Whistler mode instability with AC electric field for relativistic Subtracted bi-Maxwellian Magneto-Plasma

R S Pandey and D K Singh

Department of Applied Physics, Amity Institute of Applied Science, ASET, Noida, Amity University U.P. India

ABSTRACT

The instability of field-aligned Whistler waves is studied for subtracted bi-Maxwellian magneto-Plasma in the presence of perpendicular AC electric field by using the method of characteristic solutions and kinetic approach. The dispersion relation has been derived. The growth rate has been calculated for magnetospheric Plasma. Calculation shows that either a loss-cone or a thermal anisotropy in the hot plasma component of the magnetosphere can lead to the generation of incoherent emission of low frequency Whistler waves. The effective parameters for the generation of Whistler mode wave are not only the temperature anisotropy but also the relativistic factor, AC field frequency, amplitude of subtracted distribution and width of the loss-cone distribution function which are discussed in result and discussion section.

INTRODUCTION

The electron temperature anisotropy driven whistler mode instability for superathermal power law tail on the electron population, such as is observed in the Earth’s electron foreshock has been investigated. The enhancement in energetic electrons significantly accentuates Whistler growth at wave numbers smaller than those expected for bi-maxwellian electron distribution of similar temperature anisotropy and brings the unstable frequencies and wavelengths closer to those expected for 1Hz Whistler waves observed in the electron foreshock by many authors [1, 2]. Due to the presence of power law tail, further a rapid increase in growth rate with increasing electron temperature anisotropy [3].

Electrons can be scattered into the Loss-cone by wave particle interaction between whistler mode radiation and energetic electrons in the magnetosphere. The whistler mode radiation responsible for this pitch angle diffusion may take the form of either incoherent or coherent emissions.
Incoherent whistler mode radiation can result from instabilities driven by anisotropies in the velocity distribution of the hot electrons [4, 5]. Coherent whistler emissions can result from phase bunching of energetic electrons triggered by external emissions or by incoherent whistler waves [6]. The scattering electron into the loss-cone by incoherent whistler radiation was considered by Kennel and Petschek [7], while the pitch angle scattering caused by coherent whistler emissions was treated by Inan et al [8]. Whistler waves are believed to play an important role in the generation of the pulsating aurora. It is now firmly established that the glow of the pulsating aurora is caused by the quasi-periodic filling of the loss-cone and the subsequent precipitation of energetic electrons into the atmosphere [9]. Based on electron energy measurements obtained from sounding rockets experiments, the scattering mechanism is believed to occur in the equatorial plane. The ionosphere is believed to play some role in determining the spatial characteristics of the pulsating aurora because the drift rate of the auroral patches appears to be related to the movement of the neutral atmosphere [10]. Oguti [11] proposed that local enhancements in the ionospheric plasma density may explained to fill adjacent flux tubes with different plasma densities, thereby creating the conditions needed at the equator to trigger auroral pulsations.

The measurements of large-amplitude whistler waves by the electric field instruments aboard the STEREO (Solar Terrestrial Relations Observatory) satellite in the Earth’s radiation belt [12] and THEMIS (Time History of Events and Macro scale Interactions during Substorms) [13] has stimulated the investigation of mechanisms by which large amplitude obliquely propagating whistler waves can be generated. Electron temperature anisotropy can be excluded as a free energy source since the growth rate always has its maximum for parallel propagation. Nevertheless, the simultaneous observation of parallel and obliquely propagating whistler waves has already been described in earlier papers on chorus emission [14].

