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Velocity and acceleration in prolate spheroidal coordinates

J. F. Omonile[1], D. J. Koffa[2] and S. X. K. Howusu[2]

[1]*Department of Physics, Kogi State University, Anyigba, Nigeria*
[2]*Department of Physics, Federal University Lokoja, Lokoja, Nigeria*

ABSTRACT

In this paper, we are out to derive the expression for instantaneous velocity and acceleration entirely in terms of Prolate Spheroidal Coordinates for applications in Newtonian's Mechanics, Einstein's Special Law of Motion and Schrödinger's Law of Quantum mechanics.

Keywords: velocity, acceleration, prolate spherical coordinates.

INTRODUCTION

The prolate spheroidal coordinates (η, ξ, ϕ) are defined in terms of the Cartesian coordinates x, y, z [Howusu, 2004], by;

$$x = a\,(1-\eta^2)^{1/2}(\xi^2-1)^{1/2}\cos\phi \quad (1)$$

$$y = a\,(1-\eta^2)^{1/2}(\xi^2-1)^{1/2}\cos\phi \quad (2)$$

$$z = a\eta\xi \quad (3)$$

where a is a constant and
$$0 < \xi < \infty, -1 \le \eta \le 1, \quad 0 \le \phi \le 2\pi \quad (4)$$

Consequently, by definition, the prolate spheroidal scale factors or metrical coefficients are given by;

$$h_\eta = a\left(\frac{\xi^2-\eta^2}{1-\eta^2}\right)^{\frac{1}{2}} \quad (5)$$

$$h_\xi = a\left(\frac{\xi^2-\eta^2}{\xi^2-1}\right)^{\frac{1}{2}} \quad (6)$$

and

$$h_\phi = a(1-\eta^2)^{\frac{1}{2}}(\xi^2-1)^{\frac{1}{2}} \quad (7)$$

These scale factors define the unit vectors, line element, volume element, as well as gradient, divergence, curl and Laplacian operators in prolate spheroidal coordinates, according to the theory of orthogonal curvilinear coordinates [5, 8, 9.]. These quantities are necessary and sufficient for the derivation of the fields of all prolate spheroidal distribution of mass, charge and current. Now for the formulation of the equations of motion for test particles in these fields, we shall derive the expression for instantaneous velocity and acceleration in prolate spheroidal coordinates.

MATHEMATICAL ANALYSIS

By definition the prolate spheroidal unit vectors are given in terms of the Cartesian unit vectors as:

$$\hat{\eta} = \frac{-\eta(\xi^2-1)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}cos\phi\hat{\imath} - \frac{\eta(\xi^2-1)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}sin\phi\hat{\jmath} + \frac{\xi(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}\hat{k} \tag{8}$$

and

$$\hat{\xi} = \frac{-\xi(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}cos\phi\hat{\imath} + \frac{\xi(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}sin\phi\hat{\jmath} + \frac{\eta(\xi^2-1)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}\hat{k} \tag{9}$$

and

$$\hat{\phi} = -sin\phi\hat{\imath} + cos\phi\hat{\jmath} \tag{10}$$

Hence denoting time differentiation by a dot, it follows from (8), (9), (10) and some manipulation that;

$$\dot{\hat{\eta}} = \frac{-1}{\xi^2-\eta^2}\left[\frac{(\xi^2-1)^{\frac{1}{2}}}{(1-\eta^2)^{\frac{1}{2}}}\xi\dot{\eta} + \frac{(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-1)^{\frac{1}{2}}}\eta\dot{\xi}\right]\hat{\xi} - \frac{\eta(\xi^2-1)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}\dot{\phi}\hat{\phi} \tag{11}$$

Similarly, it follows from (9) , (8) and (10) that:

$$\dot{\hat{\xi}} = \frac{1}{\xi^2-\eta^2}\left[\frac{(\xi^2-1)^{\frac{1}{2}}}{(1-\eta^2)^{\frac{1}{2}}}\xi\dot{\eta} + \frac{(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-1)^{\frac{1}{2}}}\eta\dot{\xi}\right]\hat{\eta} + \frac{\xi(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}\dot{\phi}\hat{\phi} \tag{12}$$

It also follows from (10), and (8), (9) that;

$$\dot{\hat{\phi}} = \frac{(\xi^2-1)^{\frac{1}{2}}}{(1-\eta^2)^{\frac{1}{2}}}\eta\dot{\phi}\hat{\eta} - \frac{(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-1)^{\frac{1}{2}}}\xi\dot{\phi}\hat{\xi} \tag{13}$$

