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Understanding the decay of atom in quantum theory of radiation using the concept of area

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ABSTRACT

In the present work we have worked out a vector model for spontaneous emission based on the principle of moving vectors. This simplified approach reflects the work of Weisskopf and Wigner of spontaneous emission.

Keywords: Spontaneous emission, zero point fluctuation, vector model.

INTRODUCTION

The Weisskopf – Wigner theory of spontaneous emission [1] is well known. The mechanism of spontaneous emission can be understood from quantum theory of radiation [2]. It is an isotropic perturbation always present and attributed in connection with the quantum theory of radiation to the all pervading zero point fluctuation of the electromagnetic field. The light excites the atoms: the zero point fluctuation de-excites them resulting in the re- radiation of light. An interesting consequence of the quantization of the radiation is the fluctuation associated with the zero point energy or the so called vacuum fluctuation. These fluctuations have no classical analogy and are responsible for many interesting phenomena including spontaneous emission. In the usual atom field interaction picture it can be shown that an atom in the upper level can make transition back and forth to the lower state in time even in the absence of an applied field. However it is seen experimentally that an atom in an excited state decays to the ground state with a characteristics life time but it does not make back and forth transitions. For a proper account of the atomic decay, a continuum of modes corresponding to a quantization cavity, which is infinite in extent needs to be included. In Weisskopf-Wigner approximation the equation of motion for the probability amplitude is given by

$$\dot{C}_a(t) = -\frac{\gamma}{2} C_a(t) \qquad (1)$$

where γ is the decay constant. A solution for equation (1) gives

$$P_{aa} \equiv |C_a(t)|^2 = \exp(-\gamma t) \qquad (2)$$

That is an atom in the excited state $a\rangle$ in vacuum decays exponentially in time with the life time $\tau = \frac{1}{\gamma}$.

In the present work we report a vector model which actually reflects the work of Wigner. In a recent work [3] this model has been applied to analyze the situation of quantum interference laser.

1. Quantization of the single mode and quantum fluctuation:

Consider a free field of frequency ω which is linearly polarized in the x direction in a cavity (a laser cavity) of length L. In this case the Maxwell equations with the boundary conditions that the electric field vector vanishes at z = o and L leads to the following expression for the cavity field [4].

$$E_x(Z,t) = \left(\frac{2\omega^2}{\epsilon_0 v}\right)^{1/2} q(t) \sin\vec{k}\, Z \qquad (3)$$

where $\vec{k}$ is the wave vector and ω is the frequency. $\vec{k} = (\frac{\omega}{c})z$, V is the volume of the cavity and ϵ_0 is the permitivity of free space. The amplitude of the electric field is governed by the time dependent factor q(t), which has the dimension of length, so that the electric field can be regarded as a kind of canonical position. The magnetic field $H_y(z,t)$ can similarly be expressed as

$$H_y(Z,t) = \left(\frac{\epsilon_0}{K}\right) \left(\frac{2\omega^2}{\epsilon_0 v}\right)^{1/2} \dot{q}(t) \cos kZ \qquad (4)$$

and its amplitude is governed by a kind of canonical momentum $\dot{q}$ (t).

The field energy H can be expressed as a sum of electric and magnetic field energy in the cavity.

$$H = \left(\frac{1}{2}\right) \int dV \{\epsilon_0 E_x^2(Z,t) + \mu_0 H_y^2(Z,t)\} \qquad (5)$$

Using equation (3) and (4), equation (5) can be written as

$$H = \frac{1}{2}(p^2 + \omega^2 q^2) \qquad (6)$$

This indicates that the field mode energy is precisely that of a unit mass harmonic oscillator, with electric and magnetic fields playing the role of position and momentum.