In the recent past whistler mode energetic electrons interaction assuming bi-Lorentzian (Kappa) particle distribution was studied by Thorne and summers [15]. These authors derived the dispersion relation in the absence of AC electric field. These studies require the understanding of plasma properties subjected to fields of oscillating nature. In addition to injection of AC fields into space [16] electric field measurements at magnetospheric heights and in shock regions have given values of AC field along and perpendicular to Earth’s magnetic field [17-21].The behavior of plasma in high frequency parallel and perpendicular AC fields have also been studied by a large number of investigators. Whistler mode emission constitutes electromagnetic waves with frequency below either electron gyro frequency or local electron plasma frequency, whatever is less [22].Though cyclotron wave particle interaction [23-28] such waves provide a viable mechanism for stochastic acceleration and pitch angle scattering loss of energetic trapped electrons in the earth radiation belt[29]. Natural whistler mode emissions have been found in radiation belt environment of all magnetized planets and analogous to other mode waves, [30] they can easily propagated over a wide range of magnetosphere. Whistler mode waves can be generated by cyclotron resonant interactions with anisotropic energetic electrons [27]. The cyclotron resonant energies are capable of exceeding the electron rest mass under certain magnetospheric conditions. For fully relativistic treatment particularly distribution function is required to better understanding of origin of waves. Energetic particle distribution in the tenuous and collisionless space plasma basically presents a pronounced non –maxwellian high energy tail that can be well modeled by a generalized lorentzian distribution [31]. Summers et al [32]
have derived general properties of dielectric tensor of the kappa distribution function. Moreover, Mace [33] has derived a dielectric tensor for the kappa distribution, which extended the result of Summers [34] and allowed further progress in evaluation of perpendicularly propagating wave [35, 36]. The result presented by Qing-Hua et al [37] for understanding of plasma wave instability particularly of field aligned whistler mode waves in space plasma has been studied by using a recently developed generalized relativistic kappa type distribution.

Motivated by these studies, in the present paper, whistler mode instabilities have been analyzed for subtracted bi-Maxwellian magneto-Plasma in the presence of perpendicular AC electric field by using the method of characteristic solutions and kinetic approach. Using details of particle trajectories, dispersion relation and growth rate have been derived in analytical form and are evaluated for plasma parameters suited to the Earth’s magnetosphere. Results have been discussed and compared with that obtained by earlier workers using Maxwellian distribution. The effective parameters for the generation of Whistler mode wave are not only the temperature anisotropy but also the relativistic factor, AC field frequency, amplitude of subtracted distribution and width of the loss-cone distribution function which are discuss in result and discussion section.

2. DISPERSION RELATIONS AND GROWTH RATE:
A homogeneous anisotropic collision less plasma in the presence of an external magnetic field $B_0 = (B_0 e_z)$ and an electric field $E_{ox} = E_0 \sin vt \ e_x$ is assumed. In interaction zone in homogeneity is assumed to be small. In order to obtain the particle trajectories perturbed distributions function and dispersion relation, the linearised Vlasov-Maxwell equations are used. Separating the equilibrium and non equilibrium parts, neglecting the higher order terms and following the techniques of Pandey et al [38,39] the linearized Vlasov equations are given as:

$$v \left( \frac{\delta f_0}{\delta r} \right) + \left( \frac{e_s}{m_e} \right) \left( E_0 \sin vt + \left( \frac{v \times B_0}{c} \right) \right) \left( \frac{\delta f_0}{\delta r} \right) = 0$$

...(1)

$$\left( \frac{\delta f_1}{\delta r} \right) + v \left( \frac{\delta f_1}{\delta r} \right) + \left( \frac{F}{M_e} \right) \left( \frac{\delta f_1}{\delta r} \right) = S(r, v, t)$$

...(2)

Where the force

$$F = e \left[ E_0 \sin vt + \left( \frac{v \times B_0}{c} \right) \right] = m \frac{dv}{dt}$$

...(3)

Where $v$ is AC frequency and

$$S(r, v, t, s) = \left( \frac{e_s}{m_e} \right) \left[ E_1 + \left( \frac{v \times B_0}{c} \right) \right] \left( \frac{\delta f_1}{\delta r} \right)$$

...(4)

where $s$ denotes the type of electrons. Subscript '0' denotes the equilibrium values. The perturbed distribution function $f_1$ is determined by using the method of characteristic, which is
f_i (r, v, t) = \int_0^\infty S(f_0 (r, v, t), v_0 (r, v, t), t - t') dt

