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Velocity and Acceleration in Elliptic Cylindrical Coordinates

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ABSTRACT

In this paper, we derive the expressions for the velocity and acceleration for bodies in Elliptic cylindrical coordinate systems. These expressions for the velocity and acceleration so derived can be applied in the formulation of Kinetic energy and Newton's equations of motion for a particle in a homogeneous elliptic cylinder.

Keywords: Velocity, Acceleration, Elliptic Cylindrical Coordinates

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INTRODUCTION

In general, the coordinates (u, v, w) of a coordinate system can be used to specify a point in that coordinate system. In the Cartesian coordinate system, the coordinates (x, y, z) is used to describe a point in the Cartesian coordinate system. This description of points can be extended to other coordinate systems, however, in this paper we are interested in the elliptic cylindrical coordinate system. In the elliptic cylindrical coordinate, the coordinates (u, v, z) is used to describe a point in space. These coordinates (u, v, z) of the elliptic cylindrical coordinates can be expressed in terms of the Cartesian coordinates and consequently, the well known formulations for velocity and acceleration in the Cartesian coordinates can thus be extended to the Elliptic cylindrical coordinates.

MATHEMATICAL ANALYSIS

The elliptic cylindrical coordinates (u, v, z) can be expressed in terms of the Cartesian coordinates (x, y, z) as [1]

$$x = a \cosh u \cos v \tag{1}$$

$$y = a \sinh u \sin v \tag{2}$$

$$z = z \tag{3}$$

where

$$u \geq 0, \qquad 0 \leq v < 2\pi, \qquad -\infty < z < \infty$$

The scale factors $(h_u, h_{v,} h_z)$ in the elliptic cylindrical coordinates are defined as

$$h_u = a\sqrt{\sinh^2 u + \sin^2 v} \tag{4}$$

$$h_v = a\sqrt{\sinh^2 u + \sin^2 v} \tag{5}$$

$$h_z = 1 \tag{6}$$

The instantaneous position vector $\underline{r}$ is given by

$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k} \tag{7}$$

In terms of the elliptic cylindrical coordinates, the instantaneous position vector is expressed as [2],[3]

$$\underline{r} = a\cosh u\cos v\hat{i} + a\sinh u\sin v\hat{j} + z\hat{k} \tag{8}$$

and the unit elliptic cylindrical unit vectors $(\hat{u}, \hat{v}, \hat{z})$ is expressed in terms of the Cartesian unit vector $(\hat{i}, \hat{j}, \hat{k})$ as

$$\hat{u} = \frac{\sinh u\cos v\hat{i} + \cosh u\sin v\hat{j}}{\left(\sinh^2 u + \sin^2 v\right)^{\frac{1}{2}}} \tag{9}$$

$$\hat{v} = -\frac{\cosh u\sin v\hat{i} + \sinh u\cos v\hat{j}}{\left(\sinh^2 u + \sin^2 v\right)^{\frac{1}{2}}} \tag{10}$$

and

$$\hat{k} = \hat{k} \tag{11}$$

Now, by definition [2], [4] the instantaneous velocity $\underline{u}$ is given as:

$$\underline{u} = \underline{\dot{r}} \tag{12}$$

where $\underline{\dot{r}}$ denotes a derivative with respect to time.

Therefore, in terms of the elliptic cylindrical coordinates, equation (12) can be expressed as

$$u = u_u\hat{u} + u_v\hat{v} + u_z\hat{k} \tag{13}$$

where u_u, u_v and u_z are the components of the instantaneous velocity in the u, v and z directions respectively.

Substituting equations (9) – (11) into (12) and simplifying further yields the components of the instantaneous velocity expressed explicitly as follows:

$$u_u = a\left(\sinh^2 u + \sin^2 v\right)^{\frac{1}{2}}\dot{u} \tag{14}$$

$$u_v = a\left(\sinh^2 u + \sin^2 v\right)^{\frac{1}{2}}\dot{v} \tag{15}$$

$$u_z = \dot{z} \tag{16}$$

Likewise, the instantaneous acceleration $\underline{a}$ is given by definition [4],[5] as

$$\underline{a} = a_u\hat{u} + a_v\hat{v} + a_z\hat{z} \tag{17}$$

where a_u, a_v and a_z are the components of the instantaneous acceleration in the u, v and z directions respectively. Therefore, substituting equations (14) – (16) into equation (17) and after simplification, we can explicitly express the instantaneous acceleration given in equation (17) in its respective components as

$$a_u = \frac{a}{\left(\sinh^2 u + \sin^2 v\right)^{\frac{1}{2}}}\left\{\begin{array}{l}\left(\sinh^2 u + \sin^2 v\right)\ddot{u} + 2\sin v\cos v\dot{u}\dot{v} \\ + \cosh u\sinh u\dot{u}^2 - \cosh u\sinh u\dot{v}^2\end{array}\right\} \tag{18}$$

$$a_v = \frac{a}{\left(\sinh^2 u + \sin^2 v\right)^{\frac{1}{2}}} \begin{Bmatrix} \left(\sinh^2 u + \sin^2 v\right)\ddot{v} + 2\sinh u \cos v \dot{u}\dot{v} \\ -\cos v \sin v \dot{u}^2 + \cos v \sin v \dot{v}^2 \end{Bmatrix} \quad (19)$$

$$a_z = \ddot{z} \quad (20)$$

The equations (18) – (20) completely describes the acceleration of a body in the elliptic cylindrical coordinates.

RESULTS AND DISCUSSION

In this paper, we have derived the velocity in elliptic cylindrical coordinates along the u, v and z directions as stated explicitly by equations (14) – (16) respectively. Also, the acceleration in the elliptic cylindrical coordinates was derived and given explicitly for the u, v and z directions by equations (18) – (20).

CONCLUSION

The velocity and acceleration in the elliptic cylindrical coordinates derived here allows for the formulation of the equations of motion for bodies in the elliptic cylindrical coordinate systems. Also, the equations derived can be extended to applications for bodies in the elliptic cylindrical coordinates.

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