Now it follows from definition of instantaneous position vector, $\underline{r}$, as

$$\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \tag{14}$$

And (8)-(10) that the instantaneous position vector may be expressed entirely in terms of prolate spheroidal coordinates as;

$$\underline{r} = \frac{a\eta(1-\eta^2)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}\hat{\eta} + \frac{a\xi(\xi^2-1)^{\frac{1}{2}}}{(\xi^2-\eta^2)^{\frac{1}{2}}}\hat{\xi} \tag{15}$$

It also follows from definition of instantaneous velocity, $\underline{u}, as$:

$$\underline{u} = \dot{\underline{r}} \tag{16}$$

and (15) , (11)-(13) that the instantaneous velocity vector may be expressed entirely in terms of prolate spheroidal coordinates as:

$$\underline{u} = u_\eta\hat{\eta} + u_\xi\hat{\xi} + u_\phi\hat{\phi} \tag{17}$$

where;

$$u_\eta = a\left(\frac{\xi^2-\eta^2}{1-\eta^2}\right)^{\frac{1}{2}}\dot{\eta} \tag{18}$$

$$u_\xi = a\left(\frac{\xi^2-\eta^2}{\xi^2-1}\right)^{\frac{1}{2}}\dot{\xi} \tag{19}$$

and

$$u_\phi = a(1-\eta^2)^{\frac{1}{2}}(\xi^2-1)^{\frac{1}{2}}\dot{\phi} \tag{20}$$

Consequently, it follows from definition of instantaneous acceleration, $\underline{a}$, as:

$$\underline{a} = \underline{\dot{u}} \tag{21}$$

And (18)-(20), (11)-(13) that the instantaneous acceleration may be expressed entirely in terms of prolate spheroidal coordinates as:

$$\underline{a} = a_\eta\hat{\eta} + a_\xi\hat{\xi} + a_\phi\hat{\phi} \tag{22}$$

where

$$\underline{a_\eta} = a\left(\frac{\xi^2-\eta^2}{1-\eta^2}\right)^{1/2}\left\{\ddot{\eta} + \frac{2\xi}{\xi^2-\eta^2}\dot{\eta}\dot{\xi} + \frac{\eta\,(\xi^2-1)}{(1-\eta^2)(\xi^2-\eta^2)}\dot{\eta}^2 + \frac{\eta\,(1-\eta^2)}{(\xi^2-1)(\xi^2-\eta^2)}\dot{\xi}^2 + \frac{\eta\,(1-\eta^2)(\xi^2-1)}{(\xi^2-\eta^2)}\dot{\phi}^2\right\}\hat{\xi} \tag{23}$$

$$\underline{a_\xi} = a\left(\frac{\xi^2-\eta^2}{\xi^2-1}\right)^{1/2}\left\{\ddot{\xi} - \frac{2\eta}{\xi^2-\eta^2}\dot{\eta}\dot{\xi} - \frac{\xi\,(\xi^2-1)}{(1-\eta^2)(\xi^2-\eta^2)}\dot{\eta}^2 - \frac{\xi\,(1-\eta^2)}{(\xi^2-1)(\xi^2-\eta^2)}\dot{\xi}^2 - \frac{(\xi^2-1)(1-\eta^2)}{(\xi^2-\eta^2)}\dot{\phi}^2\right\}\hat{\xi} \tag{24}$$

and

$$\underline{a_\phi} = a[(1-\eta^2)(\xi^2-1)]^{1/2}\left\{\ddot{\phi} - \frac{2\eta}{(1-\eta^2)}\dot{\eta}\dot{\phi} + \frac{2\xi}{(\xi^2-1)}\dot{\xi}\dot{\phi}\right\}\hat{\phi} \tag{25}$$

This is the completion of the theory of oblate spheroidal coordinates system.

RESULTS AND DISCUSSION

In this paper we have successfully derived the components of velocity and acceleration in prolate spheroidal coordinates as (18)-(20) and (23)-(25) respectively.

The results obtained in this study are necessary and sufficient for expressing all mechanical quantities (linear momentum, kinetic energy, Lagragian and Hamiltonian) in terms of prolate spheroidal coordinates

CONCLUSION

Consequently, the way is opened for expressing all dynamical laws of motion (Newton's law, Lagrange's law, Hamilton's law, Einstein's special Relativistic law of motion, and Schrodinger's law of quantum mechanics) entirely in terms of prolate spheroidal coordinates.

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