We have transformed the phase space coordinate system for (r, v, t) to (r_0, v_0, t - t'). The relativistic particle trajectories that have been obtained by solving equation (3) for given external field configuration are

\[ X_0 = X + \left( \frac{P_x \sin \theta}{\omega c m_c} \right) - \left[ \frac{\sin \left( \frac{\omega x t}{\beta} \right)}{\omega c m_c} \right] \left[ \frac{\Gamma \sin \nu t}{\beta \left( \frac{\omega_x}{\beta} \right)^2 - \nu^2} \right] - \left[ \frac{v \Gamma \sin \left( \frac{\omega x t}{\beta} \right)}{\beta \left( \frac{\omega_x}{\beta} \right)^2 - \nu^2} \right] \]

\[ Y_0 = Y - \left( \frac{P_y \cos \theta}{\omega c m_c} \right) - \left[ \frac{\cos \left( \frac{\omega y t}{\beta} \right)}{\omega c m_c} \right] \left[ \frac{\Gamma \sin \nu t}{\beta \left( \frac{\omega_y}{\beta} \right)^2 - \nu^2} \right] - \left[ \frac{1 + \nu^2 \beta^2 \cos \left( \frac{\omega x t}{\beta} \right) - \omega_y^2 \nu^2}{\beta^2 \left( \frac{\omega_y}{\beta} \right)^2 - \nu^2} \right] \]

\[ z_0 = z - \frac{P_z}{\beta m_e} \]

and the velocities are

\[ v_{x0} = \frac{P_x \cos \theta}{\beta m_e} + \left[ \frac{\sin \left( \frac{\omega x t}{\beta} \right)}{\beta m_e} \right] \left[ \frac{\Gamma \sin \nu t}{\beta \left( \frac{\omega_x}{\beta} \right)^2 - \nu^2} \right] \left[ \cos \nu t - \cos \left( \frac{\omega x t}{\beta} \right) \right] \]

\[ v_{y0} = \frac{P_y \sin \theta}{\beta m_e} + \left[ \frac{\cos \left( \frac{\omega y t}{\beta} \right)}{\beta m_e} \right] \left[ \frac{\Gamma \sin \nu t}{\beta \left( \frac{\omega_y}{\beta} \right)^2 - \nu^2} \right] \left[ \sin \nu t - \sin \left( \frac{\omega y t}{\beta} \right) \right] \]

\[ v_{z0} = \frac{P_z}{\beta m_e}, \]

\[ v_x = \frac{P_x \cos \theta}{\beta m_e}, \]

\[ v_y = \frac{P_y \sin \theta}{\beta m_e}, \]

\[ v_z = \frac{P_z}{\beta m_e} \]

\[ \Gamma_x = \frac{eE_x}{m_e}, \quad m_e = \frac{m_e}{\beta}, \quad \beta = \sqrt{1 - \frac{v^2}{c^2}}, \quad \omega_c = \frac{eB_0}{m_e} \]

P_x and P_z denote momenta perpendicular and parallel to the magnetic field. Using equation (6), (7) and the Bessel identity and performing the time integration, following the technique and method of Pandey et al [40], the perturbed distribution function is found after some lengthy algebraic simplifications as:

Scholars research library
Due to the phase factor the solution is possible when \( m = n \). Here.

\[
U^* = \left( \frac{c_1 P \nu}{\beta \lambda m_e} \right) - \left( \frac{mc_i D}{\lambda_1} \right) + \left( \frac{puc_i D}{\lambda_2} \right), \quad V^* = \left( \frac{c_1 \nu P_{n J}^2}{\beta \lambda m_e} \right) + c_1 DJ_{n J} \omega_c
\]

\[
W^* = \left( \frac{n \omega c_i D}{k_1 \nu P_{\perp}} \right) + \left( \frac{\beta m_e \omega \partial f_\omega}{\partial P_z} \right) + \frac{G}{\nu \left( \frac{p}{\lambda_2} - \frac{n}{\lambda_1} \right)}
\]

\[
C_1 = \left[ \frac{\nu}{P_{\perp}} \right] \left( \frac{\partial f_0}{\partial P_{\perp}} \right) \left( \frac{\beta m_e}{P_{\perp}} \right) + k_1 \beta m_e \left( \frac{\partial f_0}{\partial P_{\perp}} \right)
\]

\[
D = \left[ \frac{\nu}{P_{\perp}} \right] \left( \frac{\beta m_e}{P_{\perp}} \right) \left( \frac{\partial f_0}{\partial P_{\perp}} \right) + \nu \left( \frac{\beta m_e}{P_{\perp}} \right) \left( \frac{\partial f_0}{\partial P_{\perp}} \right)
\]