In the case of quantization the single mode field [4] we simply take the correspondence rule that the variables q and p are replaced by their operators equivalents $\hat{q}, \hat{p}$ satisfying the commutation rule

$$[\hat{q},\hat{p}] = i\hbar \quad (7)$$

so the electric field mode operator is written as

$$\hat{E}_x(Z,t) = \left(\frac{2\omega^2}{\epsilon_0 v}\right)^{1/2} \hat{q}\,(t) \sin\vec{k}\, Z \quad (8)$$

and

$$\hat{H}_y\,(Z,t) = \left(\frac{\epsilon_0}{K}\right)\left(\frac{2\omega^2}{\epsilon_0 v}\right)^{1/2} \hat{p}(t) \cos kZ \quad (9)$$

and the energy is written as

$$\hat{H} = \frac{1}{2}(\hat{p}^2 + \omega^2 \hat{q}^2) \quad (10)$$

The mode structure in quantum theory is identical to that of a classical theory, so different and interpretation phenomenon will have the same spatial dependant in both theories. The main points are the lake of commutability of the electric and magnetic field operators, and the discreteness of the field energy with the field states with n excitations having the energy.

$$E_n = (n+\frac{1}{2})\hbar\omega \quad (11)$$

The ground state or vacuum state is the state with no excitations (n=0) and in quantum theory possesses a residual energy of $\frac{1}{2}\hbar\omega$.

We now consider the case of annihilation and creation operators. Earlier we have characterized the electric and magnetic fields by operators $\hat{q}$, and $\hat{p}$ which are Hermitian. But it is traditional to introduce the non- Hermitian (which are therefore non-observable) annihilation ($\hat{a}$) and creation ($\hat{a}^+$)operators through the combinations.

$$\hat{a} = (2\hbar\omega)^{-1/2}\,(\omega\hat{q} + i\hat{p}) \quad (12)$$

$$\hat{a}^+ = (2\hbar\omega)^{-1/2}\,(\omega\hat{q} - i\hat{p}) \quad (13)$$

In terms of these operators we can write the field energy operators as

$$\hat{H} = \frac{1}{2}(\hat{a}^+\hat{a} + \frac{1}{2})\hbar\omega \quad (14)$$

The basic communication rule becomes

$$[\hat{a}^+\hat{a}] = 1 \quad (15)$$

and the electric field operator is

$$\hat{E}_x(Z,t) = \epsilon_0(\hat{a} + \hat{a}^+)\sin kZ \quad (16)$$

where the parameters ϵ_0 is given by

$$\epsilon_0 = \left(\frac{\hbar\omega}{\epsilon_0 V}\right)^{\frac{1}{2}} \quad (17)$$

This parameter is known as the electric field per photon.

The time dependant of the annihilation operator can be derived from the Heisenberg equation of motion

$$\frac{d\hat{a}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{a}] \quad (18)$$

so we have

$$\hat{a}(t) = \hat{a}(0)\exp(-i\omega t) \quad (19)$$

$$\hat{a}^+ = \hat{a}^+(0)\exp(i\omega t) \quad (20)$$

Although the annihilation and creation operators do not themselves describe physical variables, their "normal order" product

$$\hat{n} = \hat{a}^+\hat{a} \quad (21)$$

describe number of excitation 'n' in a single mode state $|n\rangle$

$$\hat{n}|n\rangle = n|n\rangle \quad (22)$$

The number $|n\rangle$ is the energy eigenstate of the Hamiltonian equation 14 with eigenstate E_n.

$$\hat{H}|n\rangle = \hbar\omega(\hat{a}^+\hat{a} + \tfrac{1}{2})|n\rangle = E_n|n\rangle \quad (23)$$

The lowest level $|o\rangle$ is defined through

$$\hat{a}|0\rangle = o \quad (24)$$

and the other state are given by

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \quad (25)$$

$$\hat{a}^+|n\rangle = \sqrt{(n-1)}|n+1\rangle \quad (26)$$

$|n\rangle = (n!)^{-1/2}(\hat{a}^{+})^{n}|0\rangle$ (27)

The annihilation and creation operators decreases or the excitation of the mode by one quanta and have only off diagonal matrix element between number states.

$\langle n-1|\hat{a}|n\rangle = \sqrt{n}$ (28)

$\langle n+1|\hat{a}^{+}|n\rangle = \sqrt{n-1}$ (29)

Quantum fluctuation of a single mode field:
We now consider the most important aspect of quantum fluctuations of a single mode field. We observe that although the number state describe a state of precisely defined energy, it does not describe a state of well defined field.

