curves instead spiral into the origin and the fractional oscillator "demonstrates a dissipative process stochastic by nature." For the more general dynamical systems considered herein there is no Hamiltonian with which to generate the equations of motion.

The solution to the fractional phase space equation is an inverse power-law probability density function, which describes all the phenomena mentioned earlier and many more. However, rather than focusing on mathematical formalism, this talk addresses the meaning of the mathematics and attempts to answer the question: Why is the fractional calculus entailed by complexity?

Bottom Note: *This work is partly presented at International Conference on Physics June 27-29, 2016, New Orleans, USA*