\[
G = \frac{H k_1 v \Gamma_x}{\beta \left( \frac{\omega_c}{\beta} \right) - \nu^2}, \quad J_n(\lambda_1) = dJ_n(\lambda_1), \quad J_n(\lambda_2) = dJ_n(\lambda_2)
\]

and the Bessel function arguments are defined as

\[
\lambda_1 = \frac{k_1 P_{\perp}}{\omega_c m_e}, \quad \lambda_2 = \frac{k_1 \nu \Gamma_x}{\beta \left( \frac{\omega_c}{\beta} \right) - \nu^2}, \quad \lambda_3 = \frac{k_1 v \Gamma_x}{\beta \left( \frac{\omega_c}{\beta} \right) - \nu^2}
\]

The conductivity tensor \( ||\sigma|| \) is found to be

\[
||\sigma|| = -i \sum \left( e^2 / \beta m_e \right)^2 \frac{\omega}{\omega - \left( k_1 P_{\perp} / \beta m_e \right) - \left( (n + g) \omega_c / \beta \right)} \int d^3 P |J_\delta(\lambda_3)||s|
\]
where
\[
\left\| S \right\| = \left| \begin{array}{ccc}
P_{\perp} J_{n} J_{p} \left( \frac{n}{\lambda_{1}} \right) U^{*} & iP_{\perp} J_{n} V^{*} & P_{\perp} J_{a} J_{p} \left( \frac{n}{\lambda_{1}} \right) W^{*} \\
P_{\perp} J_{a} J_{n} \left( \frac{n}{\lambda_{1}} \right) U^{*} & iP_{\perp} J_{n} V^{*} & P_{\perp} J_{a} J_{p} \left( \frac{n}{\lambda_{1}} \right) W^{*} \\
P_{\perp} J_{a} J_{n} \left( \frac{n}{\lambda_{1}} \right) U^{*} & iP_{\perp} J_{n} V^{*} & P_{\perp} J_{a} J_{p} \left( \frac{n}{\lambda_{1}} \right) W^{*} \\
\end{array} \right|
\]

By using these in the Maxwell’s equations we get the dielectric tensor,
\[
\varepsilon_{ij} = 1 + \sum \left\{ \frac{4\pi e_{0}^{2}}{(\beta m_{e})^{2} \omega} \right\} \left| \frac{d^{3} P J_{g} (\lambda_{1}) \| S \|}{\omega - k \| P_{z} \|} \right| (n + g) \omega_{c} + p \nu
\]

For parallel propagating whistler mode instability, the general dispersion relation reduces to
\[
\varepsilon_{11} = N^{2}, \quad N^{2} = \frac{k^{2}c^{2}}{\omega^{2}}
\]

The dispersion relation for relativistic case with perpendicular AC electric field for \( g = 0, p = 1, n = 1 \) is written as:
\[
\frac{k^{2}c^{2}}{\omega^{2}} = 1 + \frac{4\pi e_{0}^{2}}{(\beta m_{e})^{2} \omega^{2}} \int \frac{d^{3} P_{z}}{\beta} \left[ \frac{P_{z}}{2} - \frac{\nu \Gamma_{m_{e}}}{\beta} \right] \left[ \left( \beta \omega - k_{p} \right) \partial_{0}^{2} + \frac{P_{z} \partial_{0}}{m_{e}} \right] \frac{1}{\beta \omega - k_{p} - \omega_{c} + \beta \nu}
\]

The subtracted bi-Maxwellian distribution function is given as
\[
f_{0} = \frac{n_{0}}{\pi^{3/2} p_{0} \beta_{0} \lambda_{0}} \left[ \left( 1 + \frac{\beta \delta}{1 - \beta} \right) \exp \left( \frac{p_{z}^{2}}{p_{0}^{2} \beta_{0}} \right) - \frac{\delta}{1 - \beta} \exp \left( \frac{p_{z}^{2}}{\beta_{0}^{2}} \right) \right]
\]