$\langle n|\hat{E}_x(Z,t)|n\rangle = \epsilon_0 \sin kz\,[\langle n|\hat{a}^{+}|n\rangle + \text{h.c}] = 0$ (30)

The mean field is zero, but the mean square is not zero. It is, one component of the field mode energy in equation (5)

$\langle n|\hat{E}_x^2(Z,t)\rangle$

$= \epsilon_0 \sin^2 kz[\langle n|\hat{a}^{+}\hat{a}^{+} + \hat{a}^{+}\hat{a} + \hat{a}\ \hat{a}^{+} + \hat{a}\ \hat{a}\ |n\rangle] = 2\,\epsilon_0^2 \sin^2 kz\,(n+\frac{1}{2})$ (31)

The vacuum state is not an empty but represents a field of rms magnitude ϵ_0 sin kz. If we average over the spatial variation of the single mode field, we find again that the electric field per photon is $\epsilon_0 = (\frac{\hbar\omega}{\epsilon_0 V})^{\frac{1}{2}}$.

These vacuum fluctuations are not negligible quantities and may be easily verified by substituting the values of the frequencies ω for a specified volume of the cavity.

2. Moving vectors:
The usual representation of the energy level diagram for a two level atoms indicating decay rate γ_a *and* γ_b is shown in the Fig 1.

The Weisskopf- Wigner theory of spontaneous emission justifies the inclusion of the phenomenological decay rates γ_a *and* γ_b in the Schrödinger equation for the transition probability amplitude. The probability of stimulated absorption including decay is given by

$|C_a(t)|^2 = (\frac{\wp E_0}{2\hbar})^2 \exp(\gamma_b t)\frac{\sin^2\frac{(\omega-\gamma)\,t}{2}}{\frac{(\omega-\gamma)^2}{2}}$ (32)

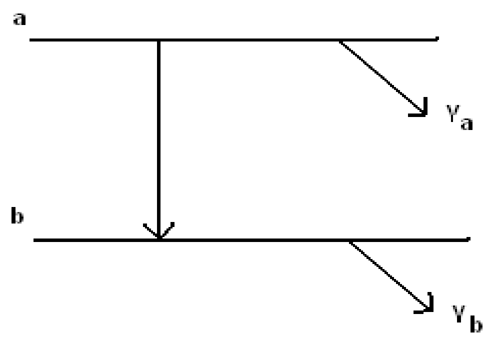

Fig 1: Energy level diagram for two level atom indicating decay rates γ_a and γ_b.

This transition probability for arbitrary value of time may be plotted for different values of detuning (ω-γ). Using the equation (3) the graph may be worked out which represents the time evaluation of population in the upper state for different values of detuning at 90,100,110.120 and 130 MHz. This shown in the Fig2.

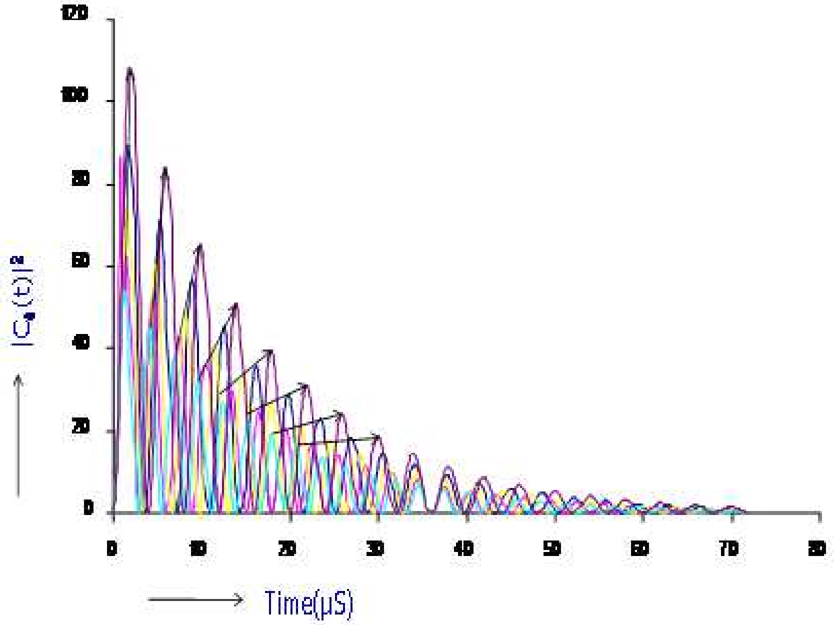

Fig 2: Transition probability for different values of detuning and γ=.005.

As may be inferred from Fig 2, we may join all the five maxima in a group of detunings as an arrow and a number of these arrows can be drawn in this way. We observe that these vectors evolve in time. These vectors can be used to represent transition probabilities for the changes of population in the upper state. The zero transition probabilities at arbitrary time of 6^2n (n=1, 2, 3…) indicate some type of collapse of wavefunction. It is worthwhile to note here that in the absence of the decay process, the semiclassical theory predicts Rabi oscillations for the atomic inversion whereas the quantum theory predicts certain collapse and revival phenomenon due to the quantum aspects of the field. The Rabi model [5] so named because of its original setting in magnetic resonance as studied by Rabi long ago. The collapse of Rabi oscillations was noted fairly early in the study of Rabi[6]. Several years later it was found that Rabi oscillations starts to revive [7] although not completely. At longer times one finds a sequence of collapses and revivals, the revival becoming less distinct as time increases. These collapse and revival behavior of Rabi oscillations in the fully quantized model is strikingly different than in the semiclassical case. In our present work we have seen that the collapses and revival of transition probabilities is similar to the quantum model which is known as Jaymes-Cummings model[8].

In this connection, it is worthwhile to note that we describe the properties of a quantum system prepared in superposition of classically distinguishable states. These states often called Schrödinger cat states are of great interest. Recently, it has been shown how they may be realized in quantum optics using non-linear interaction. It has also been demonstrated the interference properties which characterise superposition states [9] and particularly the Schrödinger cat states. In our present work, the collapse of wave function at arbitrary time of 36n, n= 0,1,2…….. represents some sort of interference resembling Schrödinger cat state.

The moving vectors can be represented by the equation

$$\frac{dk_n}{dt} = \Omega_n \times k_n \tag{33}$$

where Ω_n is the driving field vector ,
k_n is the amplitude vector.

We have worked out different phasor diagrams with different decay constants and they are shown in Fig.3(a,b).