Where \( p_{\perp} \) and \( p_{\parallel} \) are perpendicular and parallel momenta for a temperature \( T \). Substituting and using equation (9), (10) and doing integration by parts the dispersion relation is found as,
\[
D(k, \omega) = 1 + \frac{\omega_{p}^{2}}{\lambda_{0} \omega^{2}} \sum \frac{1}{J_{g} (\alpha_{2}) J_{g} (\alpha_{1})} \left( 1 - X_{1} \left( \frac{\lambda m_{e} \omega}{k_{p} p_{0} \lambda_{0}} \right) Z(\xi) + (A_{T} + X_{2} - X_{1})(1 + \xi Z(\xi)) \right)
\]

where
\[
X_{1} = \frac{\nu \Gamma_{m_{e}}}{2 \lambda_{0} \left( \frac{\omega_{c}}{\lambda} \right)^{2} - u^{2}} \frac{1}{\partial_{0}^{2}} \left[ 1 + \frac{\beta \delta}{1 - \beta} \right] - \frac{\delta \sqrt{\beta}}{1 - \beta}
\]
For real $k$ and substituting $k^2c^2/\omega^2 >> 1$

The expression for growth rate for real frequency $\omega$, in dimensionless form is found to be

$$\gamma = \frac{\sqrt{\pi}}{\lambda k_3} \frac{(k_1 - k_3)k_4^3 \exp \left\{ \left( \frac{k_3}{k_4} \right)^2 \right\}}{1 + \lambda X_4 + \frac{\tilde{k}^2(1 + \lambda X_5)}{2k_4^2} - \frac{\tilde{k}^2}{k_4} (k_1 - k_3)}$$  \hspace{1cm} \text{(14)}$$

$$X_3 = \frac{\tilde{k}^2}{\beta_1} \left[ k_2 (1 + \lambda X_4) + k_1 \frac{\beta_1}{2(1 + \lambda X_4)} \right]$$  \hspace{1cm} \text{(15)}$$

Where

$$k_3 = \frac{\lambda X_4}{1 - \lambda X_3 + \lambda X_4 + \lambda X_5}, \tilde{k} = \frac{k_4p_{0q}}{m_\gamma \omega_c}, k_4 = 1 - \lambda X_3 + \lambda X_4 + \lambda X_5, X_3 = \frac{\omega_c}{\omega_c}$$

$$X_4 = -\frac{\lambda \nu}{\omega_c}, \beta_1 = \frac{k_4 T_m n_0}{B_0^2}, k_1 = A_r + X_2 - X_1, k_2 = \frac{p_{0q}^2}{2}, X_5 = \frac{k_4 p_{0q}}{m_\gamma \omega_c}$$  \hspace{1cm} \text{(16)}$$

**RESULTS AND DISCUSSION**

For numerical evaluation of normalized growth rate of relativistic whistler mode has been analyzed for subtracted bi-Maxwellian distribution function in the magnetospheric plasma.
Following plasma parameters have been considered $n_0 = 5 \times 10^6$, $k_B T = 5 \text{keV}$, $B_0 = 2 \times 10^{-7} \text{T}$, $A_T = (T_\perp/T_\parallel) - 1 = 0.25, 0.5, 0.75$, $\beta = 0.7, 0.8, 0.9$, $\delta = 0.5, 0.7, 0.9$, relativistic factor $\lambda = 0.5, 0.7, 0.9$, $v = 0, 2\text{kHz}, 4\text{kHz}, 6\text{kHz}$. According to this choice of plasma parameters, the explanations and details of results are given as follows.

Fig 1. Variation of dimensionless growth rate $\gamma/\omega_c$ versus $\tilde{k}$ has been shown for various values of perpendicular and parallel temperature ratio $T_\perp/T_\parallel$ and other parameters are fixed. The growth rate increases with increase the value of perpendicular and parallel temperature ratio $T_\perp/T_\parallel$. It means the temperature anisotropy is free energy source. The bandwidth did not shift like the case of bi-Maxwellian distribution. The maximum peak is obtained for a fixed value of $\tilde{k} = 0.85$ for various values of perpendicular and parallel temperature ratio $T_\perp/T_\parallel$. It has been increases with increasing value of temperature anisotropy. Hence the temperature anisotropy $T_\perp/T_\parallel - 1 = A_T$ is main source of generating instability. The nature of curve is different in respect to bi-Maxwellian.