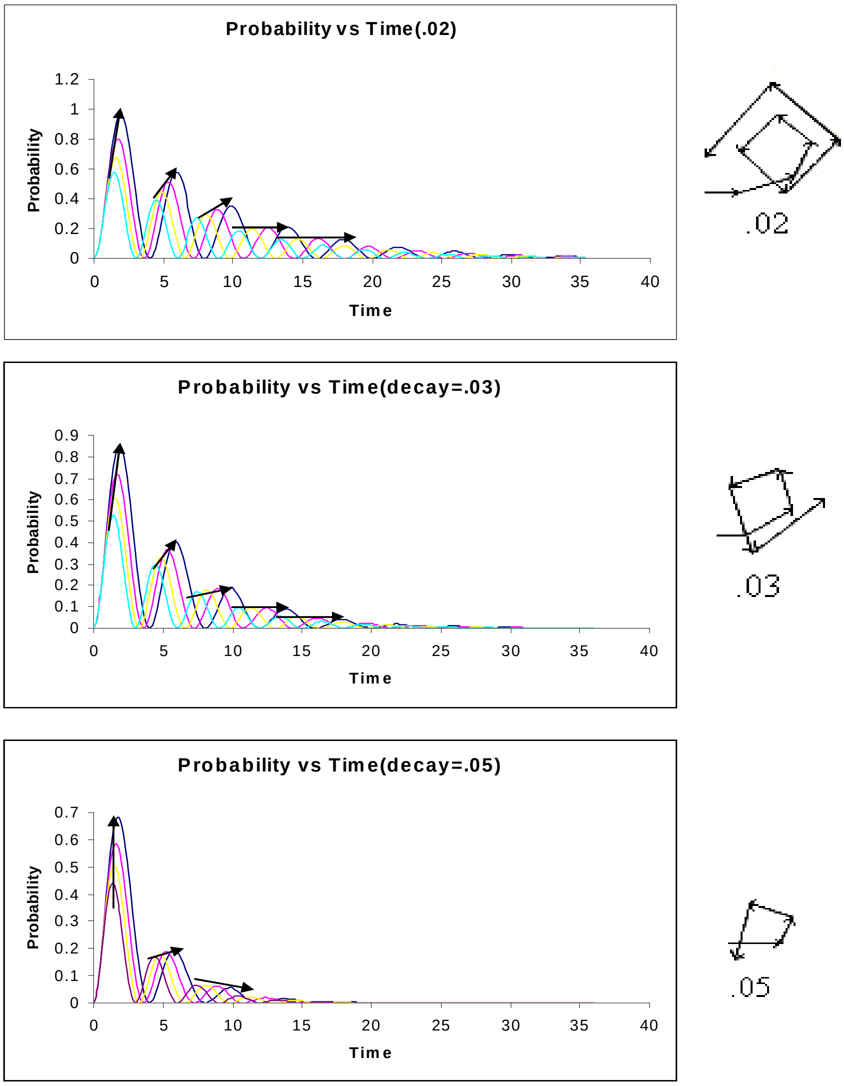

Fig 3(a): Probability versus Time graphs for decay=.001, .005, .01and their corresponding phasor diagrams.

We may note that the vectors in the phasor diagrams first closes up and uncoils again and expands. It is also observed that when decay is small the expansion is slow but for bigger decay the expansion is more. This is a general nature of all the phasor diagrams we have constructed with different decay constants and with the same set of detuning values as shown in Fig 3(a),(b).

The phasor diagrams also indicate that as the decay γ increases the area decreases exponentially which may be illustrated by plotting decay versus area graph as shown in Fig 4. As regards its physical significance it may be noted that the parameter γ is related to the lifetime as $\tau = 1/\gamma$.

This follows from the theory of spontaneous emission. In the present case the area behaves like lifetime. The result is new.

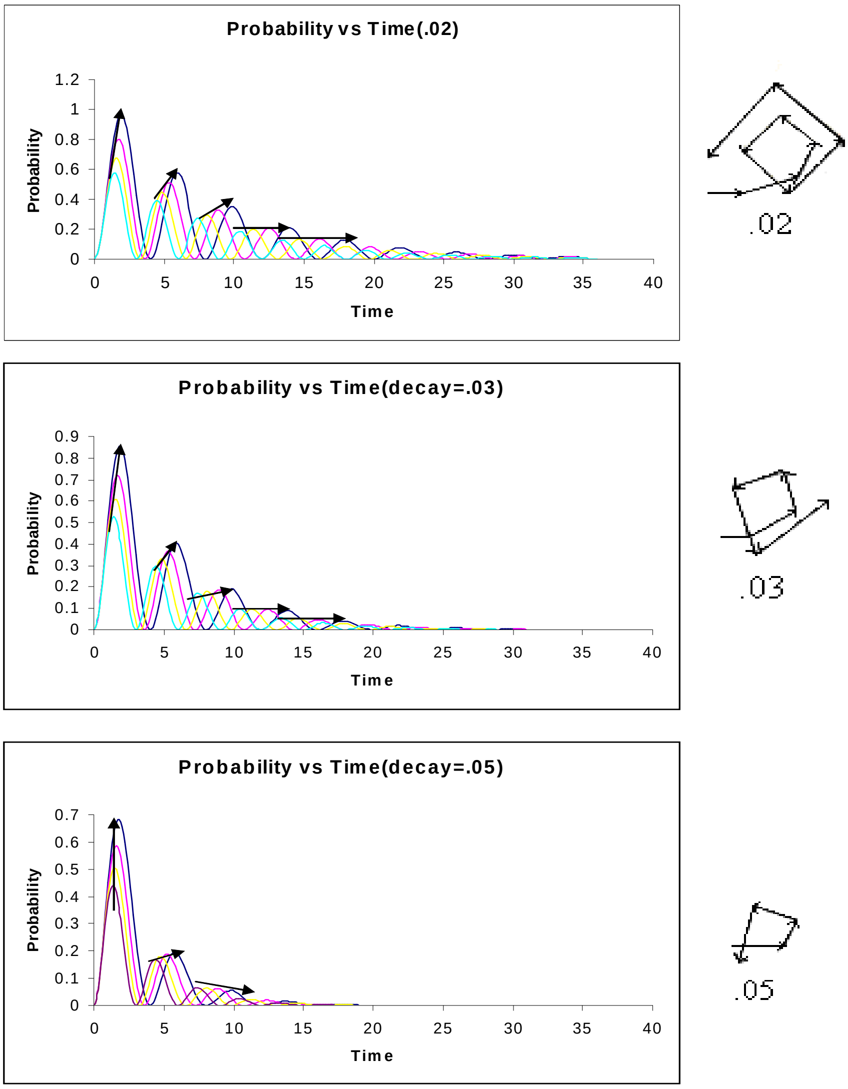

Fig 3(b): Probability versus Time graphs for decay=.02, .03, .05and their corresponding phasor diagrams.

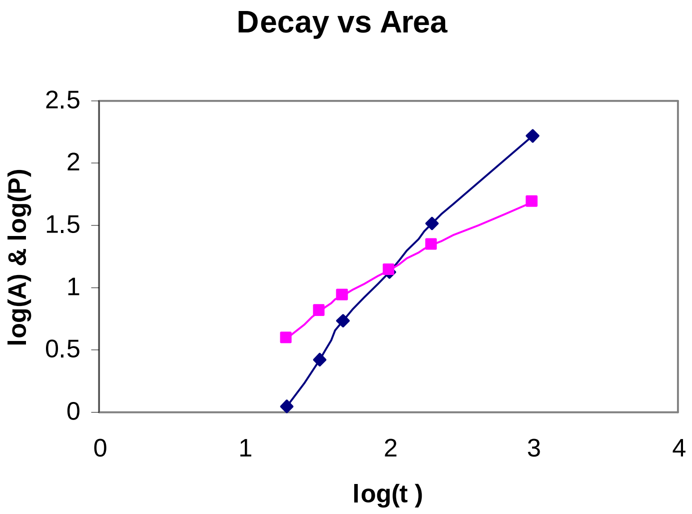

Fig 4: Decay versus Area

It is worthwhile to note here that the concept of vectors to explain the equation of motion of Galileo is a well-known concept in school level text. As for example the interesting feature of velocity time graph for any moving object is that the area under v-t graph equals displacement of the object over a given time interval. There are other examples where the concept of area is used to represent vectors.

CONCLUSION

From what has been discussed above it is reasonable to draw a conclusion. We have seen that areas of phasor diagrams at different values of decay behaves as lifetime and indicates exponential decay similar to that shown by the quantum theory of radiations. The vector model may be quite handy in explaining many phenomena in quantum optics and there is scope for improvement. We have also indicated an analogy with S.C. States with our present work.

REFERENCES

[1]. V.F. Weisskopf and E. Wigner, *Z. Physick* **1930,** 39 , 54.
[2]. M. Sargent III, M.O. Scully and W.E. Lamb Jr. Laser Physics (Addison- Wesley Publishing Company Reading, Massachusetts **1974**).
[3]. Farzana B. Hazarika, S. N. Rai and G. D. Baruah optics- Int. J. Light Electron opt. **2011**, 122 , 85.
[4]. R. London Quantum Theory of Light (Oxford U.P. oxford **1983**). 2nd Ed. M. Sargent III, M.O. Scully, and W.E. Lamb Jr. Laser Physics [Addition Wesley, Reading MA **1974**]; P Meystre and G.J. Milburn, Quantum optics (Spinger Verlag, Berlin **1994**), L. Mandel and E.Wolf, Optical coherence and Quantum Optics, (Cambridge U.P. Cambridge, **1995**)
[5]. I.I. Rabi, *Phys. Rev*. **1937**, 51, 652.
[6]. F.W. Commings *Phys. Rev*. 149 ,104.
[7]. J.H. Eberley , N.B. Narozhny and J.J. Sanchez- Mondragon, *Phys. Rev. Lett,* **1980,** 44 , 1323.

[8]. E. T. Jaynes and F. W. Cummings *Proc. IEEE*, **1963**, 51, 89
[9]. C.C. Gerry and P.L. Knight, *Am. J. phys.* Oct **1997**, 65,10.