Fig.2 Variation of dimension less growth rate $\gamma/\omega_c$ versus $\tilde{k}$ has been shown for various values of relativistic factor $\lambda = \left(\frac{1}{1 - \left(\frac{V}{c}\right)^2}\right)^{1/2}$ and other parameters are fixed. The growth rate decrease with increase the value of relativistic factor. The peaks values of growth rates are different but the value of $\tilde{k} = 0.85$. Fig.3 Variation of dimension less growth rate $\gamma/\omega_c$ versus $\tilde{k}$ has been shown for various values of $\delta$ and other parameters are fixed. The amplitude of subtracted distribution function is affected by changing the value of $\delta$. By increasing the value of $\delta$ the amplitude of subtracted distribution function has been increases if $\beta$ is fixed. Due to increase in amplitude of subtracted distribution function the growth rate has been increased. Fig.4 Variation of dimension less growth rate $\gamma/\omega_c$ versus $\tilde{k}$ has been shown for various values of $\beta$ width of loss cone and other parameters are fixed. $\beta$ is the measure of the width of loss cone relative to the width of distribution function. The growth rate increases by increasing value of $\beta$ up to 0.9 but for greater than 0.9 the growth rate decreases. In mathematically if $\beta \geq 1$ then the factor $\delta/1 - \beta$ is infinite or negative. Fig.5 The variation of dimension less growth rate $\gamma/\omega_c$ versus $\tilde{k}$ has been in this figure and others parameter being fixed. Due increase in AC frequency the Doppler shift frequency has been change. The decrease in real frequency the growth rate increases. It is also effect as a free energy source for generating the whistler mode wave. Contribution of a.c. frequency reduces the upper limit of wave number $k$ and normalized real frequency $X_3 = \omega_e/\omega_{ci}$ and increases the growth rate. The a.c. frequency appears only through modification of resonant instability.
Fig. 1 variation of dimensionless growth rate $\gamma/\omega_c$ versus $\overline{k}$ for various values of temperature anisotropy $A_T = \frac{T_c}{T_1} - 1$ and other parameters are $\lambda = 0.5, \beta = 0.7, \delta = 0.5, \nu = 2\text{kHz}, B_0 = 2 \times 10^{-7} \text{T}, E_0 = 4 \times 10^{-3} \text{mV/m}, n_0 = 5 \times 10^6 \text{m}^{-3}$ and $k_B T_1 = 5 \text{KeV}$.

Fig. 2 variation of dimensionless growth rate $\gamma/\omega_c$ versus $\overline{k}$ for various values of relativistic factor $\lambda$ and other parameters are $T_c/T_1 = 1.25, \beta = 0.7, \delta = 0.5, \nu = 2\text{kHz}, B_0 = 2 \times 10^{-7} \text{T}, E_0 = 4 \times 10^{-3} \text{mV/m}, n_0 = 5 \times 10^6 \text{m}^{-3}$ and $k_B T_1 = 5 \text{KeV}$.
\( k \)

Fig. 3 variation of dimensionless growth rate \( \gamma/\omega_c \) verses \( k \) for various values of \( \delta \) and others parameters are \( \lambda = 0.5, \beta = 0.7, T_\perp/T_i = 1.25, \nu = 2 \text{kHz}, B_0 = 2 \times 10^{-7} \text{T}, E_0 = 4 \times 10^{-3} \text{mV/m}, n_0 = 5 \times 10^6 \text{m}^{-3} \) and \( k_B T_i = 5 \text{KeV} \).

\( k \)

Fig. 4 variation of dimensionless growth rate \( \gamma/\omega_c \) verses \( k \) for various values of \( \beta \) and others parameters are \( \lambda = 0.5, T_\perp/T_i = 1.25, \nu = 2 \text{kHz}, B_0 = 2 \times 10^{-7} \text{T}, E_0 = 4 \times 10^{-3} \text{mV/m}, n_0 = 5 \times 10^6 \text{m}^{-3} \) and \( k_B T_i = 5 \text{KeV} \).
CONCLUSION

Whistler mode instabilities have been analyzed for substracted bi-Maxwellian magneto-Plasma in the presence of perpendicular AC electric field by using the method of characteristic solutions and kinetic approach. The effective parameters for the generation of Whistler mode wave are not only the temperature anisotropy but also the relativistic factor, AC field frequency, amplitude of substracted distribution and width of the loss-cone distribution function which are discuss in result and discussion section.

REFERENCES